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MAIN UNCERTAINTIES IN CHARACTERIZING PLANET HOST MAIN SEQUENCE SOLAR -LIKE OSCILLATING STARS

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Abstract. Characterization of exoplanets requires the determination of masses, radii and ages of their host stars. Seismology provides these quantities with a high level of precision. In such a case, systematics due to uncertainties in the physical description of stellar interiors starts to dominate the error budget. The main sources of uncertainty are briefly discussed for main-sequence stars with detected solar-like oscillations.

Keywords: asteroseismology, stars: interiors, stars: fundamental parameters, planets and satellites: fundamental parameters

1 Introduction

Proper characterization of exoplanets requires a good knowledge of their host stars, especially stellar masses, radii, ages. Determination of these properties relies on stellar modelling. One major source of uncertainty in deriving stellar properties then come from the uncertainties in the physical description of the stellar interiors. This report gives a brief summary of the main uncertainties in stellar modelling which impact the determination of the stellar masses, radii and ages (MRA). Nowadays highly precise MRA inferences are quite successful due to the tight observational constraints provided by seismology. We therefore restrict the discussion to main sequence (MS) stars in the mass range $[0.8 - 1.5\ M_\odot]$ roughly corresponding to stars showing solar-like oscillations. One main source of uncertainty in MRA inferences arises from the use of free parameters. Several such free parameters are involved in the approximate modelling of a physical process such as convection. Other free parameters such as the initial chemical composition are specific to each star. MRA are most often inferred by ajusting these free parameters together with the MRA. The ajustement is carried out so that the observational (classical and seismic) constraints are satisfied. In the case of binaries, additional constraints can come from assuming that they have same age and initial chemical composition. Uncertainties are usually assessed by changing one physical ingredient at a time and inferring the resulting changes in the MRA values. We illustrate the discussion with four seismically well studied stars or binary systems hosting at least one exoplanet.

2 Four benchmark cases

The stellar parameters of the close binary system α CenAB are quite well determined (Kervella et al. 2017). α Cen A (G2V, HD 128620) has a quasi solar mass $1.1055 \pm 0.0039 M_\odot$ but is more metallic ($[Fe/H] = 0.24 \pm 0.03$, Porto de Mello et al. (2008)) than the Sun. The seismic properties of both stars were obtained from ground observations (de Meulenaer et al. 2010; Kjeldsen et al. 2005). *The possible existence of a tiny convective core for α Cen A is still debated. This can significantly impact the age determination for the binary system, that-is also α Cen B (K1V, HD 128621) which hosts a planet.*

HD52265 was observed by the CoRoT mission (Baglin et al. 2009). This a G0V star more metallic than the Sun with $[Fe/H] = 0.22 \pm 0.05$. Seismic observations (Ballot et al. 2011) and subsequent inferences (Escobar et al. 2012; Lebreton & Goupil 2014) provide its seismic mass in the range $1.14 - 1.32\ M_\odot$. *Its mass uncertainty has a direct impact on its planet mass precision.*

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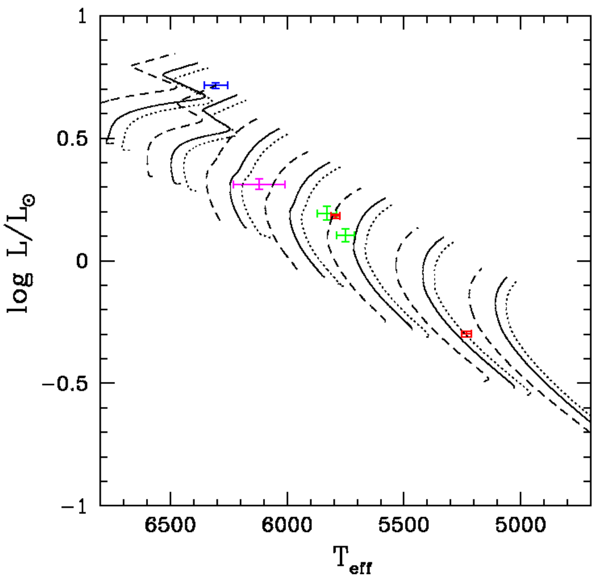

Fig. 1. Evolutionary tracks for main sequence stellar models in the mass range [0.7-1.3]$M_\odot$ with initial chemical composition $Y = 0.28, Z = 0.012$ (solid lines), $Y = 0.27, Z = 0.012$ (dotted lines) and $Y = 0.27, Z = 0.010$ (dashed lines). Crosses correspond to the benchmark stars : α CenAB (red), HD52265 (magenta), Kepler 21 (blue), 16 CygAB (green)

