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# GIANT PULSAR GLITCHES IN FULL GENERAL RELATIVITY

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**Abstract.** We present recent numerical simulations of giant pulsar glitches, as observed in the emblematic Vela pulsar, based on a two-fluid model, including for the first time all general-relativistic effects and realistic equations of state. In particular, we focus on modelling the vortex-mediated transfer of angular momentum that takes place during the spin-up stage from the neutron superfluid to the charged particles through dissipative mutual friction forces. Taking general relativity into account does not only modify the structure of the star but also leads to a new coupling between the fluids arising from frame-dragging effects. As a consequence, general relativity can strongly affect the global dynamics of pulsar glitches : the errors on the value of the characteristic rise time incurred by using Newtonian gravity are thus found to be as large as $\sim 40\%$ for the models considered.

Keywords: pulsar glitches, neutron stars, superfluidity, general relativity, equation of state, entrainment

## 1 Introduction

Pulsars are neutron stars spinning rapidly with very stable periods. Nevertheless, some irregularities called glitches have been detected in the long-term evolution of the angular velocity $\Omega$ of some pulsars, during which the pulsar suddenly spins up, with relative amplitude $\Delta\Omega/\Omega$ ranging between $\sim 10^{-11}$ and $\sim 10^{-5}$, before slowly relaxing on time scales up to months or years (Espinoza et al. 2011). To date, 482 glitches have been observed in 168 pulsars*.

Since the first detections of glitches (Radhakrishnan & Manchester 1969; Reichley & Downs 1969), several mechanisms have been suggested to explain these phenomena (see the review by Haskell & Melatos (2015)). A glitch is nowadays commonly thought to be the manifestation of an internal process, except possibly for highly magnetised neutron stars for which some evidence of magnetospheric activity have been found (e.g. Antonopoulou et al. (2015)). The interior of neutron stars can thus be probed using glitch observations (Ho et al. 2015; Pizzochero et al. 2017).

The presence of superfluids inside neutron stars was first suggested by Migdal (Migdal 1959), more than sixty years ago. This idea was later supported by the very long time scales observed during post-glitch relaxations (Baym et al. 1969). Nowadays, neutron stars are expected to contain a neutron superfluid in the inner crust and in the outer core, and possibly other superfluid species in the inner core (see, e.g., Chamel (2017) and references therein). Superfluidity is likely to affect the dynamics of neutron stars. In particular, giant glitches with $\Delta\Omega/\Omega \sim 10^{-6} - 10^{-5}$, as observed in the emblematic Vela pulsar, are generally interpreted as the result of a rapid transfer of angular momentum between the neutron superfluid and the rest of the star, triggered by large-scale vortex unpinning events (Anderson & Itoh 1975). This vortex-mediated scenario is supported by both laboratory experiments (Tsakadze & Tsakadze 1980) and the ability of the vortex creep model to reproduce the post-glitch relaxations in different pulsars (Alpar et al. 1984c,a, 1996; Gügercinoğlu & Alpar 2014).

So far, most global numerical simulations of pulsar glitches have been performed in Newtonian gravity (e.g., Sidery et al. (2010); Haskell et al. (2012)). Seveso et al. (2012) and Antonelli & Pizzochero (2017) have recently developed a non-relativistic hydrodynamic model for describing the different stages of the glitch phenomenon

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*See the Jodrell Bank Observatory glitch catalog : `http://www.jb.man.ac.uk/pulsar/glitches.html`.

based on the static structure of the neutron star computed in general relativity. However, general relativity is also likely to play an important role for the global dynamics of glitches.

