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THE DEVELOPMENT OF NEUTRINO-DRIVEN CONVECTION IN CORE-COLLAPSE SUPERNOVAE: 2D VS 3D

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Abstract. A toy model is used to study the non-linear conditions for the development of neutrino-driven convection in the post-shock region of core-collapse supernovae. Our numerical simulations show that a buoyant non-linear perturbation is able to trigger self-sustained convection only in cases where convection is not linearly stabilized by advection. Several arguments proposed to interpret the impact of the dimensionality on global core-collapse supernova simulations are discussed in the light of our model. The influence of the numerical resolution is also addressed. In 3D a strong mixing to small scales induces an increase of the neutrino heating efficiency in a runaway process. This phenomenon is absent in 2D and this may indicate that the tridimensional nature of the hydrodynamics could foster explosions.

Keywords: hydrodynamics – instabilities – accretion – shock waves – supernovae: general

1 Introduction

The explosion of massive stars in core-collapse supernovae (CCSNe) relies crucially on the action of the multi-dimensional dynamics in the core of the progenitor during the first hundreds milliseconds after the core collapses into a proto-neutron star (Foglizzo et al. 2015; Janka et al. 2016 for recent reviews). The delayed neutrino mechanism is considered as the most suitable scenario to explain a majority of core-collapse supernovae (Bethe & Wilson 1985). Detailed calculations showed that imposing spherical symmetry (1D) does not lead to an explosion (Liebendörfer et al. 2001) except for the lightest progenitors (Kitaura et al. 2006). Axisymmetric simulations (2D) revealed that two distinct instabilities can lead to a shock revival (Müller et al. 2012; Fernández et al. 2014). Both neutrino-driven convection (Herant et al. 1994; Burrows et al. 1995) and the Standing Accretion Shock Instability (SASI) (Blondin et al. 2003) break the spherical symmetry of the collapse, generate large scale non-radial motions, increase the advection timescale through the gain region and enhance the energy deposition. However in a majority of cases, 2D simulations generate under-energetic explosions (Müller 2015). Most three-dimensional (3D) simulations even produce less optimistic results with fewer explosions and a slower growth of the explosion energy (e.g. Melson et al. 2015a; Lentz et al. 2015). The different directions in which the turbulent energy cascade acts may explain the discrepancies between 2D and 3D (Hanke et al. 2012).

In this study a toy model is used to investigate the impact of the dimensionality on the dynamics of the gain region. This approach is complementary to global simulations which are still computationally challenging and for which it is complex to disentangle the hydrodynamics from other physical ingredients. We focus on the neutrino-driven convection, seen in most CCSN simulations, and develop an idealized description of the advection through the gain region. Foglizzo et al. (2006) showed that convection can be stabilized linearly by advection when the perturbations are rapidly advected outside of the heating region. Even if a perturbative analysis predicts linear stability, convection could in principle be triggered non-linearly by a perturbation with a large enough amplitude (Scheck et al. 2008).

We begin in Section 2 with a presentation of the idealized model. In section 3 we examine the linear and the non-linear onsets of convection. The impact of the dimensionality is studied in section 4. Finally we summarize our findings and conclude in section 5.

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2 Physical and numerical setup

The advection of matter through the gain region is modeled by a stationary and subsonic flow, parallel in the vertical direction (z), which experiences a constant gravity and heating proportionally to the density in the central region of the simulation domain. Upstream and downstream of the gain region, gravity and heating are turned off and the flow is uniform. These uniform regions alow us to place the boundaries far enough in the vertical direction to avoid numerical reflections that would impact the dynamics in the central region where convection can develop. The shock wave and the cooling layer are absent from our model which can be seen as a fraction of a realistic gain region.

A perturbative analysis similar to the one of Foglizzo et al. (2006) shows that the flow is linearly unstable to convection if

$$\chi \equiv \int_{r_{\mathrm{g}}}^{r_{\mathrm{sh}}} \omega_{\mathrm{BV}} \frac{dr}{|v_r|} \sim \frac{t_{\mathrm{adv}}}{t_{\mathrm{conv}}} > 2, \tag{2.1}$$

where r_{g}, r_{sh}, t_{adv} and t_{conv} respectively refer to the gain radius, the shock radius, the advection timescale across the gain region and the buoyancy timescale related to the Brunt-Väisälä frequency ω_{BV}.

