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MULTI-BODY FIGURES OF EQUILIBRIUM IN AXIAL SYMMETRY

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Abstract. We present an efficient multi-body code devoted to self-gravitating polytropic stars and rings in mutual gravitational interaction. The code implements the Self-Consistent Field method which captures solutions in an iterative manner. It works for any positive polytropic index, rotation law and configuration (axis ratios and relative separations). The number of bodies is free. We have investigated a wide range of equilibria involving 2 up to 8 bodies. A model for the disk around HL Tau is currently under progress.

Keywords: Methods: numerical, Stars: rotation, Equation of state, Gravitation

1 Introduction

There is a wide litterature devoted to figures of equilibrium involving one or two bodies, with a main interest in stellar structure and tides in binaries (Horedt 2004). Multibody configurations (with more than 2 components) are much less investigated (Hachisu 1986). New bifurcations from the Maclaurin sequence have been unveiled by Ansorg et al. (2003). In particular, these authors have computed configurations where the central ellipsoid is on the verge of splitting into several rings. We have modified the DROP code (Huré & Hersant 2017), in order to treat multi-body systems. A particular motivation is the ring structure of the HLTau circumstellar disk.

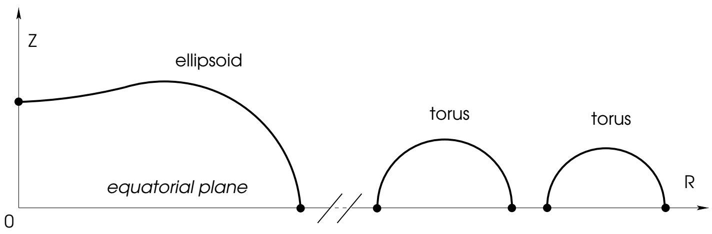

Fig. 1. Typical configuration for a multi-body system made of an ellipsoid (optional) and concentric tori.

2 Physical model and numerical treatment

The typical theoretical framework for investigating the structure and shape of self-gravitating bodies in rotation is the Bernoulli equation combined with the Poisson equation. For several bodies in interaction (see Fig. 1), the coupling is ensured by the gravitational potential, which is global, while pressure and centrifugal terms are rather local. The equation-of-state (EOS) linking the pressure to the mass density is polytropic. As often done, the centrifugal potential is to be prescribed. We solve the equation set from a modified version the Self-Consistent Field method (Hachisu 1986). In particular, the mass density contrast between bodies is computed self-consistently. We use one computational box per body, with a linear radial stretching, appropriate for oblate structures. Individual Poisson equations are solved with multigrid at second-order in the mesh spacing, with Dirichlet and Neumann boundary conditions. Fluid boundaries are detected with a Freeman-chain code and accounted for in the determination of any global quantity, for a better accuracy. The number of bodies is a priori free and limited only by the computing capabilities. When an equilibrium is found for a given parameter

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set (EOS, axis-ratios and relative positions), the code outputs all physical properties such as masses, volumes, pressure and mass density fields. In general, all fields converge close to the computer precision. The Virial Parameter depends mainly on the polytropic index and numerical resolution. It is typically $\sim 10^{-4}$ for a 128×128 cylindrical grid, while performances are reduced in the incompressible case.

3 A few examples. The case of HLTau ?

Figure 2 gives three examples obtained with the code for 2, 4 and 7 bodies in interaction. The polytropic index is 1.5, and rigid rotation is assumed. As for unary systems, fluid sections are roughly elliptical or circular. However, next to critical rotations (i.e. the mass shedding limit), shapes may become oval at edges.

The protoplanetary disk around the star HL Tauri exhibits several nearly axisymmetrical overdensities organized into ~ 7 detached ring-like substructures. The mass of the orbiting matter could be as high as 25% of the central mass from Carrasco-González et al. (2016), if gas is still present. This system seems a good candidate to test self-gravity which can probably not be neglected. We are currently using our code to check this hypothesis. The radial extension of each ring estimated from ALMA images at 1.3mm is used on input. We have no details about the EOS. We are in particular interested in setting constraints on the mass of individual rings and the rotation law. If such a disk-to-central mass ratio is confirmed, a departure from the Keplerian profile is expected.

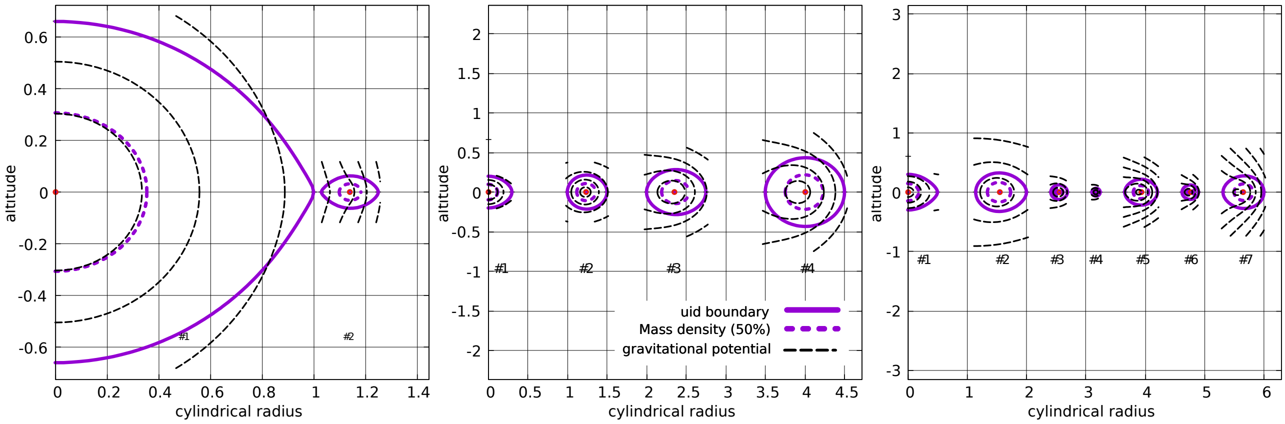

Fig. 2. Three different systems at equilibrium.

4 Conclusions

With the new version of the DROP code (Boutin-Basillais & Huré, 2019, in prep.), we can model a self-gravitating systems made of a central ellipsoid (optional) surrounded by several rings, in the polytropic assumption. It is especially well suited to investigate the HLTau ring system, in particular to study the mass-velocity relationship.

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