Here, we present global numerical simulations of vortex-mediated pulsar glitches performed within a fully general relativistic framework, focusing on the spin-up stage regardless of the glitch triggering mechanism (Novak et al. 2016; Sourie et al. 2017). A glitch event can actually be decomposed into distinct stages (*i.e.* the pre-glitch evolution, the spin up, and the post-glitch relaxation), which can be – in a first attempt – modelled separately in view of the different associated time scales suggesting different physical mechanisms. During the spin up, stellar dynamics are mainly governed by the mutual friction force between the superfluid and the rest of the star. This force acts on a characteristic time scale corresponding to the glitch rise time $\tau_{\rm r}$, which has not been fully observationally resolved yet. The most stringent observational constraint on $\tau_{\rm r}$ comes from the 2000 and 2004 Vela glitch observations: $\tau_{\rm r} < 30 - 40$ s (Dodson et al. 2007). In the following, we shall assume that $\tau_{\rm r}$ is much longer than the typical time $\tau_h \sim 0.1$ ms for the star to go back to equilibrium once being driven out of it by a change in its rotation rate. Doing so, the dynamical evolution of the pulsar can be reasonably well described by a sequence of quasi-stationary equilibrium configurations.

## 2 Stationary configurations

### 2.1 Model assumptions

For years, only the neutron superfluid contained in the inner crust of the star was generally thought to be responsible for giant glitches (see, e.g., Alpar et al. (1984b, 1993); Link et al. (1999)). Still, recent studies indicate that the crust can not provide enough reservoir of angular momentum (Andersson et al. 2012; Chamel 2013; Akbal et al. 2015; Delsate et al. 2016). This suggests that the core superfluid plays a more important role than previously thought.

In this work, we thus focus on the dynamics of superfluid neutron-star cores modelled by two fluids coupled by (non-dissipative) mutual entrainment effects (Andreev & Bashkin 1976): (i) a "normal" fluid made of protons and electrons, which are locked together by magnetic effects and essentially corotate with the crust and the magnetosphere at the observed angular velocity $\Omega$, and (ii) a neutron superfluid extending in the whole core. Quantities related to the two fluids will be labelled by indices "p" and "n" respectively. We do not account for any other effects of the magnetic field on the global dynamics of the star, which can be safely ignored for the ordinary pulsars considered here (Chatterjee et al. 2015).

Our simulations are based on the general relativistic equilibrium configurations of rotating superfluid neutron stars computed by Sourie et al. (2016), within the free and publicly available Lorene library[†]. The associated spacetime is assumed to be asymptotically flat, stationary, axisymmetric and circular. Regarding perfect fluids, this last assumption implies that the two fluids are rotating around a common axis, with possibly different rotation rates $\Omega_{\rm n}$ and $\Omega_{\rm p}$. Similarly to the model developed by Sidery et al. (2010) in Newtonian gravity, the angular velocities of the two fluids are taken to be uniform within the star. Moreover, we consider two equations of state (EoSs), DDH and DDH$\delta$, derived from density-dependent relativistic mean-field models, including $\sigma$, $\omega$, $\rho$ and possibly $\delta$ mesons (Typel & Wolter 1999; Avancini et al. 2009). They were adapted to a system of two fluids at zero temperature coupled by entrainment for arbitrary compositions. More details can be found in Prix et al. (2005); Sourie et al. (2016).

### 2.2 Fluid couplings

Let $J_{\rm n}$ and $J_{\rm p}$ be the neutron superfluid and normal fluid angular momenta respectively (see Langlois et al. (1998) and Sourie et al. (2016) for definitions and expressions). Introducing the partial moments of inertia as

$$I_{XX} = \left(\frac{\partial J_X}{\partial \Omega_X}\right)_{\Omega_Y} \quad \text{and} \quad I_{XY} = \left(\frac{\partial J_X}{\partial \Omega_Y}\right)_{\Omega_X}, \tag{2.1}$$

where $X, Y \in \{\rm n, p\}$, any changes in the angular momentum of a fluid will be simply given by

$$\mathrm{d}J_X = I_{XX}\,\mathrm{d}\Omega_X + I_{XY}\,\mathrm{d}\Omega_Y = \hat{I}_X\,\mathrm{d}\Omega_X + \hat{\varepsilon}_X \hat{I}_X\left(\mathrm{d}\Omega_Y - \mathrm{d}\Omega_X\right), \tag{2.2}$$

[†]http://www.lorene.obspm.fr/.

