

DOES MAGNETIC FIELD MODIFY TIDAL DYNAMICS IN THE CONVECTIVE ENVELOPE OF SOLAR MASS STARS?

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Abstract. The energy dissipation of wave-like tidal flows in the convective envelope of low-mass stars is one of the key physical mechanisms that shape the orbital and rotational dynamics of short-period planetary systems. Tidal flows, and the excitation, propagation, and dissipation of tidally-induced inertial waves can be modified by stellar magnetic fields (e.g., Wei 2016, 2018, Lin and Ogilvie 2018). It is thus important to assess for which stars, at which location of their internal structure, and at which phase of their evolution, one needs to take into account the effects of magnetic fields on tidal waves. Using scaling laws that provide the amplitude of dynamo-generated magnetic fields along the rotational evolution of these stars (e.g., Christensen et al. 2009, Brun et al. 2015), combined with detailed grids of stellar rotation models (e.g., Amard et al. 2016), we examine the influence of a magnetic field on tidal forcing and dissipation near the tachocline of solar-like stars. We show that full consideration of magnetic fields is required to compute tidal dissipation, but not necessarily for tidal forcing.

Keywords: Magnetohydrodynamics – waves – planet-star interactions – stars: evolution – stars: rotation – stars: magnetic fields

1 Introduction

Star-planet tidal interactions are thought to play a key role in the dynamical evolution of close-in exoplanets, by circularizing their orbit, synchronizing the rotational and orbital periods, and possibly leading to some evolution of the spin-orbit (mis-)alignment angle (e.g., McQuillan et al. 2013; Albrecht et al. 2012; Bolmont & Mathis 2016; Damiani & Mathis 2018). Tides generally fall into two categories (Zahn 1977; Ogilvie 2014): a quasi-static equilibrium tide, which corresponds to the emergence of an equatorial bulge via large-scale tidal flows, and a non-static wave-like component called the dynamical tide, which is associated with energy dissipation of tidally-induced waves. Dynamical tides in low-mass stars manifest as inertial waves in their convection zone and as gravito-inertial waves in their radiative zone. The dissipation of these waves, via turbulent friction and thermal diffusion, is part of what shapes the rotational and orbital properties of a two-body system (Hut 1981).

A body of recent works have explored the impact of various physical processes on tidal dissipation in the convective envelope of late-type stars, such as differential rotation (Ogilvie & Lin 2004; Baruteau & Rieutord 2013; Guenel et al. 2016a), turbulent friction (Ogilvie & Lesur 2012; Mathis et al. 2016), and the effects of stellar evolution and metallicity (Mathis 2015; Bolmont & Mathis 2016; Gallet et al. 2017; Bolmont et al. 2017). These works have shown that tidal dissipation varies strongly with the star’s and planet’s physical properties. It is only recently that the question of how stellar magnetism influences the propagation and dissipation of tidal inertial waves has been addressed (Wei 2016, 2018; Lin & Ogilvie 2018). Low-mass stars, which have a radiative zone under the convective envelope (from 0.4 to 1.4 solar masses), host indeed a powerful dynamo that is sustained by turbulent convection and differential rotation in their envelope (Brun & Browning 2017). In this context, Wei (2016) and Lin & Ogilvie (2018) have shown that strong magnetic fields can significantly affect

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the propagation of inertial waves, and cause Ohmic dissipation of the (magneto-)inertial waves to dominate over their turbulent viscous dissipation. Still, the question of how magnetic fields impact the tidal forcing of inertial waves via the large-scale equilibrium tide flows remains an open question, which we address in this communication, and which is the subject of a paper in preparation by Astoul et al.

To perform this analysis, we compare the magnitudes of the Lorentz and Coriolis forces in the effective tidal forcing driven by the equilibrium tide flows, which features a dimensionless number called the Lehnert number (it is the ratio of the Alfvén speed and the rotational velocity, Lehnert 1954). Various scaling laws are explored to estimate the magnitude of the Lehnert number throughout the convective envelope, which depends on the surface rotation period of the star. A parametric study using the 1D stellar evolution code STAREVOL (Amard et al. 2016) is carried out to evaluate the Lehnert number at the base of the convective envelope of a solar-mass star along its lifetime, which we apply to a few observed star-planet systems. The same parametric study is used to assess the relative magnitudes of the Ohmic dissipation and the viscous turbulent dissipation for inertial waves, which is also traced by the Lehnert number (Lin & Ogilvie 2018).

