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FOLLOWING THE ROSSBY WAVE INSTABILITY INTO THE KERR METRIC

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Abstract. The Rossby Wave Instability (RWI) has been proposed to explain the High-Frequency Quasi-Periodic Oscillation thought to occur in the close vicinity of black holes but this early work was done in the pseudo-Newtonian approach. Here we present the first general relativistic hydrodynamic simulations of this instability not only proving its theorized existence in a full GR environment but also studying the effect of strong gravity on the instability and how it relates to observations.

Keywords: disk, instability, GR

1 Introduction

Since their first detection there have been a long string of efforts to understand the source of the variability observed in microquasars. This is especially true for the origin of the rather elusive High-Frequency Quasi-Periodic Oscillation (HFQPO) in systems containing (or thought to contain) a black hole (BH). Indeed, their frequencies hint at a relation to the spin of the black hole. Here we focus on the model based on the Rossby-Wave Instability (Tagger & Varnière 2006; Varnière et al. 2011; Varnière et al. 2012) which is predicted to occur when the inner edge of the disk is close to the last orbit. It has been previously proposed to explain several phenomena occurring in the vicinity of black holes (Vincent et al. 2013, 2014; Casse et al. 2017) but its existence in an extreme Kerr environment had never been demonstrated.

Here we will be looking into more details at how its position closer to the last stable orbit of the black hole does influence the physics of the RWI and what changes, if any, we should be looking for in observations. For that we will be using a clean setup looking only for differences related to the GR-effects. Indeed, rather than starting with the study of the full dynamical system where it would be difficult to differentiate between GR and dynamical effects from the initial condition, we choose to follow a similar setup taken deeper and deeper in the potential well of a maximally spinning Kerr black hole to study the minute changes in the instability behavior.

2 Down the rabbit hole

The RWI requires having an extremum of $\mathcal{L} = (\nabla \times \mathbf{v})_\perp / \Sigma$ in which the instability could develop, where $\mathbf{v}$ is the velocity of the gas while Σ is its height-integrated density ($\perp$ stands for the component perpendicular the the disk plane). The position of the extremum is called the corotation radius of the wave, meaning that at this radius the wave and the gas have the same velocity. In a previous publication (Casse et al. 2017) we showed that the behavior of the instability from Newtonian to a Schwarzschild black hole stays very similar. Here, we followed the same methods but changed not only the position of the extremum of $\mathcal{L}$ located at r_b but also the spin. Having a spin closer to one allows us to get even closer to the black hole than the last stable orbit in the case of a Schwarzschild black hole.

On Fig. 1 we compare four snapshots of the developed RWI with an increasing spin/decreasing extremum position from left to right. On the far left, we have the case of a Schwarzschild metric with the inner edge of

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the disk at its last stable orbit ($a = 0, r_b = 6r_S$), then a mildly relativistic case with a spin $a = 0.5$ with the inner edge of the disk close, but not at its last stable orbit ($r_b = 5r_S$). We follow with a spin of $a = 0.95$ with the inner edge of the disk at its last stable orbit ($r_b = 1.94r_S$) and finally the extremely relativistic case of $a = 0.995, r_b = 1.45r_S$, with the inner edge of the disk close to its last stable orbit.

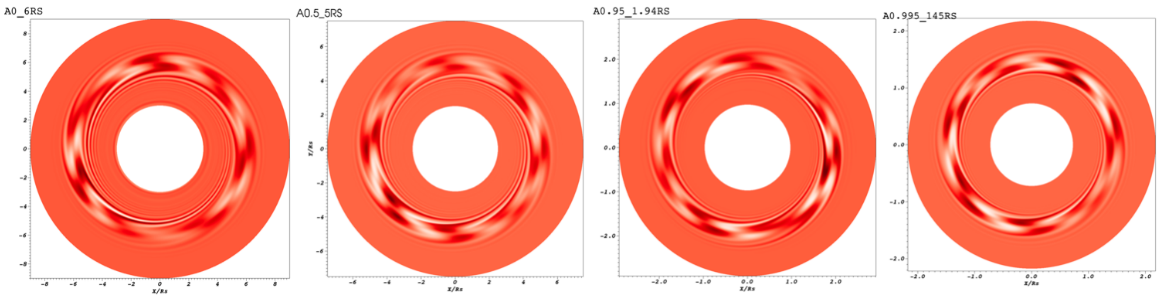

Fig. 1. Four snapshots of the developed RWI with from left to right $a = 0, r_b = 6r_S$, $a = 0.5, r_b = 5r_S$, $a = 0.95, r_b = 1.94r_S$, and $a = 0.995, r_b = 1.45r_S$

While the time, in orbital period at the corotation radius where the RWI happens, is not exactly the same for all the snapshots as it is difficult to obtain with so many different spins, we see that, qualitatively, the four disks are very similar and cannot be distinguished.