The other stars were observed by the Kepler mission (Borucki et al. 2010). Kepler 21 is a F6IV star (Howell et al. 2012; López-Morales et al. 2016) with a quasi solar metallicity ($[Fe/H] = -0.03 \pm 0.010$). The seismic data (Lund et al. 2016) enabled Silva Aguirre et al. (2015) to derive its seismic mass $1.408^{+0.021}_{-0.030} M_\odot$. *The star is at the end of main sequence or in the early subgiant phase depending on the extension of its convective core. Mass and age of Kepler 21 then depend on the adopted (or adjusted) value for the overshoot parameter.*

The binary system 16 CygAB is considered as a benchmark case for studying solar analogues. 16 CygB is known to host a giant planet. 16 CygA (HD186408, G1.5V) and 16 CygB (HD186427, G3V) have masses, metallicities and ages close to those of the Sun. They are bright enough that interferometric radii (White et al. 2013) and seismic data (Davies et al. 2015) are available in both cases (Davies et al. 2015). *Despite their binarity, masses of 16 CygAB must be determined seismically and remain model dependent to some extent.*

In all four cases, the stars are close enough that their distances are precisely known and thereby their luminosities as well. The location of these stars in a HR diagram is displayed in Fig.1. They approximatively cover the mass range of main sequence solar-like oscillators.

3 Main uncertainties in stellar modelling and their impact on star characterization

Initial helium content The initial helium relative abundance, Y_{ini} is not an observable but is an input parameter for stellar modelling. A mass - initial helium abundance degeneracy exists (Lebreton & Goupil 2014; Silva Aguirre et al. 2015). Any uncertainty on Y_{ini} then transfers into a systematic mass uncertainty. As an exemple, the inferred seismic mass for HD52265 was found to increase linearly from 1.10 to 1.30 $M_\odot$ when Y_{ini} was decreased from 0.38 to 0.24. The uncertainty on the inferred mass is then up to 17% due to uncertainties on Y_{ini}. As a consequence the age determination can be inacurate as well. Its impact is significant : for $\Delta Y_{ini} = 0.02$, the age varies by 15 to 35 % depending on the mass of the star (Lebreton et al. 2014). A tight correlation between the seismically inferred mass and age relative errors was shown to exist for a sample of simulated stars: a relative error of 10% in mass causes a relative error in age of 100% (late MS) to 400% (early MS) depending on the age of the star (Valle et al. 2015). The initial helium abundance value is often taken as the calibrated solar value $Y_{ini,\odot}$ although with no real justification. Another option is to use an enrichment law $\Delta Y/\Delta Z$ so that $Y_{ini} = Y_p + (\Delta Y/\Delta Z)Z_{ini}$ where Y_p is the primordial helium abundance and Z_{ini} the initial metal content of the star. For instance, Silva Aguirre et al. (2015) used $\Delta Y/\Delta Z = 1.4$ to infer the masses and ages of the Kepler candidate exoplanet host stars. But the scatter in $\Delta Y/\Delta Z$ value is large (Gennaro et al. 2010) and can usually be assumed in the range [1-3] leading to large uncertainties on the inferred mass. $\Delta Y/\Delta Z$ can be deduced from seismic inferences. Joyce & Chaboyer (2018) derived the seismically adjusted value $\Delta Y/\Delta Z = 0.90 \pm 0.12$ assuming it is the same for both α Cen A and α Cen B. However individual inferences led to different values for α Cen A (1.08) and for α Cen B (0.72). The ajusted values of Y_{ini} and $\Delta Y/\Delta Z$ are also found to depend on the adopted chemical abundance of heavy elements for the star.

Such uncertainties can hamper an accurate mass determination. Nevertheless, seismic inferences by adjusting Y_{ini} decreases the impact of its uncertainty on MRA inferences. For HD52265, this enabled to restrict Y_{ini} in the range 0.24 - 0.28 and therefore the mass range for this star becomes $M/M_{\odot} = 1.18 - 1.28$ i.e. $\Delta M/M \sim 10\%$. Further, a determination of Y_{ini} is possible using seismic glitches (Houdek & Gough 2007) in favorable cases such as 16 CygA (Verma et al. 2014). Buldgen et al. (2016) used the additional constraint on glitch-measured surface helium abundance and found a lower mass and older age with much less scattered values $M/M_{\odot} = 0.97 - 1.0$ (i.e. 2%), age $= 7.0 - 7.4$ Gyr (i.e. 3%). Without the helium constraint, the mass and age uncertainties for this star amount to 3% and 8% respectively.

