SF2A 2021
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MULTIPLE STELLAR EVOLUTION: A POPULATION SYNTHESIS ALGORITHM TO MODEL THE STELLAR, BINARY, AND DYNAMICAL EVOLUTION OF MULTIPLE-STAR SYSTEMS

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Abstract. In recent years, observations have shown that multiple-star systems such as hierarchical triple and quadruple-star systems are common, especially among massive stars. They are potential sources of interesting astrophysical phenomena such as compact object mergers, leading to supernovae, and gravitational wave events. However, many uncertainties remain in their often complex evolution. Here, we present the population synthesis code *Multiple Stellar Evolution* (MSE), designed to rapidly model the stellar, binary, and dynamical evolution of multiple-star systems. MSE includes a number of new features not present in previous population synthesis codes: (1) an arbitrary number of stars, as long as the initial system is hierarchical, (2) dynamic switching between secular and direct N-body integration for efficient computation of the gravitational dynamics, (3) treatment of mass transfer in eccentric orbits, which occurs commonly in multiple-star systems, (4) a simple treatment of tidal, common-envelope, and mass transfer evolution in which the accretor is a binary instead of a single star, (5) taking into account planets within the stellar system, and (6) including gravitational perturbations from passing field stars. MSE, written primarily in the C++ language, will be made publicly available and has few prerequisites; a convenient PYTHON interface is provided. We give a short description of MSE and illustrate how to use the code in practice. We demonstrate its operation in a number of examples.

Keywords: binaries: general, stars: kinematics and dynamics, methods: statistical, gravitation,planets and satellites: dynamical evolution and stability, stars: evolution

1 Introduction

Multiple-star systems, stellar systems containing three or more stars, are common. They are usually arranged in a hierarchical configuration, since they would otherwise be short lived. The simplest hierarchical configuration occurs in triple systems in which two stars are orbited by a more distant, tertiary star. If the inner and outer orbits in such a configuration are initially mutually highly inclined, then the gravitational torque of the outer orbit can induce high-amplitude eccentricity oscillations in the inner binary, known as von Zeipel-Lidov-Kozai (ZLK) oscillations, which have important implications for a large variety of triple systems (von Zeipel 1910; Lidov 1962; Kozai 1962; see Naoz 2016; Shevchenko 2017; Ito & Ohtsuka 2019 for reviews). The dynamics become more complex for higher-order multiplicity systems. In hierarchical quadruples, secular evolution can be more efficient compared to triples (Pejcha et al. 2013; Hamers et al. 2015; Vokrouhlický 2016; Hamers & Lai 2017; Grishin et al. 2018a), and this can have implications for, e.g., short-period binaries (Hamers 2019), and Type Ia Supernovae (SNe Ia; Hamers 2018a; Fang et al. 2018). This trend carries over to higher-multiplicity systems (quintuples, sextuples, etc.), in which the likelihood for strong interactions due to secular evolution is even higher (Hamers 2020a).

Population synthesis codes, have been used extensively during the past several decades to study the evolution of predominantly binary stars (e.g., Whyte & Eggleton 1985; Portegies Zwart & Verbunt 1996; Izzard et al. 2009; Toonen et al. 2012). In particular, BSE (Hurley et al. 2002, hereafter HTP02), based on the rapid evolution algorithm SSE (Hurley et al. 2000, hereafter HPT00), has been an industry standard for nearly two decades. Also, BSE, and the SSE analytic stellar evolution tracks on which it is based, have formed the basis for many

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other codes. More recently, population synthesis codes have been developed that can model the evolution of triple stars taking into account both stellar/binary evolution, and gravitational dynamics (Hamers et al. 2013; Toonen et al. 2016). Current triple population synthesis codes face a number of limitations: mass transfer is not taken into account self-consistently, dynamics are often approximated, 'triple' interactions such as mass transfer from the tertiary onto the inner binary are not included, and objects are limited to be stars (i.e., planets cannot be included).

In these proceedings, we present a new population synthesis code, Multiple Stellar Evolution (MSE), aimed at modelling the stellar evolution, binary interactions (such as mass transfer and common-envelope, CE, evolution), and gravitational dynamics of multi-body systems. The core components of MSE are the stellar evolution fits of HPT00, several aspects of binary evolution adopted from HTP02, a self-consistent treatment of eccentric mass transfer, and accurate dynamical modelling using either secular or direct N-body integration. We briefly present the new evolution algorithm (Section 2; for more details, we refer to Hamers et al. (2021)) and give an example in Section 3. We conclude in Section 4.