2 Impact of the star’s magnetic field on the tidal forcing of inertial waves

2.1 An analytical criterion

In the linearised Navier-Stokes equation for the dynamical tide, the tidal force arises from an effective forcing driven by velocity and displacement fields associated with the equilibrium tide (see Appendix B from Ogilvie 2005, 2013). In the presence of a magnetic field, the Lorentz force comes into play, affecting both dynamical and equilibrium tides. Lin & Ogilvie (2018) have investigated the action of uniform and dipolar magnetic fields on the propagation and dissipation of inertial waves, without taking into account, in their simulations, the impact of the magnetic field on the tidal forcing. When doing so, the effective tidal force density is the sum of a classical hydrodynamical part f_1 , due to the hydrostatic equilibrium tide in the fluid’s rotating frame, and of the linearised Lorentz force density f_2 (see Appendix B from Lin & Ogilvie 2018, Astoul et al. in prep.):

$$\begin{cases} f_1 = -\rho(\partial_t \mathbf{u}_{\text{nw}} + 2\Omega \mathbf{e}_z \wedge \mathbf{u}_{\text{nw}}) \\ f_2 = \frac{\nabla \wedge \mathbf{B}}{\mu_0} \wedge \mathbf{b}_{\text{nw}} + \frac{\nabla \wedge \mathbf{b}_{\text{nw}}}{\mu_0} \wedge \mathbf{B} \end{cases}, \quad (2.1)$$

where \mathbf{u}_{nw} and \mathbf{b}_{nw} are the non wave-like flow and magnetic field (referring to the equilibrium tide), respectively, and ρ , B , and Ω denote the mean density, the magnetic field maximum amplitude (\mathbf{B} being the field itself), and the equatorial rotational frequency, respectively. The non wave-like magnetic field $\mathbf{b}_{\text{nw}} = \nabla \wedge (\boldsymbol{\xi}_{\text{nw}} \wedge \mathbf{B})$, with $\boldsymbol{\xi}_{\text{nw}}$ the equilibrium tide/non-wave like displacement, is deduced from the equation of induction, where we neglect the action of magnetic diffusivity. The ratio of both components of the tidal forcing reads (Astoul et al. in prep.):

$$\frac{f_2}{f_1} = \mathcal{O}\left(\frac{\text{Le}^2}{\text{Ro}_t}\right) \quad \text{with} \quad \begin{cases} \text{Le} = \frac{B}{\sqrt{\rho\mu_0}2\Omega R} \\ \text{Ro}_t = \frac{|\sigma_t|}{2\Omega} \end{cases}, \quad (2.2)$$

where we have introduced the Lehnert number Le , the Doppler-shifted Rossby number Ro_t , and the tidal frequency in the rotating frame σ_t (for its definition, see section 2.4 in the case of circularised and synchronised systems). Estimate of the time-dependent ratio Le^2/Ro_t will allow us to know how the forcing through the linearised Lorentz force induced by the equilibrium tide compares to the forcing through the Coriolis acceleration applied on this flow, and whether the Lorentz force needs to be taken into account in the tidal forcing of inertial waves.

2.2 Estimate of the magnetic field with simple scaling laws

To estimate how the radial profile of the Lehnert number varies with time in the convective envelope of a $1M_\odot$ star, we have used the 1D stellar evolution code called STAREVOL (Amard et al. 2016). The code does not include the evolution of the star’s magnetic field, which, however, may play a key role in transporting angular momentum, especially around the tachocline in solar-type stars (e.g. Strugarek et al. 2011; Barnabé et al. 2017). In table 1, we list simple prescriptions giving rough approximations of the magnetic field’s strength as well as the Lehnert number expressed with quantities computed by STAREVOL (Astoul et al. in prep.). These dynamo

prescriptions are based upon different reservoirs of magnetic energy: viscous diffusion for the weak scaling, kinetic energy for the equipartition, and gravitational energy for the buoyancy dynamo, or forces balance as is the case of the magnetostrophic regime where the Coriolis acceleration equates to the Lorentz force. In the Sun, the toroidal magnetic field at the tachocline is expected to be strong (about one Tesla, e.g. Charbonneau 2013), which is checked by the super-equipartition or the magnetostrophic regimes. This can be seen in the left panel of Fig. 1, where we display the magnetic field at the base of the convective zone against time, deduced from the different scaling laws listed in Table 1 and applied thanks to our STAREVOL calculations. The buoyancy dynamo model works well for fast and young stars like T-Tauri stars, as well as fast rotating giant planets like Jupiter (Christensen 2010).

