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AMPLITUDE OF THE PLUME-INDUCED SOLAR GRAVITY MODES: IMPLICATIONS REGARDING THEIR DETECTION

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Abstract. The observation of the low-frequency gravity oscillation modes in the Sun is expected to bring unprecedented constraints on the solar interior. However, despite several claims, their detection has not been confirmed yet. Within this context, theoretical estimates of their amplitude can help guide the observational strategies and the design of future measuring devices. In this short paper, we report the recent results of Pinçon et al. (2021) who considered the penetration of convective plumes at the top of the radiative interior as the driving mechanism, hence completing previous estimates. Accounting for the uncertainties in the plume modeling, the surface disk-integrated apparent mode radial velocity that would be measured with the GOLF instrument on the spacecraft SoHO is estimated on the order of 0.5 cm s^{-1} in the most plausible favorable case, which still requires about 25 years of observation for a robust detection.

Keywords: Sun – helioseismology – gravity modes

1 Introduction

Solar gravity modes have buoyancy as the restoring force: they thus can propagate in the central radiative zone of the Sun. As a consequence, the oscillation spectrum of the gravity modes contains information on the inner stratification of the Sun and thus appears very promising to constrain the microphysics used to build solar models, as for instance the opacity, the nuclear reaction rates, or the chemical mixture. The potential of combining these seismic constraints with the recent measurements of the neutrino flux produced by the CNO cycle in the nuclear solar core is even more promising (Borexino Collaboration et al. 2020; Salmon et al. 2021). It is expected for instance to give important insights into the solar metallicity problem (e.g., Buldgen et al. 2019). Moreover, the gravity modes have also the potential to inform us on the central rotation of the Sun, which will permit to put stringent constraints on the internal angular momentum redistribution and induced mixing processes during its whole past life. In addition, considering the Sun as a particular target to calibrate stellar evolution codes, the amount of information expected from the observation of the solar gravity modes represents a goldmine for the study of the stellar structure and evolution in general.

Nevertheless, although the detection of the solar gravity modes has already been claimed by several previous works, it has not been robustly confirmed yet (e.g., Appourchaux et al. 2010; Fossat et al. 2017; Schunker et al. 2018). Within this context, theoretical estimates of their amplitude can help implement new observational strategies and guide the design of future instruments dedicated to their search. While the current numerical simulations can provide interesting hints about the generation of gravity modes in stars, the values of their control parameters remain far from the expected stellar regimes (e.g., Dintrans et al. 2005; Alvan et al. 2014). A complementary approach then consists in studying the excitation mechanism from a semi-analytical point of view (e.g., Belkacem 2011). The previous models mainly considered the turbulent Reynolds stress in the convective envelope as the source of the gravity modes. Guided by numerical simulations of the envelope of the Sun to model the turbulent properties of the convective eddies, the most recent estimate of Belkacem et al. (2009) predicted GOLF apparent mode radial velocities close to about 0.5 cm s^{-1} for a typical mode frequency $\nu \sim 100$ μHz, which is at the limit of the detection with the current GOLF data. However, these previous estimates were incomplete since they did not account for the potential excitation of the gravity modes by the penetration of downward convective plumes into the top layers of the radiative zone, or penetrative convection.

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This mechanism was shown to be very efficient in the Sun to generate very-low-frequency progressive gravity waves by Pinçon et al. (2016). The application to the case of gravity modes in a higher frequency range has been undertaken only recently by Pinçon et al. (2021). In this short paper, we briefly summarize their main results.

2 Analytical expression of the mean mode energy generated by penetrative plumes

Pinçon et al. (2021) could estimate the mean energy of the plume-induced gravity modes using three main assumptions. (1) The source term in the wave equation corresponds to the dynamical ram pressure exerted by an ensemble of incoherent penetrating plumes uniformly distributed over the sphere at the base of the convective region. (2) The plume Péclet number at the base of the convective zone is much higher than unity, meaning that the density contrast between the plumes and the surrounding is high at the top of the radiative region so that the buoyancy braking of the plumes when the penetrate into the stably-stratified radiative layers is very efficient. (3) The frequency range is comprised between 10 μHz and 100 μHz for the sake of the simplicity. Indeed, in this frequency range, the spatial behavior of the oscillation field can first be easily obtained using the JWKB asymptotic method; second, the mode damping is dominated by radiative diffusion and the mode lifetime is much larger than the mode period, which makes the coupling between the momentum and the heat equation tractable analytically using a two-timing method. Based on these assumptions, Pinçon et al. (2021) found that the mean energy over time of a mode with a radial order n, an angular degree ℓ, an azimuthal number m and an angular frequency $\omega_{n\ell m}$ is provided by the final expression

$$\langle E_{n\ell m}\rangle \approx \frac{\left[(\omega_{n\ell m}\Delta\Pi_\ell/\pi^2)\ \overline{L_{\rm p}}\ F_{{\rm d},\ell}\ e^{-\ell(\ell+1)b^2/2r_{\rm b}^2}\ \mathcal{C}_{n\ell m}\right]}{2\eta_{n\ell m}}\ , \tag{2.1}$$