A density perturbation is added to the stationary flow to trigger convection. Such perturbations may arise from entropy-vorticity waves produced by shock oscillations driven by SASI (Foglizzo et al. 2007), progenitor asphericites resulting from convection in Si,O shells (Müller et al. 2016) or originate from numerical artifacts such as embedded Cartesian grids using several refinement levels (Ott et al. 2013). The perturbation characteristics are set so that the unstable mode with the highest growth rate is triggered. The perturbation has a two-dimensional structure and a random noise of the order of 0.1% in density is added in the transverse direction to enable the development of the instability in 3D.

The time-dependent simulations are performed with the RAMSES code (Teyssier 2002; Fromang et al. 2006). The dimensions of the Cartesian domain are: $-150\,\mathrm{km} \leq \mathrm{x}, \mathrm{y} \leq 150\,\mathrm{km}$ for the horizontal directions and $-450\,\mathrm{km} \leq \mathrm{z} \leq 450\,\mathrm{km}$ for the vertical one. The gain region is located in the central part: $-50\,\mathrm{km} \leq \mathrm{z} \leq 50\,\mathrm{km}$. The heating and the gravity are smoothened at the edges of the gain region to avoid discontinuities. The details of the setup will be discussed in a forthcoming paper (Kazeroni et al., in preparation).

3 Competition between advection and convection

In the regime of linear stability, it was proposed that convection could be triggered by a perturbation with a high enough amplitude (Scheck et al. 2008). We test that hypothesis by performing a set of 2D simulations with different perturbation amplitudes ($\delta\rho/\rho$) and an initial value of χ below the instability threshold. We observe that the criterion of Scheck et al. (2008), giving a threshold of about 1%, is robust to predict the presence of upward motions in the gain region (Fig. 1, left). Nevertheless that is not a sufficient condition to generate self-sustained convection because buoyant motions are suppressed within an advection timescale in the case of $\delta\rho/\rho = 1\%$. Regardless the perturbation amplitude, the convective instability is always suppressed in the linear stability regime. The damping timescale increases with higher perturbation amplitudes and the proximity to the instability threshold $\chi = 2$ (Fig. 1). This shows that a single excitation by a strong perturbation is not sufficient to trigger self-sustained convection in situations where convection is linearly stabilized by advection.

The robustness of the linear instability threshold proposed by Foglizzo et al. (2006) is assessed using a second set of parametric simulations. The self-sustained instability is only obtained in cases where the χ value of the stationary flow is above the instability threshold (Fig. 1, right). This suggests that the criterion (2.1) can also be applied in the non-linear regime for which the perturbative analysis is not suitable.

4 The impact of the dimensionality

In this section, we examine the discrepancies between 2D and 3D in a case where convection is linearly unstable. The dynamics is already impacted by the dimensionality in the early non-linear phase. We observe that some 3D buoyant bubbles rise faster than their 2D counterparts. Besides, 3D simulations exhibit a higher growth of the entropy after the linear phase (Fig. 2). These discrepancies are consistent with the properties of the Rayleigh-Taylor instability. For the latter, it was shown that 3D non-linear penetration depths are higher in the case of an incompressible flow without advection (Young et al. 2001).

While the instability saturates after 4-5 advection timescales in 2D, a very distinct dynamics is observed in 3D. In the latter case, the entropy increases in a runaway process. The downflows are decelerated due to a

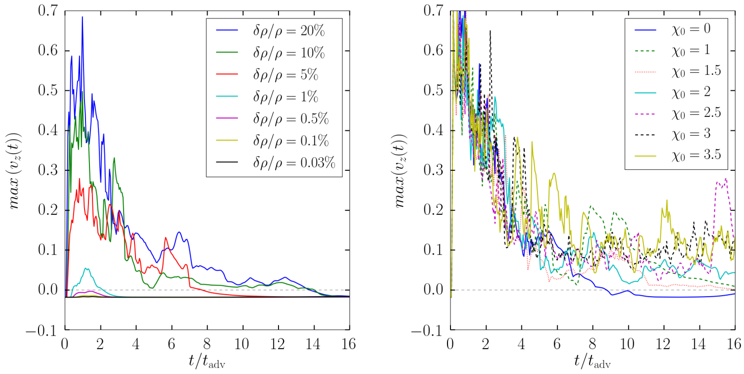

Fig. 1. Left: Time evolution of the maximum upward velocity in the gain region of 2D simulations with $\chi_0 = 1.5$. In all cases, this quantity becomes negative showing that buoyant motions are suppressed. **Right:** Time evolution of the maximum upward velocity in the gain region for a perturbation amplitude of $\delta\rho/\rho = 30\%$ and several values of χ_0. Convective motions are suppressed and the flow returns to a stable state only in cases where $\chi_0 < 2$. The instability is self-sustained beyond that threshold.

