SEISMIC DIAGNOSIS FOR RAPIDLY ROTATING G-MODE UPPER-MAIN-SEQUENCE PULSATORS: THE COMBINED EFFECTS OF THE CENTRIFUGAL ACCELERATION AND DIFFERENTIAL ROTATION

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Abstract. We generalise the traditional approximation of rotation (TAR) to take into account simultaneously the centrifugal deformation in a non-perturbative way and a general 2D differential rotation. We show how they affect the pulsation-period spacings between consecutive g-mode pulsations and we discuss their detectability using high-precision asteroseismic data.

Keywords: hydrodynamics, waves, stars: rotation, stars: oscillations

1 Introduction

The traditional approximation of rotation (TAR) is a treatment of the hydrodynamic equations of rotating and stably stratified fluids in which the horizontal projection of the rotation vector is neglected. This treatment makes it possible to preform intensive seismic forward modelling. This gives access to properties of chemical stratification and to the rotation rate near the convective core/radiative envelope interface of rapidly rotating intermediate-mass stars thanks to the high precision of space-based photometric observations (Aerts 2021). This approximation is applicable for low-frequency gravito-inertial waves (GIWs) propagating in strongly stratified zones of uniformly rotating spherical stars (Bildsten et al. 1996; Lee & Saio 1997). However, it has been generalised first to include the effects of general differential rotation (Mathis 2009; Van Reeth et al. 2018) and then to take the centrifugal acceleration into account for slightly deformed stars using a first-order perturbative approach (Mathis & Prat 2019; Henneco et al. 2021). Here, we generalise the TAR to take into account simultaneously the centrifugal deformation in a non-perturbative way and the general differential rotation.

2 Generalised TAR

Spheroidal geometry To account for the centrifugal deformation in a non-perturbative manner, we use the spheroidal coordinate system (ζ, θ, φ) proposed by Bonazzola et al. (1998) linked to the usual spherical one via a mapping, where ζ is the pseudo-radius, θ the colatitude and φ the azimuth.

To treat the wave dynamics in differentially rotating, strongly deformed stars, we first derive the complete adiabatic inviscid system of equations in spheroidal coordinates (we adopt the Cowling and the anelastic approximations). Then, by assuming the hierarchies of frequencies: $2\Omega \ll N$ and $\omega \ll N$ (where Ω is the angular velocity, N the Brunt-Väisälä frequency and ω the frequency of the waves in the rotating frame) and the resulting velocity scales ($|v^\zeta| \ll \{|v^\theta|, |v^\varphi|\}$), we built the generalised framework for the TAR. In fact, we derive the generalised Laplace tidal equation (GLTE) for the normalised pressure $w_{\omega^{\rm in}km}$ (the generalised Hough functions) by adopting the JWKB approximation where m is the azimutal order, k is the index of an eigenmode, $\Lambda_{\omega^{\rm in}km}$ are the eigenvalues, ν the spin parameter and $x = \cos\theta$ the reduced latitudinal coordinate

$$\mathcal{L}_{\omega^{\rm in}m}\left[w_{\omega^{\rm in}km}\right] = \omega\partial_x\left[\frac{1}{\omega}\frac{1}{\mathcal{B}(\mathcal{E}-\mathcal{F})}\left(\mathcal{E}+\frac{\nu^2\mathcal{C}^2}{\mathcal{B}}\right)\left(1-x^2\right)\partial_x\right]w_{\omega^{\rm in}km} - \frac{m\mathcal{F}}{\nu\mathcal{C}(\mathcal{E}-\mathcal{F})}\partial_x w_{\omega^{\rm in}km}$$
$$+\left(m\omega\partial_x\left(\frac{\nu\mathcal{C}}{\omega\mathcal{B}(\mathcal{E}-\mathcal{F})}\right) - \frac{m^2}{(\mathcal{E}-\mathcal{F})\left(1-x^2\right)}\right)w_{\omega^{\rm in}km} = -\Lambda_{\omega^{\rm in}km}(\zeta)w_{\omega^{\rm in}km}, \quad (2.1)$$

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where $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ and $\mathcal{E}$ are the coefficients that describe the geometry and the rotation. This equation allows us to derive the asymptotic frequencies of GIWs which is a great asset to perform detailed seismic modelling.

3 Application to 2D ESTER models

To implement our equations, we use the ESTER model (Espinosa Lara & Rieutord 2013) the most advanced 2D model of stellar structure and evolution that takes both the centrifugal and Coriolis accelerations into account.

Validity domain: The generalised TAR is applicable in all the space domain to early-type stars rotating up to $\Omega/\Omega_{\rm K} = 0.2$ (Fig. 1, left panel). This formalism can be applied to all rapidly rotating bodies (stars of all types and planets) as long as we have a 2D model with access to the quantities that we used to solve the GLTE.

Period spacing pattern and detectability: The centrifugal acceleration and the differential rotation cause a shift in the period spacing pattern (Fig. 1, middle panel). In fact, the centrifugal effect is not important when assessed within the validity domain of the generalised TAR whereas the differential rotation effect is, theoretically, largely detectable using TESS and *Kepler* observations (Fig. 1, right panel). Despite their theoretical detectability, these effects will be subtle to detect in the real data because of their small amplitudes.

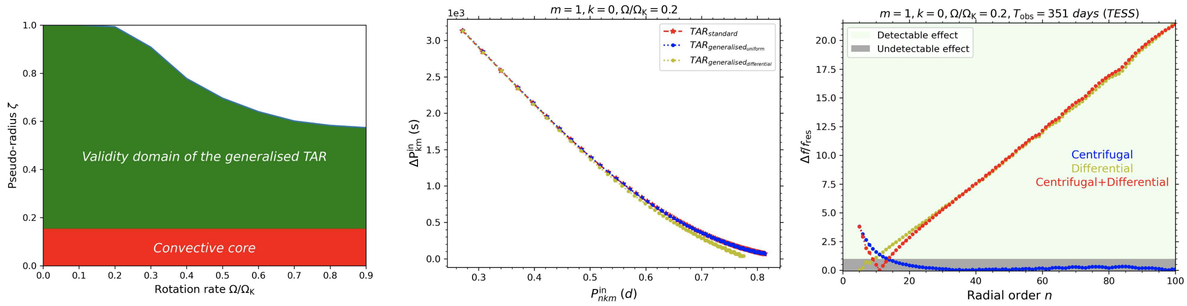

Fig. 1. Left: Validity domain of the generalised TAR for a $3\,{\rm M}_\odot$ ESTER model. **Middle:** Period spacing pattern in the inertial frame. **Right:** Detectable radial orders n for a $\{k = 0, m = 1\}$ mode using TESS.

4 Conclusion

A new generalisation of the TAR designed for 2D stellar models like ESTER, which takes into account simultaneously the differential rotation and the centrifugal acceleration in a non-perturbative way, is derived. This generalisation allows us to study the detectability and the signature of the centrifugal effects on GIWs in differentially rotating deformed early-type stars which are theoretically detectable in asteroseismic data.

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