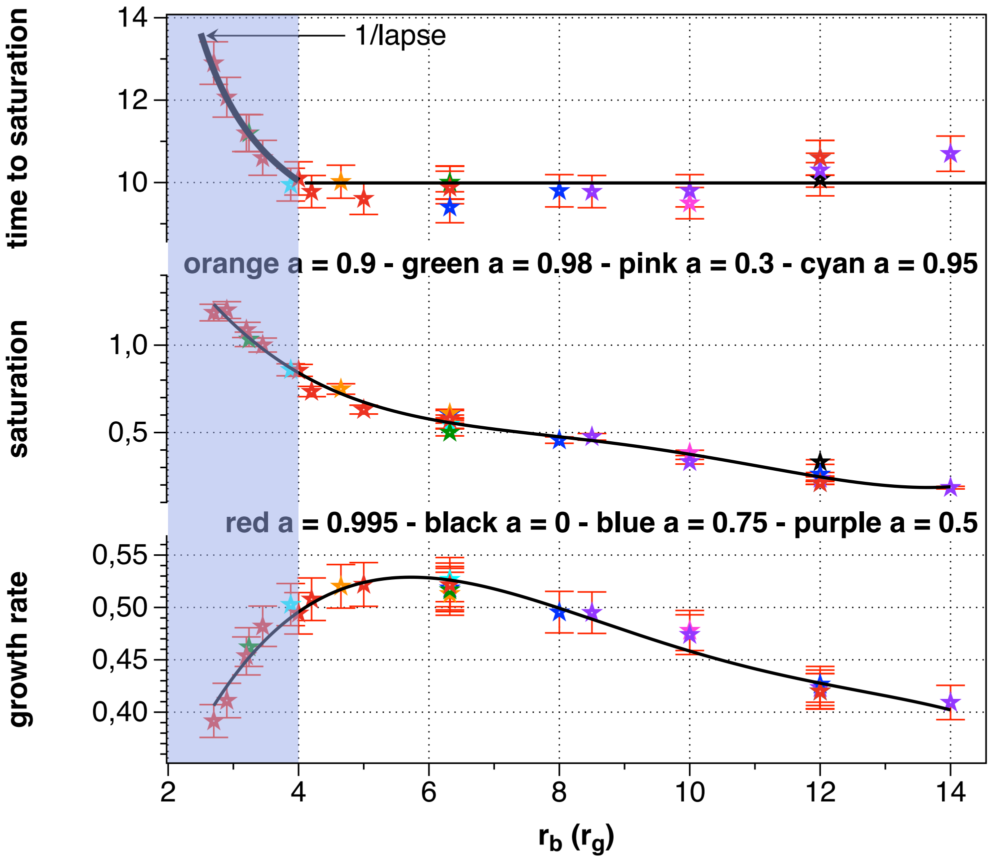

Fig. 2. Evolution of the growth rate of the instability, its saturation level and the time to reach saturation for different spin and position of the instability

To see the differences we need to look more quantitatively, namely by looking at the growth rate of the instability, its saturation level and the time required to reach saturation. On top of Fig.2 we see that the time to reach saturation, while relatively constant at larger r_b, the small differences coming mostly from the differences in the setup we used, increased rapidly when the corotation radius is inside about $4r_g$. This rise shows the impact of the lapse on the propagation of the instability. While the change is about 40% of the corotation period, the actual numerical value is extremely small as those objects are very fast. It is not possible to detect such differences in observations.

At the bottom of Fig.2 we see that the growth rate of the instability goes through a maximum for a corotation of about $5r_g$. This, once again is too small of a change to have any detectable effect as one needs too long of an observation to detect HFQPOs, so we cannot follow the 'instantaneous' evolution of its rms.

