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P. Di Matteo, O. Creevey, A. Crida, G. Kordopatis, J. Malzac, J.-B. Marquette, M. N'Diaye, O. Venot (eds)

TOWARDS ULTRA SPEED UP FOR DUST GROWTH SCHEMES

M. Lombart[1] and G. Laibe [1]

Abstract. Current 3D simulations of dusty protoplanetary discs do not include dust growth and fragmentation comprehensively since the computational cost associated to traditional algorithms is prohibitive. We show that it is possible to overcome this difficulty by using a high-order discontinuous Galerkin solver.

Keywords: Planets and satellites: formation, Dust, extinction, Methods: numerical.

1 Introduction

Recent spatially resolved observations of protoplanetary discs by SPHERE/VLT and ALMA have revealed that structures such as rings, spirals, horseshoes and gaps are ubiquitous (e.g. Avenhaus et al. 2018; Andrews et al. 2018). A novel generation of numerical models including differential dynamics between the gaseous nebula and dust grains has subsequently been developed to interpret these observations (e.g. Dipierro et al. 2015). However, no 3D dust/gas simulation has included dust coagulation and fragmentation comprehensively so far, although 1+1D modelling or 3D simulations with toy models of growth have shown that its effect can not be neglected (e.g. Brauer et al. 2008). This is mainly due to the prohibitive computational cost associated to the resolution of the so-called Smoluchowski equation with existing numerical algorithms. Indeed, any bin added to improve the resolution in size rises up the computational time drastically since one has to pay the cost of the integration of the associated dynamics at every grid point and for each time step. Typically, we aim for $\sim$15 size-bins from 1μm to 1cm with an accuracy of order $\lesssim 0.1\%$ to be consistent with hydro solvers. We show that this challenging difficulty can be overcome by using a well-designed high-order discontinuous Galerkin algorithm.

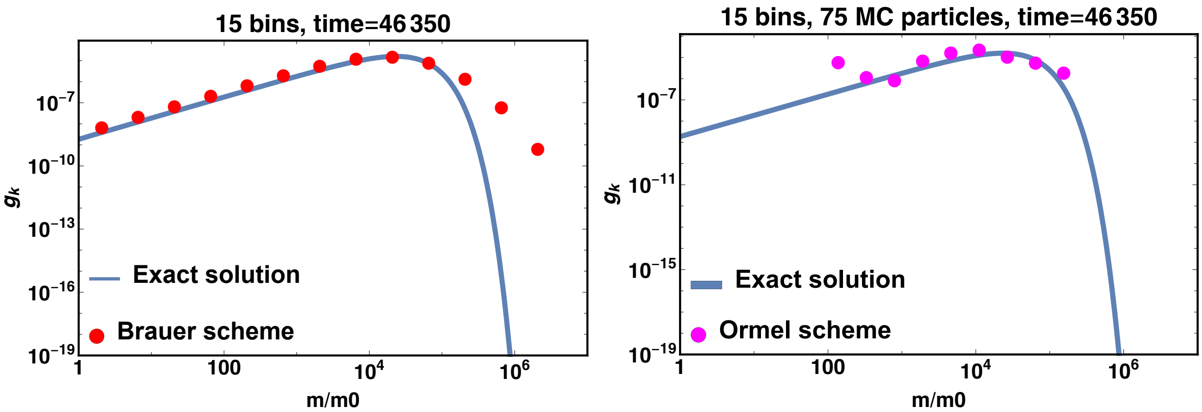

Fig. 1. Numerical solution with a constant kernel $K = 1$ with the algorithms described in Ormel & Cuzzi (2007) (**left**) and in Brauer et al. (2008) (**right**). The desired accuracy of $\lesssim 0.1\%$ is not achieved.