Regime	Weak scaling ¹	Equipartition ^{1, 2}	Buoyancy dynamo ⁴	Magnetostrophy ^{1, 3}
Balance	$F_L = F_\nu$	ME = KE	$\frac{ME}{KE} = \text{Ro}^{-1/2}$	$F_L = \rho a_c$
$\text{Le} \times R/l_c$	$\text{Ro}/\sqrt{3}$	Ro	$\text{Ro}^{3/4}$	$\sqrt{\text{Ro}}$

¹ Brun et al. (2015) ² Brun & Browning (2017)
³ Curtis & Ness (1986) ⁴ Augustson et al. (2017)

Table 1. Scaling laws for the Lehnert number (defined in Eq. 2.2) obtained with different assumptions for the strength of the magnetic field force or energy densities. In the 'balance' row, F_L denotes the Lorentz force density, F_ν the viscous force density, a_c the Coriolis acceleration, ME and KE are the magnetic and kinetic energy densities, and $\text{Ro} = u_c/(2\Omega l_c)$ the convective Rossby number with convective speed u_c and length l_c . The turbulent viscosity is defined as $\nu = u_c l_c/3$. The angular frequency Ω , u_c , and l_c are outputs of the stellar code evolution STAREVOL.

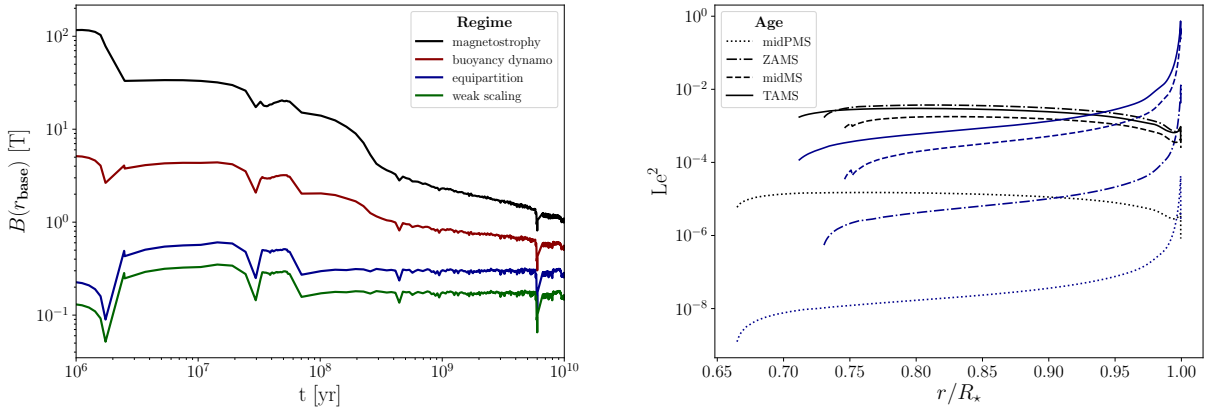


Fig. 1. **Left:** Magnetic field (in Tesla) at the base of the convective zone against time for a $1M_\odot$ star, obtained from the different scaling laws introduced in Table 1. Results are obtained for a fast initial rotation ($P = 1.6\text{d}$). **Right:** Lehnert number squared against radius (normalized to the star radius R_*) in the magnetostrophic (*black*) and equipartition (*blue*) regimes, for different evolutionary stages: amid the pre-main sequence ("midPMS"; $\sim 30\text{Myr}$), at zero-age main sequence ("ZAMS"; $\sim 70\text{Myr}$), amid main sequence ("midMS"; $\sim 5.3\text{Gyr}$) and towards the end of the main sequence ("TAMS"; $\sim 10.5\text{Gyr}$).

2.3 Parametric study of the Lehnert number

The right panel of Fig. 1 shows the Lehnert number squared, Le^2 , versus the normalized radius inside the convective zone, in the equipartition (in blue) and magnetostrophic (in black) regimes (see Table 1). Results are shown at different evolutionary stages from the middle of the pre-main sequence ($\sim 30\text{Myr}$; in dotted lines) to the termination of the main sequence ($\sim 10.5\text{Gyr}$; in solid lines). We stress that Le^2 increases with time

in the equipartition regime. In this regime, there is a sharp variation of Le^2 at the base and the top of the convective zone but it remains fairly flat in between, which comes about because of the radial profile of the fluid Rossby number (see Fig. 4 from Mathis et al. 2016). In the magnetostrophic regime, Le^2 is rather uniform in the bulk of the convective zone, fairly constant over time from the ZAMS to the TAMS, and decreases near the surface. Overall, we see that Le^2 is always less than unity, in accordance with previous estimations (Lin & Ogilvie 2018; Wei 2018). In the following, we will quote the Lehnert number at the base of the convective zone, keeping in mind that for all prescriptions, except for magnetostrophy, Le can significantly increase towards the surface of the star. This choice corresponds to focus on the effects of large-scale magnetic fields.