More interestingly, we see on the middle plot, that the closer to r_g the corotation is, the higher the saturation is. This quantity is related to the maximum strength a QPO can attain, hence to how much modulation there is of the observed flux. While we cannot directly test that with observations, using all the published HFQPO data we find that the higher the rms, the higher the spin of black hole. The limitation being the very low number of spin determination for HFQPO source. This is something we will be exploring, especially with NICER.

3 Looking In the time domain

Another way to look at our data is in the time-domain, which is the main way to study HFQPOs at the moment. In order to do that we created synthetic lightcurves from our simulations using the ray-tracing routine Gyoto (Vincent et al. 2011) and then performed a Fourier Transform and a 'fit' with Lorentzian as is done with real data.

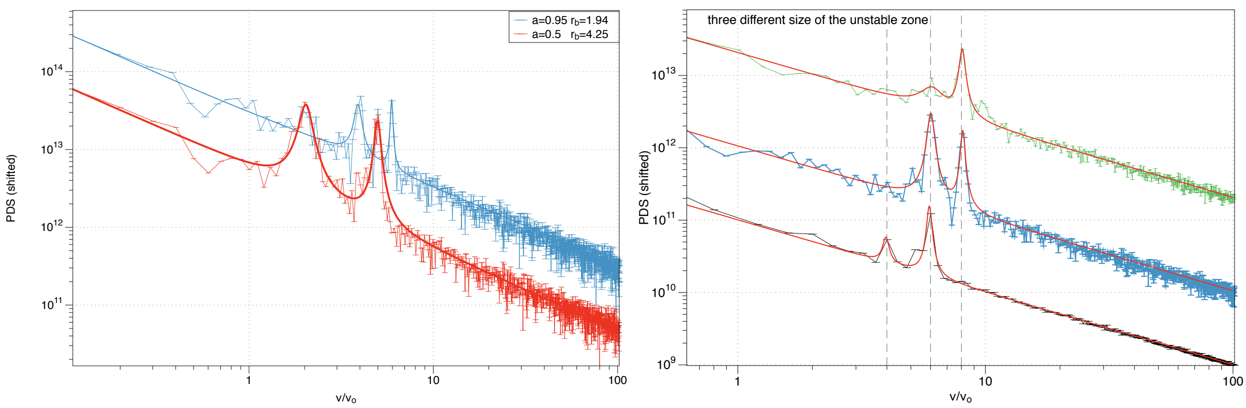

Fig. 3. Left: Two examples of synthetic PDS for two different spins and positions ($a = 0.5, r_b = 4.25$ and $a = 0.95, r_b = 1.94$) but a similar unstable zone. **Right:** Three examples of the same spin and position but with different unstable zones.

We see in on the left of Fig.3 two examples of synthetic PDS for two different spins and positions one mildly relativist ($a = 0.5, r_b = 4.25$) and one closer to maximal spin ($a = 0.95, r_b = 1.94$) but a similar unstable zone, while on the right there are three examples of the same spin and position but with different unstable zones. The abscissa of the plots are normalized to the fundamental frequency of the unstable zone so that we can compare different cases where the fundamental frequencies are different. Here we are interested in the mixture of frequencies detected and if any integer ratio exists between them. While changing the spin and position does give a different 'mix' of modes no general pattern seems to emerge as function of the spin of the central objects. Indeed, the size of the unstable zone has more impact on which modes emerge and how the ratio between the modes behaves. The three cases shown on the right of Fig.3 have at most a 30% difference in the dimension of the unstable zone (between the black and blue curve it is about 11%). Using this, a relatively small change in the local disk condition can change rapidly the dimension of the unstable zone hence creating a different observable mix of frequencies between observations.

4 Conclusions

The RWI has been previously proposed to explain the High-Frequency QPOs occurring in the vicinity of black holes (Vincent et al. 2013, 2014) but its existence in a full-GR environment had never been demonstrated up to now. Here we remedy this by performing 2D and 3D GR-HD simulations of the RWI in an extreme Kerr environment proving at the same time:

i) that, as predicted, the instability actually still exists when taking into account the full effects of a Kerr black hole.

ii) that its behaviour is only mildly affected by relativistic effects and only when the unstable zone is inside $5r_g$. Nevertheless, some of those effects might lead to observables.

iii) and that it is able to create a variety of PDS that encompass the observed one while explaining the origin of those different behaviours.

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