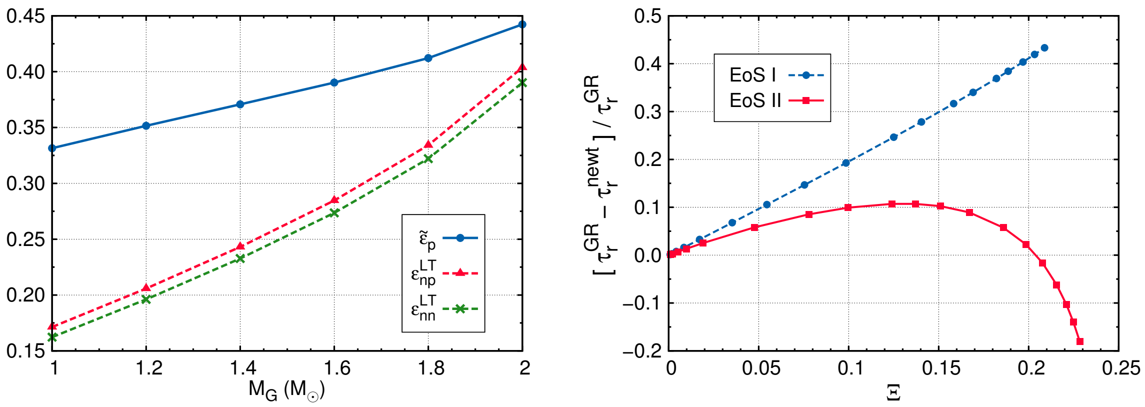


**Fig. 1. Left:** The quantities $\tilde{\varepsilon}_{\rm p}$, $\varepsilon_{\rm np}^{\rm LT}$ and $\varepsilon_{\rm nn}^{\rm LT}$ are plotted as functions of the gravitational mass $M_{\rm G}$ of the star for the DDH EoS, assuming $\Omega_{\rm n} = \Omega_{\rm p} = 2\pi \times 11.19$ rad.s$^{-1}$ (which corresponds to the rotation rate of the Vela pulsar). **Right:** Relative differences between general relativistic and Newtonian rise times for a glitch amplitude $\Delta\Omega/\Omega = 10^{-6}$ as functions of the (relativistic) compactness parameter, for a star spinning at 10 Hz. Results are shown for two different polytropic EoSs described in Sourie et al. (2017).

where $\hat{I}_X = I_{XX} + I_{XY}$ and $\hat{\varepsilon}_X = I_{XY}/\hat{I}_X$. The cross moment of inertia $I_{XY}$ (and thus $\hat{\varepsilon}_X$) contains any possible (non-dissipative) couplings between the fluids. In the Newtonian limit, entrainment happens to be the main coupling mechanism at low angular velocities. Quite remarkably, a new coupling arises in the general relativistic context from the so-called Lense-Thirring or frame-dragging effect (Carter 1975). In the slow-rotation approximation ($\Omega_{\rm n}, \Omega_{\rm p} \ll \Omega_{\rm K}$, where $\Omega_{\rm K}$ is the Keplerian angular velocity) and to first order in the lag $\delta\Omega = \Omega_{\rm n} - \Omega_{\rm p}$, the total coupling coefficient $\hat{\varepsilon}_X$ reads (Sourie et al. 2017)

$$\hat{\varepsilon}_X = \frac{\tilde{\varepsilon}_X - \varepsilon_{YX}^{\rm LT}}{1 - \varepsilon_{YX}^{\rm LT} - \varepsilon_{XX}^{\rm LT}}, \tag{2.3}$$

where the term $\tilde{\varepsilon}_X$ characterizes entrainment effects averaged over the star, whereas $\varepsilon_{YX}^{\rm LT}$ and $\varepsilon_{XX}^{\rm LT}$ represent respectively the frame-dragging effect on fluid $X$ caused by the second fluid and fluid $X$ itself. As can be seen from Eq. (2.3), the Lense-Thirring effect is found to act in an opposite way to entrainment in the core, where $\tilde{\varepsilon}_X > 0$ (Chamel & Haensel 2006; Sourie et al. 2016). As a consequence, in the absence of entrainment, the total coupling coefficient is still expected to be non-vanishing and negative. Although entrainment is likely to be small in the outermost regions of the core of neutron stars, its overall effect on the whole star is not necessarily negligible and therefore $\hat{\varepsilon}_X$ could be positive or negative.