Chemical composition of heavy elements The relative abundances of heavy elements of a star (often referred as chemical mixture or simply mixture) are usually assumed to follow the same ratios than for the Sun. Revisions of the solar chemical composition over the years led to a significant decrease of the solar metallicity. However, although more physically justified, the newly chemical mixture (AGSS09) destroys the overall good agreement between the seismic Sun and the standard solar model which was obtained with the previous high metallicity solar mixtures (GN93,GS08). Although some improvment can come from some foreseen increase of opacity, the issue remains essentially unsolved today. The impact of a change in the adoption of the chemical mixture was assessed with seismic inferences for 32 stars from the Kepler legacy sample (Nsamba et al. 2018b) : the relative error on the mass remains of the order of 5% or below.

Mixing length parameter One dimensional stellar models use the mixing length prescription (MLT; Böhm-Vitense (1958)) for the temperature gradient in presence of turbulent convection. This prescription involves an unknown free parameter, the mixing length parameter α_{MLT}. If one imposes the solar value to the α_{MLT} parameter, the errors on the mass and age inferences for another star are found artificially small. For the 16 CygAB binary system, Metcalfe et al. (2012) actually found that α_{MLT} ought to differ from the solar value. On the other extreme, inferring the age interval of a star by varying α_{MLT} within a range of reasonable values leads to a prohibitively large scatter in age. For instance allowing a range $\Delta\alpha_{MLT} = 20\%$ around $\alpha_{MLT,\odot}$, the age changes by more than 50 % with a classical inference. For HD52265, the adjusted α_{MLT} range of values led to uncertainties at the level 7 % on mass, 3 % radius, 13 % age (Lebreton & Goupil 2014). Joyce & Chaboyer (2018) found an age dispersion of 2-8 Gyr for α CenAB, despite their well-known stellar parameters. On the other hand, letting α_{MLT} be free to adjust so that all observational constraints including the seismic ones are satisfied, inference shows that $\Delta\alpha_{MLT} <\sim 4\%$ around $\alpha_{MLT,\odot}$ and the age changes decrease to 10 %. For α CenA, Joyce & Chaboyer (2018)'s study shows that the age scattering drops from 2-8 Gyr to 4.8-5.7 Gyr. The authors also found that the optimal α_{MLT} values are found to increase with mass and are in agreement with the values provided by empirical relations (Viani et al. 2018; Creevey et al. 2017) but in disagreement with 3D simulations of convection (Magic et al. 2015) for the sign of the metallicity dependence of α_{MLT}.

Atomic diffusion has a significant impact on MRA determination (Valle et al. 2014, 2015; Silva Aguirre et al. 2015) : $\Delta M/M <\sim 6\%; \Delta A/A <\sim 20\%$. It reduces the age at turn-off of low-mass stars by a few per cent compared to models without diffusion (Lebreton et al. 2014).

For masses in the range $M < 1.1 - 1.2 M_{\odot}$, atomic diffusion is now most often included but a 20 % uncertainty on its efficiency remains. On the other hand, radiative accelerations are negligible. Seismic inferences for α Cen A,B favor inclusion of atomic diffusion over the option of no diffusion (Miglio & Montalbán 2005). Seismic inferences also favor models with standard diffusion and models with suppressed diffusion over models with (artificially) enhanced diffusion (Joyce & Chaboyer 2018). This is also the case for 16 CygA (Buldgen et al. 2016) for which this results in 5% changes in mass, 3% changes in radius and up to 8% changes in age. For masses $M > 1.2 M_{\odot}$, atomic diffusion drains the thin outer convective region of its heavy elements and helium. However radiative accelerations can hamper the seeking of heavy elements. Then remains the problem of helium depletion which is usually thought to be counteracted by some turbulent mixing.

Onset of Convective core Stars with masses above a critical value develop a convective core. The mass at the onset of core convection is $M_{cc} \approx 1.1 M_\odot$ but depends on the solar mixture. The convective core appears at higher mass for the lower metallicity solar mixture (AGSS09) than for the older high metallicity solar mixture (GN93/GS98). It also depends on the efficiency of nuclear reactions in the CNO cycle. For α Cen A, Bazot et al. (2016) report a 40% chance of a convective core for models computed with the GN93 mixture while Nsamba et al. (2018b) found a 70% chance of core convection when assuming the GS98 mixture. In contrast Joyce & Chaboyer (2018) using the AGSS09 mixture found that a convective core can exist only when diffusion is enhanced compared to models with standard diffusion and when core overshoot in included; the corresponding models are however not those which match best the observational constraints. The derived ages of models with a convective core are lower by ~ 0.4Gyr than those of models without a convective core.

