SCATTERING TRANSFORMS FOR INTERSTELLAR ASTROPHYSICS AND BEYOND

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Abstract. Scattering transforms are a set of novel tools that provide a general ensemble of statistical descriptors able to characterize the multi-scale geometry of images by encoding the couplings between oriented scales. They are inspired by the architecture of convolutional neural networks, but do not require any training stage. Their applications range from the statistical description of interstellar emission maps in total intensity and polarization, or that of large-scale structures in the cosmic web, to statistical denoising and component separation, through data augmentation. They are accessible thanks to two python packages.

Keywords: Interstellar medium, large-scale structure, statistics

1 Introduction

The complex filamentary patterns observed in the interstellar medium (ISM) are prime examples of highly non-Gaussian structures emerging from the non-linear interactions between a variety of physical processes at play (turbulence, gravity, magnetic fields, thermodynamics, ...). The large-scale filaments of the cosmic web in the non-linear regime set another example of such non-Gaussian structures. Our understanding of how these structures form and evolve now largely relies on the statistical analysis of observations and their comparison with numerical simulations, in order to assess physical processes and their interactions, across a vast range of spatial scales. The quantitive comparison of simulation results to sets of observational data therefore requires an adequate statistical description of non-Gaussian structures that goes beyond the power spectrum. Ideally, such a description would allow for a simple connection between statistical geometrical descriptors and physical properties of the systems under study.

2 Scattering transforms

Scattering transforms are a set of tools developed in the field of data science (Mallat 2012; Bruna & Mallat 2012), originally designed to reproduce and understand the results obtained by convolutional neural networks (CNN) in the field of supervised machine learning. They are based on a local, multi-scale description of images through wavelet transforms, using a discrete family of Morlet or bump-steerable wavelets, $\{\psi_{j,\theta}\}$ covering the entire Fourier plane through a set of scales (j) and angles (θ), together with non-linear operators aimed at coupling spatial scales.

2.1 *Wavelet Scattering Transform (WST)*

The WST implements a layered architecture of successive convolutions with these wavelets, followed by non-linear (modulus) operators. Although formally similar to a CNN, the WST only uses a few layers and the convolution kernels are not adapted, so that there is effectively no training stage. Combined with simple linear classifiers, the WST has been shown to achieve state-of-the-art results in various classification problems (e.g. Sifre & Mallat 2013). Typically, a $256{\times}256$ image may be adequately described by some ~ 1000 WST coefficients.

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2.2 Reduced Wavelet Scattering Transform (RWST)

Images arising from physical processes are expected to exhibit some statistical regularity, which translates into regular patterns emerging when plotting their WST coefficients. The angular dependency of these coefficients may be modeled via simple trigonometric functions, resulting in the RWST, which typically reduces the number of necessary statistical descriptors by an order of magnitude - without any loss of informational content. The RWST model has been shown to apply equally well to ISM maps in total intensity (Allys et al. 2019) and in polarization (Regaldo-Saint Blancard et al. 2020).

2.3 Wavelet Phase Harmonics (WPH)

Another approach to describe statistical interactions across oriented scales in a given image lies in computing the covariance of different wavelet transforms $\mathrm{Cov}(I \star \psi_{j_1,\theta_1}, I \star \psi_{j_2,\theta_2})$. To ensure that this is non-zero, the "phase harmonics" operator $[z]^p : z = |z|e^{i\phi(z)} \mapsto |z|e^{ip\phi(z)}$ is applied to the wavelet transforms. It accelerates the phases to line them up for the different bandpasses. Thus altered, covariances yield statistical information on the couplings between scales (Mallat et al. 2019). The many different WPH coefficients may be used as a basis of generative models, starting from Gaussian random noise and performing a gradient descent constrained by the target WPH coefficients. This approach was used in the context of cosmological simulations of the large-scale structures of the Universe, demonstrating that WPH-constrained random realizations ("syntheses") present the same statistics (power spectra, probability density functions, bispectra, Minkowski functionals, ...) as the original image (Allys et al. 2020).

3 Application : denoising and component separation

The scattering transforms have been used to perform statistical component separation, given observational data $d = s + n$ that is the sum of a signal s and a noise n. Assuming that the statistics of the nuisance signal n are known, and that they are different from those of the target signal s, it is possible to obtain an estimate $\hat{s}$ of the signal through the minimization of a loss function $\mathcal{L}(s) \propto \sum_i ||\phi(s + n_i) - \phi(d)||$ where the n_i are different realizations of the noise. This was demonstrated on simulated polarization data and applied to observational data from *Planck* in the Chamaeleon-Musca molecular cloud (Regaldo-Saint Blancard et al. 2021).

A follow-up analysis has shown that such an approach may also be applied to the separation of Galactic thermal dust and cosmic infrared background (CIB) emission. Using a simulation of the CIB and 21 cm line emission as a proxy for a pure Galactic thermal dust emission map, a CIB-contaminated Galactic dust signal is built. The WPH statistics of the CIB map are used to generate multiple random realizations of the CIB, which are then used in the same way as in Regaldo-Saint Blancard et al. (2021) to statistically separate the Galactic dust from the total signal, effectively cleaning it from CIB contamination.

4 Conclusions

The WST, RWST, and WPH are novel statistical tools aimed at describing complex structures with non-Gaussian statistics such as appear in interstellar medium images. They provide sufficient statistics to serve as a basis for realistic generative models and for a metric to compare observational and simulation data sets. Finally, they usher in a new avenue towards statistical component separation, with potentially vast applications in astrophysics and cosmology. They are released for wide use through two public python packages, `PyWST` and `PyWPH`, available through `https://github.com/bregaldo`.

References

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