In the left panel of Fig. 2, Le^2 is displayed against time for our $1M_\odot$ model for the different scaling laws detailed in table 1. Results are shown for two different initial periods: 1.6 days (solid curves) and 9 days (dashed curves). Until about 1 Gyr, Le^2 is greater for slower initial rotation, and its variations mostly reflect those of the surface rotation period of the star as modeled in Gallet & Bouvier (2013). Beyond 1 Gyr, Le^2 increases monotonically with time as a consequence of the wind braking mechanism described by the Skumanich relationship (Weber & Davis 1967; Skumanich 1972). One can see for instance that Le^2 increases as $t^{1/2}$ in the magnetostrophic regime.

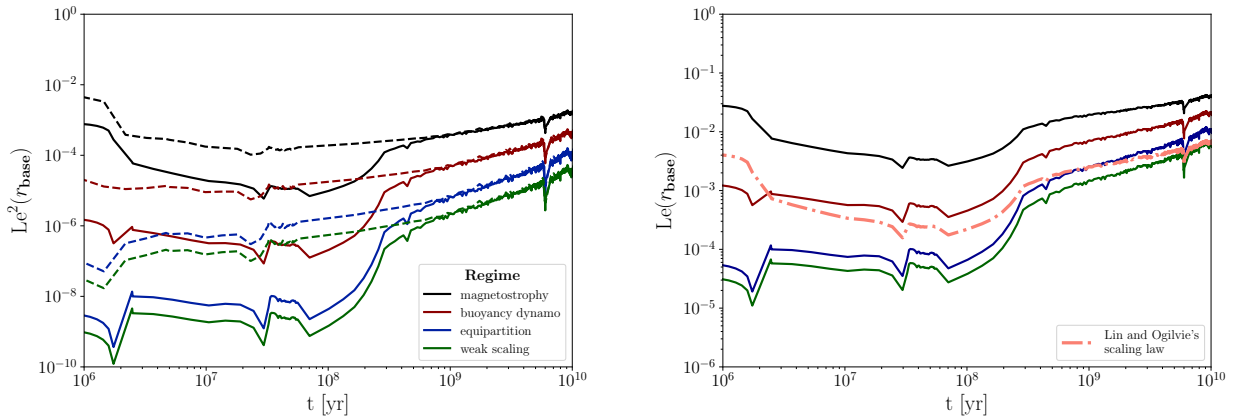


Fig. 2. Left: Lehnert number squared at the base of the convective zone against the age of a $1M_\odot$ star, for the four different scaling laws introduced in Table 1 to estimate the star’s magnetic field amplitude. Results are shown for an initial rotation period of 1.6 days (solid curves) and 9 days (dashed curves). **Right:** Lehnert number at the base of the convective zone versus the age of a $1M_\odot$ star for fast initial rotation (1.6 days), and for the same scaling laws as in the left panel. The criterion derived by Lin & Ogilvie (2018) to assess when Ohmic dissipation dominates over viscous turbulent diffusion of (magneto-)inertial waves, reads $Le_{crit} = Em^{2/3}$, with $Em = \eta/(2\Omega R_\star^2)$ the magnetic Ekman number and η the magnetic diffusivity. This scaling is overplotted by the dash-dotted curve, and we chose a turbulent magnetic diffusivity (e.g. $\eta = u_c l_c/3$).

2.4 Does magnetic field matter for the tidal forcing in observed star-planet systems?

The Doppler-shifted Rossby number $Ro_t = |\sigma_t|/(2\Omega)$ introduced in Eq. (2.2) can be estimated for observed planetary systems with a solar-mass star. For the tidal frequency σ_t , we assume for simplicity the case of circular and coplanar orbits, so that $\sigma_t = 2\Omega_o - \Omega$ with Ω_o the orbital frequency of the planet. The Doppler-shifted Rossby number can be recast as $Ro_t = |P_\star/P_o - 1|$ with P_\star and P_o the rotational and orbital periods, respectively. We see that the closer these periods are, the higher the ratio Le^2/Ro_t , and the larger the effect of the linearised Lorentz force when compared to those of the Coriolis acceleration on tidal forcing.