2 The MSE algorithm

2.1 Dynamics

The MSE code models the evolution of an arbitrary number of stars in an initial hierarchical configuration. Gravitational dynamical evolution is taken into account via two methods: (1) secular (orbit-averaged) integration for sufficiently hierarchical systems, based on the formalism of Hamers & Portegies Zwart (2016); Hamers (2018b, 2020b), and (2) direct N-body integration for cases in which the secular approach breaks down using the algorithmic chain regularization code MSTAR (Rantala et al. 2020). The secular integrations include tidal evolution under the assumption of equilibrium tides (Hut 1981; Eggleton et al. 1998), where the efficiency of tidal dissipation is computed using the prescription of HTP02. The direct N-body integrations currently do not include additional acceleration terms that describe tidal evolution. However, collision detection is implemented. Both secular and direct N-body codes include post-Newtonian terms.

Often, the system is dynamically stable, and the secular approximation applies. However, evolutionary processes such as mass loss from stellar evolution can destabilise a system, or could void the validity of the secular approximation. In MSE, it is continuously checked whether the system is still stable using the stability criterion of Mardling & Aarseth (2001). However, other, less stringent conditions exist in which the secular approximation breaks down, namely when the secular time-scale becomes comparable to one of the orbital periods in the system (e.g., Antonini & Perets 2012; Luo et al. 2016; Grishin et al. 2018b; Lei et al. 2018; Hamers 2020b). The MSE code checks for this 'semisecular' regime by comparing the instantaneous time-scale for secular evolution to change the specific angular momentum of any orbit to the orbital period. If either a dynamical instability occurs or the semisecular regime is entered, MSE switches to direct N-body integration. Subsequently, the N-body system is analysed and if (and only if) a stable hierarchical (sub)configuration is identified for the entire system (this can include unbound objects), it will switch back to secular integration.

2.2 Stellar evolution

Stellar evolution in MSE is modelled using fast analytic fitting formulae to detailed stellar evolution models from HPT00. These tracks include information on large-scale parameters such as total mass, radius, luminosity, and global properties of the core (if present). Also included is mass loss due to stellar winds, and spin-down due to magnetic braking. Mass loss due to stellar winds is assumed to affect the orbits adiabatically, i.e., with $m_{\rm enc}a_i$ and e_i constant, where $m_{\rm enc}$ is the enclosed mass, and a_i and e_i are the orbital semimajor axis and eccentricity, respectively.

When stars evolve to become NSs or BHs, we assume that mass is lost instantaneously (i.e., the opposite from the adiabatic regime), and take into account the effect of the mass loss on all orbits in the system, assuming no interaction with the lost mass. We also take into account natal kicks for NSs and BHs, by adopting several models for sampling the velocity kick when compact objects are formed (see Hamers et al. 2021). In the default model, 'kick distribution model 1', the natal kick speeds are drawn from a Maxwellian distribution with dispersion $\sigma_{\rm kick} = 265\,{\rm km\,s^{-1}}$ for NSs, Hobbs et al. 2005) and $\sigma_{\rm kick} = 50\,{\rm km\,s^{-1}}$ for BHs.

2.3 *Binary interactions*

The MSE code includes a number of binary interactions. We check for the condition of Roche lobe overflow (RLOF) for all stars onto companions. If the companion is a single star, 'binary' mass transfer applies; otherwise, 'triple' mass transfer or triple CE could occur (see Section 2.4). In the binary case, details of the mass transfer process such as the mass transfer rate, aging/rejuvenation, and the conditions for unstable CE evolution are modelled using similar prescriptions as those used by HTP02. However, an important difference from HTP02 is the orbital response to mass transfer: HTP02 assumed that tides are always efficient enough to circularise the orbit at the onset of mass transfer. This assumption can break down in triple or higher-order systems, in which eccentricity can be excited secularly (e.g., Toonen et al. 2020). MSE includes the analytic model of Hamers & Dosopoulou (2019) to describe the orbital response to mass transfer in eccentric orbits. Unstable mass transfer can lead to CE evolution; this is taken into account in MSE by adopting the α_{CE}-λ prescription, similar to HTP02. Prescriptions for the outcomes of CE evolution are also adopted from HTP02. In close orbits, accretion of material from stellar winds onto companions can be important. MSE includes this process of wind accretion by adopting the Bondi-Hoyle-Lyttleton formalism (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944).

