# A SIMPLE TOOL FOR CALCULATING CENTRIFUGAL DEFORMATION STARTING FROM 1D MODELS OF STARS OR PLANETS

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**Abstract.** We describe a tool which is able to calculate the centrifugal deformation of a rotating star or planet starting from a 1D non-rotating model, for conservative (i.e. cylindrical) rotation profiles. This tool applies an iterative approach based on the Self-Consistent Field (SCF) method while preserving the pressure profile as a function of density. The resultant model is suitable for stellar pulsation calculations, thus making this tool suitable for parametric asteroseismic investigations. It can also be used to calculate the deformation of rapidly rotating planets such as Jupiter and Saturn which contain internal discontinuities.

Keywords: centrifugal deformation, stars, planets, oscillations

## 1 Introduction

Interferometry has revealed the shortcomings of 1D spherically symmetrical models in describing a number of stars such as Achernar, Vega, or Altair (e.g. Domiciano de Souza et al. 2003; Peterson et al. 2006; Monnier et al. 2007). Likewise, telescope and close-up observations of Jupiter and Saturn have shown that these planets are considerably deformed by the centrifugal acceleration (e.g. Iess et al. 2018; Guillot et al. 2018). Ideally, 2D models which fully take into account the effects of rotation throughout the star's or planet's evolution should be used to model them in a self-consistent way. For instance, the ESTER code is the first to self-consistently take into account centrifugal deformation and baroclinic effects in static rapidly rotating stellar models (e.g. Rieutord et al. 2016). However, such models typically prove to be expensive to calculate and may not currently be the most suitable for a parametric study with, for instance user-defined rotation profiles, or a  $\chi^2$  minimisation to fit a set of observations. Furthermore such models do not reach the same degree of realism as 1D models when it comes to modelling stellar or planetary evolution. A solution to this problem is to deform 1D stellar/planetary models using a given rotation profile. Here, we develop a code capable of doing this for conservative (i.e. cylindrical) rotation profiles.

# 2 Self-Consistent Field method

Our approach is based on the Self-Consistent Field (SCF) method (see Jackson et al. 2005; MacGregor et al. 2007). It consists in calculating the total (gravitational plus centrifugal) potential for a given density distribution and rotation profile, using this to find a new mapping composed of level surfaces, subsequently redistributing density and pressure profiles to this new mapping, and reiterating till convergence. The relation between pressure and density is preserved\* by preserving the relation between density and total potential to within an additive constant.

Two variants of this method have been produced:

• a first version which consists in interpolating Poisson's equation onto a spherical grid prior to solving it. Such an approach is computationally fast as Poisson's equation decouples according to different spherical harmonics. However, it is unable to handle discontinuities in the density profile as these line up with level surfaces rather than spherical surfaces.

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<sup>&</sup>lt;sup>2</sup> Università © CÃ 'te d'Azur, Laboratoire Lagrange, Observatorie de la CÃ 'te d'Azur, CNRS UMR 7293, 06304 Nice, France \*This turns out to approximate fairly well the more realistic baroclinic models from the ESTER code.

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• a second version which solves Poisson's equation directly in a coordinate system based on the level surfaces. Although slower as a result of coupling between the spherical harmonics, this approach is typically more accurate and can handle discontinuous models, which is more appropriate for models of gaseous planets with a solid core, such as Jupiter. The left panel of Fig. 1 illustrate a discontinuous model which has been deformed with this version of the code.

Once such models have been produced, it is possible to study their pulsation modes using the TOP pulsation code (e.g. Reese et al. 2006). The right panel of Fig. 1 illustrates one such mode for a deformed model of Jupiter.



Fig. 1. Left: Deformed N = 1 polytropic model with a discontinuity (indicated by the light blue line). Right: Pulsation mode in a deformed model of Jupiter.

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