In Table 2, the ratio Le^2/Ro_t is shown for actual star-planet systems of various ages and periods. These systems have stellar masses close to $1M_\odot$, and nearly circular orbits with small projected stellar obliquities. We see that Le^2/Ro_t is much smaller than unity for these systems, meaning that the Lorentz force has nearly no impact on tidal forcing. This is not really surprising since these systems are far from synchronization. A state close to synchronization could be obtained for younger stars with close-in companions. It may also be obtained as a result of strong differential rotation in the star’s convective envelope (Guenel et al. 2016b; Benomar et al. 2018).

Systems	age [Gyr]	P_o [d]	P_* [d]	Le^2/Ro_t
HAT-P-36 (b)	6.6 ± 1.8	1.3	15.3 ± 0.4	10^{-4}
WASP-5 (b)	3.0 ± 1.4	1.6	16.2 ± 0.4	9×10^{-5}
WASP-16 (b)	2.3 ± 2.2	3.1	18.5	10^{-4}

Table 2. Estimate of Le^2/Ro_t at the base of the convective zone for three star-planet systems with a $\sim 1M_\odot$ host star and a hot Jupiter-like planet. Ages and orbital periods (P_o) were found in <https://www.exoplanet.eu>. The age of the stars has been used to get the rotation period P_* for the star WASP-16 via STAREVOL, and Le^2 for the magnetostrophic regime in all three systems. The rotation periods of the host stars HAT-P-36 and WASP-5 have been found in Mancini et al. (2015) and Maxted et al. (2015), respectively.

3 Impact of the star’s magnetic field on tidal dissipation of (magneto-)inertial waves

In Sect. 2, we have estimated the impact of a star’s magnetic field on the effective tidal forcing that leads to wave excitation in the convective zone. However, the magnetic field can also directly affect the propagation and dissipation of these waves. For large magnetic fields, the Lorentz force can thwart the Coriolis acceleration, thereby exciting magneto-inertial waves in the convective zone (Wei 2016; Lin & Ogilvie 2018). The energy dissipation of these waves can be controlled by either Ohmic dissipation or viscous turbulent diffusion, depending on the value of the Lehnert number. More specifically, Lin & Ogilvie (2018) showed that Ohmic dissipation dominates when $Le \geq (Em)^{2/3}$, where $Em = \eta/(2\Omega R_*^2)$ denotes the magnetic Ekman number and η the magnetic diffusivity. In the right panel of Fig. 2, we display $Le = (Em)^{2/3}$ against the star age, assuming $\eta = \nu = u_c l_c/3$ (that is, a turbulent magnetic Prandtl number of unity). Comparison with the solid curves underlines that, depending on the scaling law used to estimate the star’s magnetic field, Ohmic dissipation can dominate or not. Awaiting constrained magnetic field strengths, both viscous and Ohmic dissipation should be taken into consideration.

4 Conclusions

We have investigated the influence of the amplitude of stellar magnetic fields on star-planet tidal interactions. We have first derived a simple analytical criterion to quantify the importance of the Lorentz force, when compared to the Coriolis acceleration, on the effective tidal forcing of (magneto-)inertial waves in the convective envelope of a solar-type star. We have used a 1D stellar evolution model to examine how the impact of the Lorentz force in the tidal forcing depends on the age of the star, its initial rotation period, and the position in the convective envelope. For coplanar and circularised star-planet systems, we have shown that the impact of large-scale magnetic fields on tidal forcing is likely small at the base of the convective envelope (that is, near the tachocline), except for near-synchronised systems. We have also used the results of our stellar evolution model to assess the importance of Ohmic diffusivity on the dissipation of (magneto-)inertial waves, by using the criterion derived by Lin & Ogilvie (2018). Our results indicate that Ohmic and viscous turbulent dissipations have similar magnitude at the base of the convective zone in solar-mass stars, so that both diffusion processes should be taken into account for the calculation of the dynamical tide. In a paper in preparation, we will expand and detail these preliminary results, and will also consider lower-mass stars. Moreover, the study of the impact of magnetic field on tides must be combined with the analysis of planet-star interactions through magnetic couples (Strugarek et al. 2017).

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