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A HYBRID NUMERICAL APPROACH TO MODEL PULSAR MAGNETOSPHERES

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Abstract. The study of pulsar magnetospheres has developed quickly in recent years thanks to the development of high-performance computing. Two complementary numerical methods have been used to model these objects thus far: the magnetohydrodynamic (MHD) and the particle-in-cell (PIC) techniques. The MHD approach is well-suited to describe the plasma at large scales, while the PIC method is appropriate to capture the microphysics but it is computationally expansive. Our objective is to combine the strengths of both approaches into the same numerical framework in order to achieve a larger scale separation and magnetic field strength. This approach will allow us to make realistic predictions of particle and electromagnetic spectra for pulsar, therefore bridging the gap between observations and ab-initio models. In this contribution, we present the first application of this approach to the aligned pulsar magnetosphere.

Keywords: pulsars: general, magnetosphere, relativistic, Methods: numerical

1 Introduction

A pulsar is a rotating and highly magnetized neutron star. It has a radius $r_* \approx 10\text{km}$, a mass $M_* \approx 1.4 M_\odot$, a surface magnetic field $B_* = 10^9 - 10^{14}\text{G}$ and a spin period $P = 2\pi/\Omega = 1\text{ms} - 1\text{s}$. In these objects, the misalignment between the magnetic and rotation axis leads to a pulsed emission. This emission is associated with phenomena happening close to the pulsar in what is called the magnetosphere. The magnetosphere can be loosely decomposed into two main regions: (i) a corotating zone where most field lines are closed, and (ii) the wind zone composed of open, toroidal-dominated field lines. Opened field lines participate in the activity of the magnetosphere and spin down the pulsar. The transition between both regions occurs at the light-cylinder radius, $R_{\rm LC} = c/\Omega$, where the corotation velocity equals the speed of light c. Charges are extracted from the neutron star surface by the electric field induced by the fast rotation of the star. Copious pair production ensures that the magnetosphere is filled with plasma. Because of the strong magnetic field, all forces other than the Lorentz force can be neglected such that the magnetosphere is close to being force-free. The force-free approximation breaks down within the current sheet that forms in the wind at the interface between both magnetic polarities. Magnetic reconnection within the current sheet leads to particle acceleration and non-thermal radiation. Thus, studying the dynamic of the magnetosphere is of interest to connect the model with observations.

Numerical simulations are required to model the magnetosphere. Two numerical approaches are currently being used in the community: Force-Free (FF) simulations and particle-in-cell (PIC) simulations. The first one directly translates the electromagnetic nature of the problem by using magnetohydrodynamic equations in the force-free limit. This type of simulation is computationally cheap but dissipation and all the microphysics are lacking. In contrast, PIC simulations capture all the microphysics (particle acceleration, pair creation, radiation) accurately but they are computationnaly expansive. Therefore, combining the strengths of both approaches is attractive, and would allow one to perform larger simulations and increase the predictive power of current models.

In this paper, we present a new hybrid method combining PIC and FF in the same numerical framework using the *Zeltron* code (Cerutti et al. 2013). In the section 2, we introduce the concept of the hybrid setup and how the interface between both approaches is done. In the section 3, we show preliminary results of this new approach applied to the magnetosphere of an aligned pulsar.

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2 Hybrid method

The hybrid method couples the FF and the PIC approaches. In both methods, the electromagnetic fields ($\mathbf{E}$,$\mathbf{B}$) are evolved in time through Maxwell-Ampère and Maxwell-Faraday equations. These equations are numerically solved with the standard finite-difference time-domain algorithm (Yee 1966). They differ in the way the current density $\mathbf{J}$ is computed. In FF, the current density is inferred from the force-free condition,

$$\rho\mathbf{E} + \frac{1}{c}\mathbf{J} \times \mathbf{B} = 0,$$

where $\rho = \nabla \cdot \mathbf{E}/4\pi$ is the charge density. In the PIC approach, $\mathbf{J}$ is directly reconstructed from the motion of a large number of charged particles. Hence, the coupling between these two methods can be achieved via the current.