The relative importance of these two effects on the total coupling parameter $\hat{\varepsilon}_{\rm p}$ is studied in the left panel of Fig. 1. Since general relativistic effects are the strongest for the most massive stars, $\varepsilon_{\rm np}^{\rm LT}$ increases with the stellar mass. Interestingly, $\tilde{\varepsilon}_{\rm p}$ and $\varepsilon_{\rm np}^{\rm LT}$ are found to be roughly of the same order of magnitude, making the Lense-Thirring contribution to the total coupling very important. Similar conclusions are reached for the two EoSs.

## 3 The glitch spin-up

### *3.1 Evolution equations*

Neglecting any external torque, the dynamics of the two fluids during the spin up is simply governed by

$$\begin{cases} \dot{J}_{\rm n} = + \Gamma_{\rm mf}, \\ \dot{J}_{\rm p} = - \Gamma_{\rm mf}, \end{cases} \tag{3.1}$$

where overdot denotes time derivative. A covariant expression for the (relativistic) mutual friction torque $\Gamma_{\rm mf}$ was derived by Langlois et al. (1998), considering straight vortices parallel to the rotation axis. Neglecting the small contribution of the non-circular motion of the vortices and any dissipation related to chemical reactions, one has

$$\Gamma_{\rm mf} = -\bar{\mathcal{B}} \times 2\zeta \hat{I}_{\rm n} \Omega_{\rm n} \times \delta\Omega, \tag{3.2}$$

where $\delta\Omega = \Omega_{\rm n} - \Omega_{\rm p}$. The value of the quantity $\zeta$, which reduces to 1 in the Newtonian limit, depends on the stellar structure and can be obtained from stationary configurations. Given the current lack of knowledge on the microscopic origin of the mutual friction force, the averaged mutual friction parameter $\bar{\mathcal{B}}$ is taken as a free input parameter in our numerical simulations and is moreover assumed to be time-independent for simplicity.

Plugging (2.2) into (3.1) gives the equation governing the time evolution of the lag $\delta\Omega$. Recalling that $\Delta\Omega/\Omega \ll 1$, the lag approximately evolves as $\delta\Omega(t) \propto \exp(-t/\tau_{\rm r})$, with a characteristic time scale given by

$$\tau_{\rm r} = \frac{\hat{I}_{\rm p}}{\hat{I}} \times \frac{1 - \hat{\varepsilon}_{\rm p} - \hat{\varepsilon}_{\rm n}}{2\zeta\bar{\mathcal{B}}\Omega_{\rm n}}, \tag{3.3}$$

where $\hat{I} = \hat{I}_{\rm n} + \hat{I}_{\rm p}$. Expression (3.3) is very similar to that obtained in Newtonian framework (see Sidery et al. (2010) or Sourie et al. (2017)). It should be stressed however that general relativistic effects are not only included in the coefficient $\zeta$ but also modify the moments of inertia (2.1) and the coupling coefficients (2.3).