2.4 *Triple interactions*

In some systems, an outer star can fill its Roche lobe around an inner companion consisting of two stars (in contrast to 'binary' mass transfer). This type of evolution, which can result in stable transfer onto the companion binary, or unstable 'triple CE' evolution, is still poorly understood. MSE models this phase by adopting a number of simplified prescriptions, motivated by more detailed simulations (de Vries et al. 2014; Comerford & Izzard 2020; Glanz & Perets 2021).

2.5 *Fly-bys*

The effects of passing stars in the field (i.e., low-density environments) are taken into account in MSE by sampling interloping stars during the simulation with a Monte Carlo approach. Based on the stellar density and relative velocity dispersion, perturbers are sampled that impinge on an 'encounter sphere' with a large radius. The effects of the perturber on the multiple system are then computed for impulsive encounters (the latter are most important in the field, although secular encounters dominate in dense stellar systems such as globular clusters, see, e.g., Heggie & Rasio 1996; Hamers & Samsing 2019).

3 Example system

We show an example system evolved with MSE involving RLOF and CE evolution in a stellar triple. In Fig. 1, we show the evolution of the masses (top row), orbital separations and stellar radii (middle row), and stellar types (bottom row). We also show important events during the evolution of the system in the form of a mobile diagram (Evans 1968) in Fig. 2. During the MS, high-amplitude ZLK oscillations are induced in the inner binary, but they are not sufficiently strong to induce interaction. As the $3\,\mathrm{M}_\odot$ primary star evolves to an AGB star, it fills its Roche lobe around its $2\,\mathrm{M}_\odot$ companion. The donor is then stripped of its envelope, and the core turns into a CO WD which is orbiting the companion (still an MS star) in a more compact orbit. In this example, we focus on the early evolution. However, at later times, the inner orbit could undergo further interaction (e.g., produce a cataclysmic variable).

4 Conclusions

MSE is a new population synthesis code which can be used to quickly model the stellar, binary, and gravitational dynamical evolution of hierarchical multiple systems with any number of stars. The gravitational dynamics are taken into account using either a secular approach (Hamers & Portegies Zwart 2016; Hamers 2018b, 2020b), or direct N-body integration (Rantala et al. 2020). Stellar evolution is taken into account by adopting the SSE fitting functions (HPT00), whereas binary interactions are modeled using semi-analytic models and prescriptions. New features of MSE in comparison to previous population synthesis codes include (1) an arbitrary number of stars, as long as the initial system is hierarchical, (2) dynamic switching between secular and direct N-body integration for efficient computation of the gravitational dynamics, (3) treatment of mass transfer in eccentric orbits, which occurs commonly in multiple-star systems, (4) a simple treatment of tidal, common-envelope, and

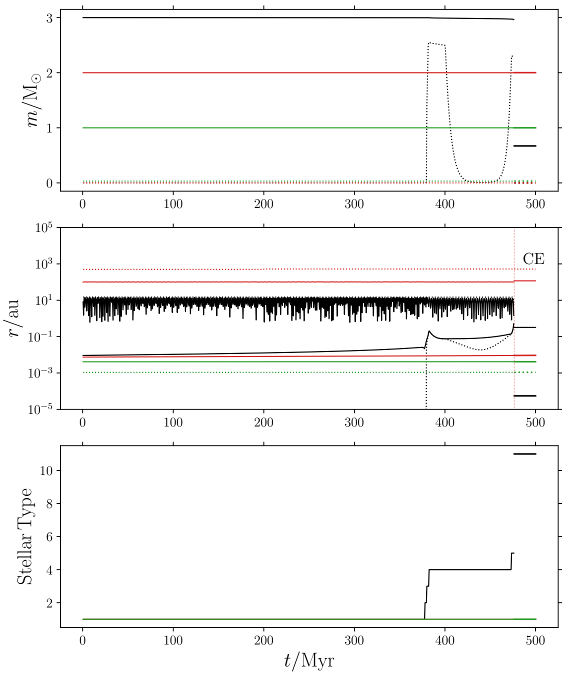

Fig. 1. Evolution of the masses (top row), orbital separations and stellar radii (middle row), and stellar types (bottom row) for the example triple system discussed in Section 3. In the top panel, the three masses are shown with solid black, red, and green lines, respectively. The convective core radii of the corresponding stars are shown with dotted lines. In the middle panel, the bottom solid lines show the stellar radii, with the same colours used as in the top panel. The black and red lines in the top and middle part of the panel show the orbital separations (solid: periapsis distances; dotted: semimajor axes) of the inner and outer orbits, respectively. The onset of a CE event in the inner binary is indicated. The bottom panel shows the evolution of the stellar types (see HPT00), with the same colours used as in the top panel.