[1] École normale supérieure de Lyon, CRAL, UMR CNRS 5574, Université de Lyon , 46 Allée d'Italie, 69364 Lyon Cedex 07, France

2 Method

Dust coagulation is described by the Smoluchowski equation $\frac{\partial g}{\partial t} + \frac{\partial F[g]}{\partial x} = 0$ where t and x denote time and mass respectively, and g is the mass density of the grain distribution (Tanaka et al. 1996). The mass flux F is a function of the coagulation kernel K that models microscopic growth:

$$F[g](x,t) = \int_0^x \int_{x-u}^{\infty} \frac{K(u,v)}{v} g(u,t)\, g(v,t)\, \mathrm{d}u\mathrm{d}v. \tag{2.1}$$

The key idea consists in approximating the mass distribution on a bin size I_j *not* by a constant function but by a high-order polynomials, separating the variables t and x i.e. $g(x,t) \simeq g_j(x,t) = \sum_{i=0}^{k} c_j^i(t)\phi_i(\xi_j(x))$ – see Fig. 2. Choosing ϕ_i as the Legendre polynomials and projecting Eq. 2.1 onto this basis, one obtains an ordinary system of differential equations for the c_i's. Fluxes are computed using by Gauss Quadrature (Liu et al. 2019).

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} c_j^0 \\ \ddots \\ c_j^k \end{bmatrix}(t) = \frac{2}{\Delta x_j}\begin{bmatrix} 1/d_0 & & \\ & \ddots & \\ & & 1/d_k \end{bmatrix}\left(\int_{I_j} F[g](x,t)\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix} \phi_0 \\ \ddots \\ \phi_k \end{bmatrix}(\xi_j(x))\,\mathrm{d}x - F[g](x,t)\begin{bmatrix} \phi_0 \\ \ddots \\ \phi_k \end{bmatrix}(\xi_j(x))\,|_{\partial I_j}\right). \tag{2.2}$$

3 Results

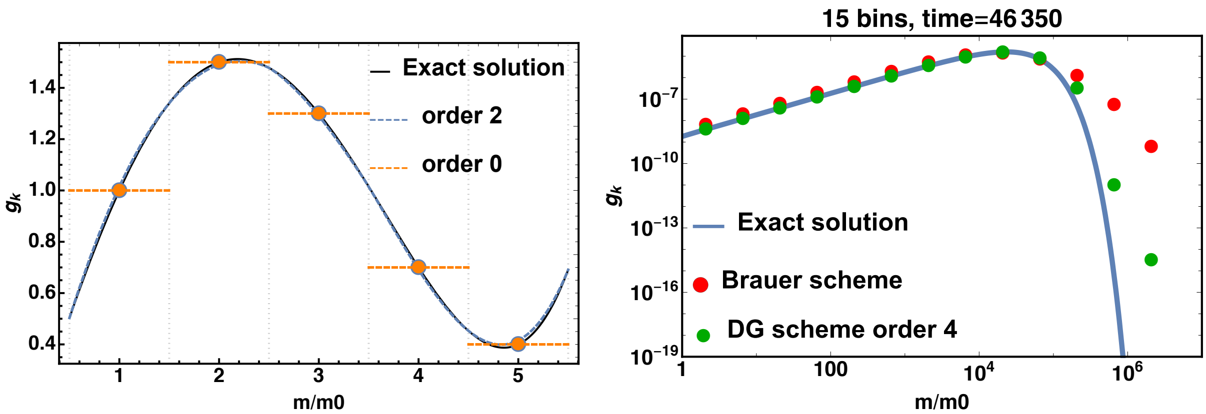

Fig. 2. Left: Low-order *vs.* high-order approximation. **Right:** Numerical solution obtained for an integration of order 4 with a constant kernel $K = 1$ over 15 size bins and 75 degrees of freedom for discontinuous Galerkin algorithm.

For the seminal test with $K = 1$, an error of $\lesssim 0.1\%$ in L_1-norm is achieved under the conditions required by hydrodynamical simulations (Fig. 2). Further refinements of this algorithm have been performed but are not presented here (improved projection basis, refined binning, time stepping, limiter for positivity).

4 Conclusions

We have shown that high-order discontinuous Galerkin algorithms pave the way to include the full dust coagulation equation in realistic 3D dust/gas simulation of protoplanetary discs.

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