where $\Delta\Pi_\ell$ is the asymptotic period spacing between two consecutive gravity modes of degree ℓ, $\overline{L_{\rm p}}$ is the mean plume kinetic luminosity at the base of the convective zone (at radius $r_{\rm b}$), $F_{{\rm d},\ell} = V_{\rm b}k_{h,{\rm b}}/N_{\rm t}$ is the Froude number, with $V_{\rm b}$ the plume velocity, $k_{h,{\rm b}} = \sqrt{\ell(\ell+1)}/r_{\rm b}$ the horizontal wavenumber of the mode and $N_{\rm t}$ the value of the Brunt-Vaisala frequency at the top of the radiative zone, b is the plume radius, and $\eta_{n\ell m}$ is the damping rate per unit of time. Finally, the $\mathcal{C}_{n\ell m}$ term measures the temporal correlation between the plumes and the modes, and thus depends on the plume time evolution profile inside the penetration zone. Given the lack of knowledge on this question and its complexity, the authors considered the two limiting cases of an exponential and a Gaussian law, that is, in the form of $e^{-|t|/\tau_{\rm p}}$ and $e^{-t^2/\tau_{\rm p}^2}$ with $\tau_{\rm p}$ the plume lifetime. In the considered frequency range, it can be shown that $\eta_{n\ell m} \ll \nu_{\rm p} \ll \omega_{n\ell m}$, where $\nu_{\rm p} = 1/\tau_{\rm p}$, and $\mathcal{C}_{n\ell m}$ reduces in the case of a Gaussian and and exponential plume time profile to, respectively,

$$\mathcal{C}^{\rm G}_{n\ell m} \approx 4\sqrt{\pi}\ \frac{\eta_{n\ell m}}{\nu_{\rm p}}\ \frac{\nu_{\rm p}^3}{\omega_{n\ell m}^3} \quad \text{and} \quad \mathcal{C}^{\rm E}_{n\ell m} \approx 16\frac{\nu_{\rm p}^3}{\omega_{n\ell m}^3}\ . \tag{2.2}$$

In the considered frequency range, the temporal correlation is thus expected to be much smaller in the Gaussian case than in the exponential case, i.e. $\mathcal{C}^{\rm G}_{n\ell m} \ll \mathcal{C}^{\rm E}_{n\ell m}$, so does the mean mode energy.

3 Amplitude of the plume-induced solar gravity modes and concluding remarks

In order to compute Eq. (2.1), a standard calibrated solar model was used to estimate the structure parameters (e.g., $\Delta\Pi_\ell$, $\eta_{n\ell m}$, $r_{\rm b}$, $N_{\rm t}$). The plume width and velocity, b and $V_{\rm b}$, were estimated using the turbulent model of plumes of Rieutord & Zahn (1995), and the plume lifetime was chosen around the turnover timescale of the convective eddies above the base of the convective zone such as predicted by the mixing length theory in the considered solar model. The result was then converted into a GOLF apparent mode radial velocity applying two multiplication factors: first, the mode mass that can be computed from the considered solar model using a pulsation code (e.g., Samadi et al. 2015); second, a visibility factor taking into account the effect of the limb darkening and the line-of-sight projection over the solar disk (e.g., Dziembowski 1977). For comparison, a theoretical detection threshold with the GOLF instrument as a function of the observation duration was estimated using the analytical development of Appourchaux et al. (2000) with a false alarm probability of 1%.

Without going into mode details, Pinçon et al. (2021) found that the result is very sensitive to the assumption made on the plume time evolution profile. In the case of a Gaussian time evolution, the gravity modes turn out to be undetectable because the temporal correlation between the plumes and the modes is definitely too small.

In the case of an exponential time evolution, the apparent mode radial velocity is estimated around 0.05 cm s^{-1} for $\nu \sim 100$ μHz, which is still one order of magnitude lower than the current GOLF detection threshold and the mode amplitude estimate considering the turbulent Reynolds stress as the driving mechanism by Belkacem et al. (2009). Considering uncertainties in the plume parameters, reasonable variations in their values in the most plausible favorable case (i.e., b and $\nu_{\rm p}$ are multiplied by a factor of about 2) can lead to an increase of the apparent mode radial velocity to 0.4 cm s^{-1} at $\nu \sim 100$ μHz, which still requires at least 25 years of GOLF observation to be detected.

Overall, these estimates, considering the excitation both by turbulent pressure and penetrative convection, indicate that the solar gravity modes are currently at most at the limit of the detection with the GOLF instrument in the asymptotic frequency range. Nevertheless, it is worth mentioning that a large amount of data other than that provided by the GOLF instrument is available too and form an important source of information to be analyzed, as for instance the observations by the GONG and BiSON ground-based telescope networks. From a theoretical point of view, it will be also important in the future to reduce the large uncertainties in the amplitude modeling in order to make a relevant comparison (e.g., the plume time evolution profile at the base of the convective zone). Another point will be to extend this result to a higher frequency range around 100 μHz and 400 μHz. While this is challenging since it requires to fully accounting for the complex interaction between the modes and the turbulent convection (e.g., Belkacem 2011), the solar oscillation modes in this frequency range have the advantage to be mixed modes. These modes can propagate in the inner radiative zone, where they behave as gravity modes, but also in the external layers, where they behave as acoustic modes. The solar mixed modes thus represent particular target as their surface amplitude is expected to be higher than the pure gravity modes at lower frequencies that are evanescent in the external layers of the Sun.

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