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EXPONENTIAL AND FERMI'S PARADOX: LIMITS TO GROWTH

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Abstract. We now know that there are billions of (potentially habitable) terrestrial planets in the Milky Way, which only strengthens Fermi's paradox: why has no extraterrestrial civilization already conquered the Galaxy and visited Earth ? Such a conquest would be a natural consequence of an exponential development. However, every exponential phenomenon hits its limits at some point. The same should apply to the growth of the world Gross Domestic Product (GDP) and energy consumption (both being intimately linked), as numbers easily show.

A solution to Fermi's paradox is simply the limits to exponential growth. The fact that no little green man has visited us teaches us that we are soon to hit the limits (actually we already have). Meanwhile, the Covid-19 pandemic has shown that exponential phenomena require action before the situation becomes dramatic. Hence, simple maths and astronomy show that we need to stop with our exponential growth now.

Keywords: Exponential function, Fermi's paradox, growth

1 Fermi's paradox

Since 1995, more than 4000 exoplanets have been discovered, with a wide variety in mass, radius, orbital distance. From radial velocity studies (HARPS), Mayor et al. (2011) showed that about 14% of solar-type stars have a giant planet with an orbital period shorter than 10 years, and that more than half solar-type stars harbor at least one planet with an orbital period shorter than 100 days. From transit studies (*Kepler*) Bryson et al. (2021) estimate that the occurrence rate of terrestrial planets (between 0.5 and 1.5 Earth radius) in the habitable zone of solar type stars (between 4800 and 6300 K effective temperature) is of the order of 50%. This makes billions of such planets in the Milky Way, with the closest one probably within 6 parsecs of our Sun.

Looking at the conquest of space, from the first plane which flew a few hundred meters to the crossing of the Channel by Blériot, man in space, man on the Moon, *Viking* on Mars, *Cassini* at Saturn, *Voyager II* leaving the heliosphere, one finds a roughly exponential growth of the distance flown, which is multiplied by ten every decade. Extrapolating this relation (see Figure 1), we should reach 100 000 light-years (the diameter of the Milky Way) by the end of the century. Obviously this won't happen: we all know that the speed of light, c, can not be overpassed, so that traveling 100 000 light-years in less than 100 years is not possible. This is a first example of a physical limit to an exponential development.

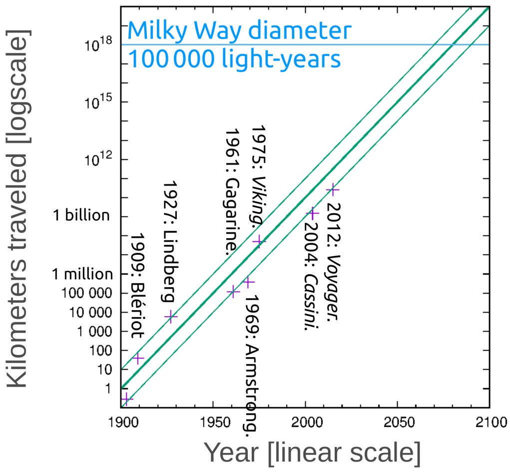

Fig. 1. Landmarks of the conquest of space showing the distance traveled increasing with time. Without using a logarithmic scale in the y-axis, the diagram would be impossible to read. The underlying trend corresponds to multiply the distance by 10 every 10 years, or equivalently double every 3 years.

Nonetheless, even if we limit ourselves to exploring the universe at $c/10$, it should take only a million years to visit the Galaxy. This is just a snap on the Universe clock. Though our planet, and our civilization, did not appear early in the history of the Milky Way, other planets and civilizations must have appeared long ago. So why didn't others do it before?

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2 The exponential function and limits

The exponential function is the solution to the simplest differential equation : $y' = ay$. Hence, many phenomena, in which the increase rate is proportional to the considered quantity, follow an exponential development. Using $e^{at} = 2^{t/T}$ where $T = \ln(2)/a \approx 0.7/a$, one can convert an exponential growth rate a into doubling time T, which is more visual and handy to conceive.

For instance, percentages describe exponential processes. To first order, a quantity that grows by $r\%$ per year doubles in $70/r$ years. The world Gross Domestic Product (the total amount of wealth produced per year) has doubled every 17 years since the industrial revolution in the mid XIX$^{\text{th}}$ century*, which corresponds to an average growth rate of 4% per year.