Core overshoot Overshoot of convective elements into the adjacent radiative regions is known to occur but the magnitude of the resulting extension of the central mixed region remains uncertain. For stars with convective cores sufficiently developped ($M >\approx 1.2 M_\odot$ or end of MS), the amount of convective overshoot significantly impacts the inferred stellar properties: ages differ by more than 10 % at turn off for a increase/decrease of $\alpha_{ov} \sim 0.2\ H_p$ (Lebreton et al. 2014). In the case of Kepler 21, inferences from grids of stellar models with and without core overshooting ($\alpha_{ov} \sim 0.2\ H_p$) give low and high mass solutions: the relative changes in mass and age are at the level of 10% in mass and 35% in age. With the additional constraint on the luminosity (from Hipparcos distance), the high mass (i.e solution with overshooting) is favored Silva Aguirre et al. (2015).

Rotationally induced transport Rotationally induced and turbulent mixing transport chemicals. For stars with masses $M/M_\odot > 1.15$, relative age differences can reach up to 10 % at turn off between models with and without rotationally induced mixing (Lebreton et al. 2014).

4 Conclusions

Seismic constraints combined with classical spectrophotometric ones have been proven to be invaluable for deriving very precise and more accurate stellar masses, radii and ages.

Uncertainties on MRA inferences from stellar modelling in a nutshell

Several in depth studies were then carried out in order to estimate uncertainties on MRA inferences: they were done either assuming a given evolutionary stage (Lebreton et al. 2014) or using samples of simulated stars (Valle et al. 2014, 2015; Reese et al. 2016) or studying samples of Kepler stars (Silva Aguirre et al. 2017; Serenelli et al. 2017; Nsamba et al. 2018a).

From these studies, each source of uncertainty discussed above can impact the mass determination up to the level of $2-5\%$, the radius one up to the level $0.5-3\%$. The age is the most affected: the level of age uncertainty depends on the source of the uncertainty, on the mass and on the evolutionary stage of the star and is most often larger than 10%. As an illustration, the results of two different studies are shown in Fig.2. We refer to the respective papers for details. When two sources of uncertainty are taken into account simultaneously, Valle et al. (2014) showed that errors obtained with one single source of uncertainty add up. For instance uncertainties in the α_{MLT} and ΔY_{ini} roughly doubles the uncertainties for the mass (4.3 %) and for the radius (2.0 %).

Impact on planet characterization

The efficiency of seismology to derive precise stellar mass, radius and age make possible an improved characterization of exoplanets when they are hosted by pulsating stars. Accuracy can be improved as well through the adjustment of free parameters although they have no predictive values for other stars. Samples of Kepler hosts stars were the subject of such characterizations (Huber et al. 2013; Silva Aguirre et al. 2015).

As an exemple, the mass of the exoplanet HD52265b was derived with an increasing effort to obtain a proper characterization of the host star $M_p = 1.13 \pm 0.03 M_J$ (Butler et al. 2000), $M_p = 1.09 \pm 0.11 M_J$ (Gizon et al. 2013) and $M_p = 1.21 \pm 0.5 M_J$ (Lebreton & Goupil 2014) where the last error bars include systematics from uncertainties on the physical input of the star modelling.

The second exemple is 16 CygB. The seismic mass, M_B, improved from $1.05 \pm 0.04\ M_\odot$ (Metcalfe et al. 2012) to $1.04 \pm 0.02\ M_\odot$ (Metcalfe et al. 2015) due to the increase of the Kepler data sample from 3 to 30 months of observation. Several studies followed (Buldgen et al. 2016; Silva Aguirre et al. 2017) which determined the mass of 16 Cyg B using essentially the same data but different approaches and physical assumptions. The internal precision on M_B determination from each individual study amounts to 1.5 to 5%. The central value from all