### *3.2 Numerical results*

To test the validity of the previous analytical approach and to assess the importance of general relativity, we have also solved Eqs. (3.1) numerically. Starting from some initial conditions at the beginning of the glitch, these equations are solved step by step so as to get the time evolution of the two angular velocities during the spin up, using the equilibrium configurations described in section 2. These simulations require the following macroscopic ingredients: the rotation rate $\Omega$ of the star, its gravitational mass $M_{\rm G}$ and the glitch amplitude $\Delta\Omega/\Omega$. In addition, the following microscopic inputs need to be specified: the EoS used to describe the interior of the star and the mutual friction parameter $\bar{\mathcal{B}}$. By fitting the time evolution of the lag $\delta\Omega$ with an exponential law, one can determine the rise time corresponding to the chosen input parameters. It should be remarked that this characteristic time corresponds indeed to the spin-up time-scale that could be measured from precise timing observations of glitches. Numerical results are found to agree with values inferred from Eq. (3.3) with a very good precision, such that the spin-up time scale can be very accurately estimated from stationary configurations only. Furthermore, since the actual value of $\tau_{\rm r}$ is found to be much longer than the hydrodynamical time scale $\tau_h$ for current estimates of the mutual friction forces, the whole dynamical evolution of the star during the spin up can be accurately computed by considering a quasi-stationary approach.

We also study the global contribution of general relativity to the glitch dynamics, by comparing the rise times obtained within both relativistic and Newtonian frameworks. For simplicity, we consider two polytropic EoSs, as implemented by Prix et al. (2005), with different entrainment contributions. These EoSs have been chosen in order to reproduce "realistic" values for the mass, radius and proton fraction of the stars. In the right panel of Fig. 1, the relative differences on $\tau_{\rm r}$ are plotted with respect to the compactness parameter $\Xi$ (defined as the dimensionless ratio of the gravitational mass of the star to its radius), obtained for a star rotating at 10 Hz by varying its mass. As expected, general relativistic corrections vanish when the compactness parameter decreases. For values of the compactness parameter relevant for neutron stars, *i.e.* $\Xi \sim 0.15 - 0.20$, these two EoSs predict that an error of the order of $\sim 20 - 40\%$ is made on the rise time by using Newtonian gravity instead of general relativity, as can be seen in Fig. 1. It is therefore necessary to account for general relativistic effects in order to get precise results on the spin-up time scales. Furthermore, it should be mentioned that these errors depend significantly on the rotation rate and the EoS under consideration.

## 4 Conclusions

We have studied in detail the impact of general relativity on the global dynamics of giant pulsar glitches as observed in Vela (Sourie et al. 2017).

First, we analytically solved the dynamical equations governing the transfer of angular momentum that takes place between the core neutron superfluid and the rest of the star during the spin up, by expressing the change in the lag as $\delta\dot{\Omega}/\delta\Omega \approx -1/\tau_{\rm r}$. The characteristic rise time $\tau_{\rm r}$ can be expressed in a form similar to that obtained in the Newtonian limit (Sidery et al. 2010). Still, general relativity not only changes the structure of the star, but also impacts the fluid dynamics. In particular, frame-dragging effects induce additional fluid couplings of the same form as the entrainment arising solely from neutron-proton interactions. General relativity can thus affect significantly the glitch dynamics.

To assess the importance of general relativity, we have also solved numerically the evolution equations relative to the angular velocity of each fluid. Numerical results are found to be very well reproduced by the analytical approximation. In particular, the glitch rise time $\tau_r$ can be precisely estimated from stationary configurations only. Moreover, we have also investigated the impact of general relativity on $\tau_r$ by using two different polytropic EoSs. Both the effects of general relativity on the stellar structure and on the fluid couplings are found to be important. Realistic simulations of the global glitch dynamics should therefore be carried out in full general relativity. The errors incurred by using Newtonian gravity instead of general relativity, however, might not be the main source of uncertainties. For instance, neutron superfluid vortices may not be simply straight and aligned with the rotation axis, as assumed here. Likewise, the interactions between superfluid vortices and proton flux tubes remain poorly known and their impact on the glitch dynamics warrant further studies.

Finally, the Low Frequency Array (LOFAR) radio telescope (Stappers et al. 2011) and the future Square Kilometer Array (SKA) (Watts et al. 2015) will be able to observe the spin up with unprecedented accuracy. It would thus lead to much more stringent constraints on the characteristic time $\tau_r$ and thereby on the underlying glitch mechanism. This calls for more realistic models of glitches including the crust magnetoelasticity and superfluidity and accounting for the local dynamics of quantised vortices.

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