But it's not so easy to keep doubling. Takes a sheet of paper and fold it in two. You now have doubled its thickness. Do it again. The thickness is four times the initial one. Do it again, and again... Any guess of the thickness after 10 folds ? You won't get there, it's impossible to fold a normal A4 paper more than 7 times. If one could do it, it would only take 42 folds to reach the Earth-Moon distance ! Yes, $2^{42} \times 0.1\,\text{mm} \approx 4 \cdot 10^5\,\text{km}$.

Another well-known example is the tale of the waterlily that produces a new leaf from every leaf every night. If it covers half the lake in 30 days, how long will it take to cover the whole lake ? The simplest (linear) answer is "an other month", but the real answer is of course "one more day". What is less known is that doubling its area everyday, the waterlily would cover the entire Earth in less than two months...

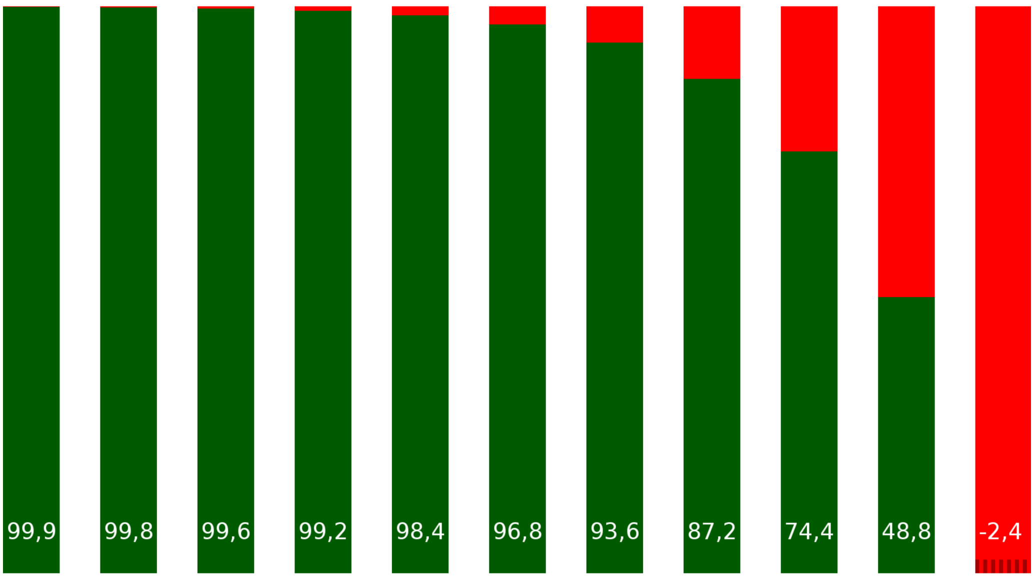

Fig. 2. Illustration of reaching the limits in an exponential development. A vertical bar represents 100, the total available for a quantity. Green (bottom) is the part left over, red (top) in the quantity used. Moving from left to right, the used quantity doubles every bar. At first, the green part is not affected, but it suddenly vanishes.

Finally, figure 2 illustrates why reaching the limits comes as a surprise. Say a quantity is limited to 100, and one takes initially 0.1 (leftmost column). Doubling the take to 0.2, what remains available hardly changes, decreasing from 99.9 to 99.8. The used quantity (in red in the figure) can keep doubling regularly many times without significantly affecting the available stock (in green). However, from the antepenultimate to the penultimate column, the rest drops by one third, from 74.4 to 48.8. One more doubling and there is nothing left.

While the behavior of the used quantity (in red) has not changed (doubling every time), the behavior of the available quantity (green) suddenly changes and collapses to zero. Similarly, folding a sheet of paper is easy, until the 6$^{\text{th}}$ fold and then it's impossible. After 31 days of quiet growth, the water lily finds itself blocked and asphyxiates the pond. Which brings us to the next section.

*As a consequence, the total amount of wealth produced in the XXI$^{\text{st}}$ century is already more than the total amount of wealth produced during all mankind history until and including year 2000 !