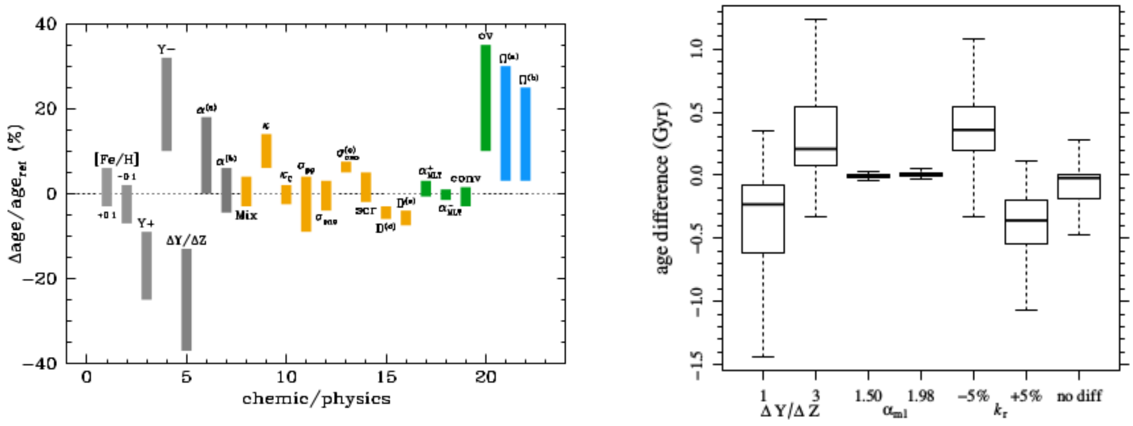

Fig. 2. Age uncertainties due to changes in one stellar physical input at a time at a given evolutonary stage: from Lebreton et al. (2014) (**left**) and Valle et al. (2015) (**right**) to which we refer for details.

these studies show a large scatter of about 11% which propagates onto the mass of the 16 CygB exoplanet. With a mass ratio $M_p \sin i/M_B = 1.697 \pm 0.071\ M_{Jup}/M_\odot$ (Wittenmyer et al. 2007), the 16 CygBb mass then lies in the range $M_p \sin i = [1.578 - 1.765]\ M_{Jup}$ i.e. showing a scatter much larger than the observational error. It must also be noted that the observed significant difference in surface lithium abundances between the two solar analogs 16CygAB is still basically unexplained, indicating that some physical processes might be missing (Deal et al. 2015) or improperly modelled.

For Kepler 21, the solution with overhoot is favoured. The derived mass was obtained with the fixed value $\alpha_{ov} = 0.2\ Hp$ (Silva Aguirre et al. 2015) and was used to derive the mass of the planet Kepler21b (López-Morales et al. 2016) with an uncertainty of $\sim 29\%$. However a reasonable value of the overshoot parameter can be anywhere in the range $[0.1 - 0.2]Hp$ which implies an uncertainty of about 5% on its mass and therefore must be added to the error budget for the Kepler21b mass.

Prospects for improvements

In the net budget of errors for characterization of exoplanet host stars, statistical (i.e. observational propagation) errors are important but can be alleviated by decreasing the systematics due to the inference techniques and -although more demanding- by the acquisition of higher quality observational data. Systematics biases due to the use of different evolutionary and oscillation codes for stellar modelling are non negligible compared to other statistical or systematic errors (Valle et al. 2015) but can easily be avoided/at least assessed. This can be done along the line used by the ESTA project (Monteiro et al. 2006). On the other hand, decreasing the systematic errors due to an improper stellar physics description as discussed above requires substantial theoretical efforts. The description of the physics of stellar interior for implementation in one-dimensional stellar models is continuously improving from the microphysics (computation and tabulation of nuclear reaction rates, equation of state and opacities for various chemical compositions) to the macrophysics (transport by turbulence, waves, etc..). Three-dimensional surface convection starts to be used in stellar evolution calculations (Jørgensen et al. 2018). Two-dimensional stellar models also become available (Rieutord 2016). At the same time, work continues in order to develop further the seismic diagnostics and to adapt inversion techniques to stars other than the Sun. The goal is to obtain a seismic measurement (and not an adjusted value) of free parameters such as surface helium abundance (Verma et al. 2017), core overshoot (Deheuvels et al. 2016) or the internal rotation profile (Benomar et al. 2015; Nielsen et al. 2017) at least for the brightest stars. These developments are motivated by the exquisite data provided by the CoRoT, Kepler and Gaia (Brown et al. 2018) missions but also by the exciting perspective of similar data for much larger samples of stars from the ongoing TESS (Campante et al. 2016) and future PLATO (Rauer et al. 2014) missions.

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