mass transfer evolution in which the accretor is a binary instead of a single star, (5) taking into account planets within the stellar system, and (6) including gravitational perturbations from passing field stars. At the time of writing, MSE is part of a private repository on `GitHub`*. Access to this repository can be requested by contacting the author. In the future, the repository will be made publicly available.

A.S.H. thanks the Max Planck Society for support through a Max Planck Research Group.

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*`https://github.com/hamers/mse`.

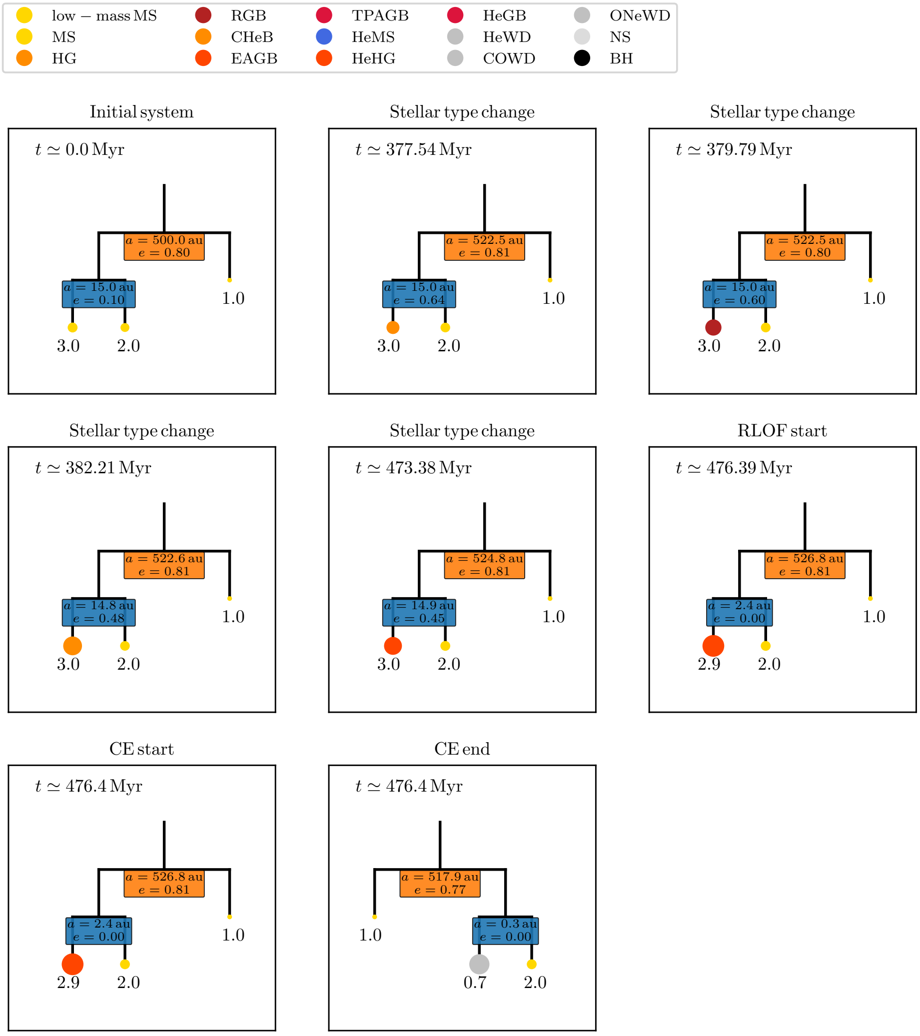

Fig. 2. Mobile diagrams for the example triple system discussed in Section 3. The title of each panel gives a description of the event that occurred. The semimajor axes and eccentricities are indicated at each orbit. Numbers next to stars show the masses of the objects (in $M_\odot$). The colours of the stars depend on the stellar type; see the legend at the top of the figure.

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