3 What about growth ?

Our economic growth has been an exponential phenomenon since the industrial revolution. But it relies on exploiting resources, and emitting waste. The capacity of the planet to provide resources and to absorb waste are finite. Hence, the limits are expected to be reached (Meadows et al. 1972, 1992, 2004, 2012). Let us focus on the energy demand, which is in linear relation with the GDP (e.g Parrique et al. 2019), and grows by 2% per year since 1990. The world total energy consumption was $5.67 \cdot 10^{20}$ J or 158 000 TWh in 2013. Increasing this number by 2% per year for 463 years multiplies it by 9600, and it reaches the whole energy the Earth receives from the Sun. In other words, if we move to renewable energies and keep our growth, we need to cover the entire planet by solar panels of 100% efficiency in a shorter time than what separates us from the Renaissance.

Assume we succeed. 1100 years later, we need the whole energy emitted by the Sun in all directions ! Very clearly, nuclear fusion plants on Earth can never provide this... Assume we manage to surround the Sun by ideal solar panels in a so-called Dyson-sphere ; 35 years later, we'll need an other star ! And 35 years later 2 new stars. In less than 3000 years, we will need all the stars of the Milky Way. Then what ?

An astrophysicist must conclude that growth can not be an everlasting phenomenon. And we see that the time scale is actually pretty short, comparable to human civilizations. So when should we stop ?

4 Exponential + delay = danger !

It is already well-known that we have overpassed the limits of Earth's capacity. But so far, so good, one could (almost) say. Unfortunately, the recent pandemic has taught us the hard way how acting too late against an exponential phenomenon can be a disaster.

In the case of the Covid-19 pandemic first wave, the development of the number of deaths was exponential, with a doubling time of 2.6 days in most countries. The way to stop the exponential propagation of the virus is the lock-down. But there is an average delay of two to three weeks between the time someone is infected by the coronavirus and the time the disease starts, develops, and eventually leads to death. Hence, after the lock-down, the number of casualties still grows exponentially, and eventually saturates in a month, as the last persons who got contaminated just before lock-down either heal or die. In every country, the final number of deaths after the first lock-down was about 80 times what it was at the moment of the lock-down. Because no one was ready to accept a lock-down when only a dozen people were killed, the final number of casualties reached several tens of thousands. Moving the time of the lock-down by only 2.6 days would have changed the number of victims by a factor 2.

In this case, in absence of tests, we could only see the consequences (the number of deaths), and act on the cause (the propagation of the virus). The delay between the cause and the consequence had disastrous effects.

Similarly, our emissions of greenhouse gases are following an exponential trend very similar to that of the world GDP and energy consumption. But the consequences of a stronger greenhouse effect (melt of polar caps and mountain glaciers, modifications of ecosystems, change in ocean currents, higher frequency of heat waves...) take decades to reach their final amplitude. We just start seeing the effects of the global warming, with more numerous and severe natural disasters. We are on the front of climate where we were on March 16, 2020 in France[†] on the front of Covid. To stop the exponential growth of our greenhouse gases emissions, action must be taken without delay.

5 Conclusion

The solution to Fermi's paradox is a *reductio ad absurdum* :

- Hypothesis (often implicit) : an everlasting exponential growth is possible without limits.
- Consequence : the conquest of the Milky Way is easy and quick.
- Contradiction : this has not been done by any of the billions of potentially habitable terrestrial planets in 12 billion years.
- Conclusion : the starting hypothesis is wrong, for some reason (e.g. energy resource limits).

[†]The lock-down started on March 17, 2020. Only 175 persons had then died from Covid-19.

Many other solutions have been proposed to Fermi's paradox in the litterature. Some are exotic, but most of them actually point to some specific limitation to the growth of a civilisation. Here, I have focused on the energy budget, which is clearly an upper limit. Anyway, if growth is limited, there is no way a civilization can leave its planetary system and communicate with, visit, or conquer planets around other stars. The paradox vanishes, but this teaches us a lesson: as our space conquest is progressing rapidly, we are soon to hit a wall.

Astronomy, because it forces to see the global picture, helps to understand that our planet is just a small pale blue dot and our civilization an epiphenomenon. With this in mind, it becomes clear that growth, which we consider as natural as the cycle of seasons because it dates back to the grand-parents of our grand-parents, is actually not granted. We have to imagine an other way of life, within the limits.

I thank the organisers of the S14 workshop "Transition environnementale: quel rôle pour la communauté astronomique ?".
Many thanks as well to the SF2A council for having allowed and encouraged this very interesting workshop to take place.
The oral presentation (in French) can be seen here (15 minutes): https://vimeo.com/560589409 .
A longer, outreach version (still in French) is available there: https://www.youtube.com/watch?v=I3JkQ7quGG8 .

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