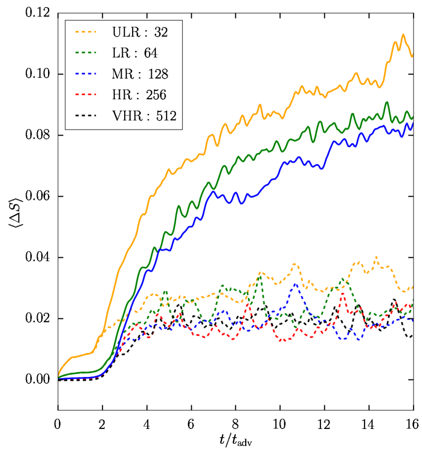

Fig. 2. Time evolution of the average entropy in the gain region of 2D (dashed curves) and 3D (solid curves) simulations. The numerical resolution is varied from 32 to 128 vertical cells in the gain region in 3D and from 32 to 512 in 2D.

more efficient mixing to small scales at their interfaces. This keeps the matter in the gain region for a longer time and thus generates more heating. On the contrary, the downflows are not disrupted in 2D and the matter is channeled more easily below the gain region (Fig. 3). Rather than being mixed, the 2D flow is stirred by large scale vortices.

The numerical resolution does not seem to affect dramatically the dynamics. An earlier growth of the instability is observed in the lowest resolution case (Fig. 2) which is rather due to the relaxation of the stationary flow on the numerical grid. Similar asymptotic entropy values are reached in 2D for the different resolutions considered. The same conclusion can be drawn from the 3D simulations keeping in mind the artificial earlier growth of in instability at low resolution. The role of the numerical resolution on global CCSN simulations is not clear yet (e.g. Janka et al. 2016). Radice et al. (2016) studied a more realistic stationary flow and concluded that despite a mixing to smaller scales with increasing numerical resolution no clear correlation exists between the evolution of global quantities, such as the shock radius and the numerical resolution.

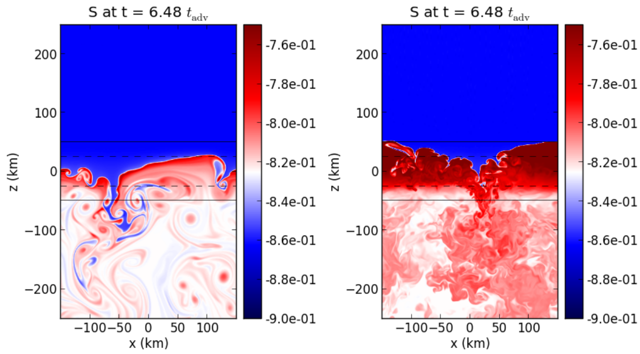

Fig. 3. Entropy snapshots of the fully turbulent dynamics. In 2D *(left)*, a downflow penetrates in the downstream layer and channels matter in a region without heating. On the contrary, the 3D *(right)* downflows are fragmented to small scales and this increases the advection time. Consequently, the heating efficiency increases in a runaway process.

5 Conclusions

The development of neutrino-driven convection was investigated using an idealized model of the gain region of CCSNe. We tested the robustness of linear and non-linear conditions to generate the instability. We found that the threshold proposed by Foglizzo et al. (2006) can be successfully applied even when the perturbed flow is already in the non-linear regime. A criterion relying on a competition between convection and advection is not sufficient to predict the development of a self-sustained instability triggered by a non-linear perturbation, because the stratification of the flow should be taken into account.

The dynamics of the gain region is severely impacted by the dimensionality. In 3D a turbulent mixing to small scales fragments the downflows. Consequently the heating efficiency increases in a runaway process. On the contrary, large scale motions in 2D channel efficiently the matter outside of the gain region preventing a continuous entropy increase as in 3D. The numerical resolution does not seem to play a significant role, at least when the instability is triggered linearly by a low perturbation amplitude.

Increasing the complexity of our model would help evaluate the relevance for global CCSN simulations of the positive 3D effects identified in this purely hydrodynamical study. The present work captures physical ingredients that seem supportive of more robust explosions in 3D as witnessed similarly by Melson et al. (2015b) and Müller (2015) in some convection-dominated low-mass progenitors. Whether these positive effects are generic characteristics of CCSNe will only be answered by performing self-consistent 3D simulations of a large set of progenitors which will become feasible within the next decade.

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