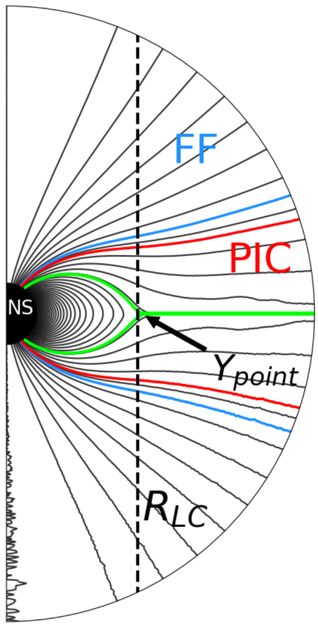

Fig. 1. Example of a domain separation in the hybrid setup. The blue line corresponds to $\Psi = \Psi_0$, the red line to $\Psi = \Psi_1$, the green line represents the separatrix and the current sheet respectively inside and outside the light cylinder (black dashed line).

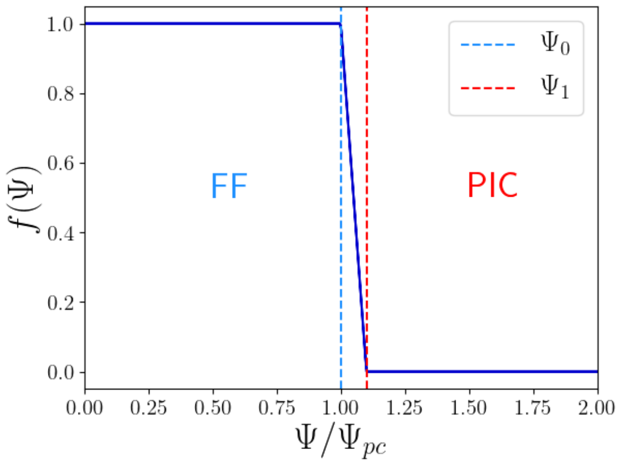

Fig. 2. Shape of the mask used for the transition layer between the two numerical domains. The blue and red dashed lines delimit respectively the start and the end of the transition layer (Ψ_0 and Ψ_1).

In Fig. 1, we provide a schematic view of a pulsar magnetosphere. In this figure, only the separatrix (at the interface between the closed and open field lines inside $R_{\rm LC}$) and the equatorial current layer regions deviate from the force-free regime. The Y-point is the location where the separatrix merge to form the equatorial current layer. At and post this point, magnetic reconnection operates and accelerates particles efficiently. The PIC approach is needed in this zone. The footpoint of the separatrix is located near the star polar cap and represents the transition between open and closed field lines. Outside of the current sheet and the separatrix regions, the full domain is in the force-free regime and well described by the FF approach. The boundary between the PIC and the FF domains is chosen as an isocontour of the magnetic flux function, Ψ, which is defined as

$$\Psi = \frac{1}{2\pi} \iint \mathbf{B} \cdot \mathrm{d}\mathbf{S}. \tag{2.1}$$

This choice is convenient in this setup because isocontours of Ψ follow poloidal magnetic field lines in MHD axisymmetric. In our hybrid method, Ψ is computed at every timestep to follow the behaviour of the magnetosphere such that the frontiers between the PIC and the FF domains evolve dynamically. In this hybrid setup, the total current density is computed as

$$\mathbf{J} = \mathbf{J}_{\rm PIC}\,(1 - f(\Psi)) + \mathbf{J}_{\rm FFE} f(\Psi), \tag{2.2}$$

where $\mathbf{J}_{\rm PIC}$ is the current density inferred from the particles, $\mathbf{J}_{\rm FFE}$ is the current density from the force-free region, and $f(\Psi)$ is the mask function. This function is equal to 1 in the FF region and is equal to 0 in the PIC zone (see Fig. 2). To avoid numerical artefacts at the boundaries, there is a buffer layer between Ψ_0 and Ψ_1 where both methods overlap. A linear interpolation suffices to ensure a smooth transmission between both domains. Ψ_0 and Ψ_1 are scaled to the value of the flux function at the polar cap $\Psi_{\rm PC} = B_* r_*^3/R_{\rm LC}$. The thickness of the layer is $\Psi_1 - \Psi_0 = 0.1\Psi_{\rm PC}$.

3 Results

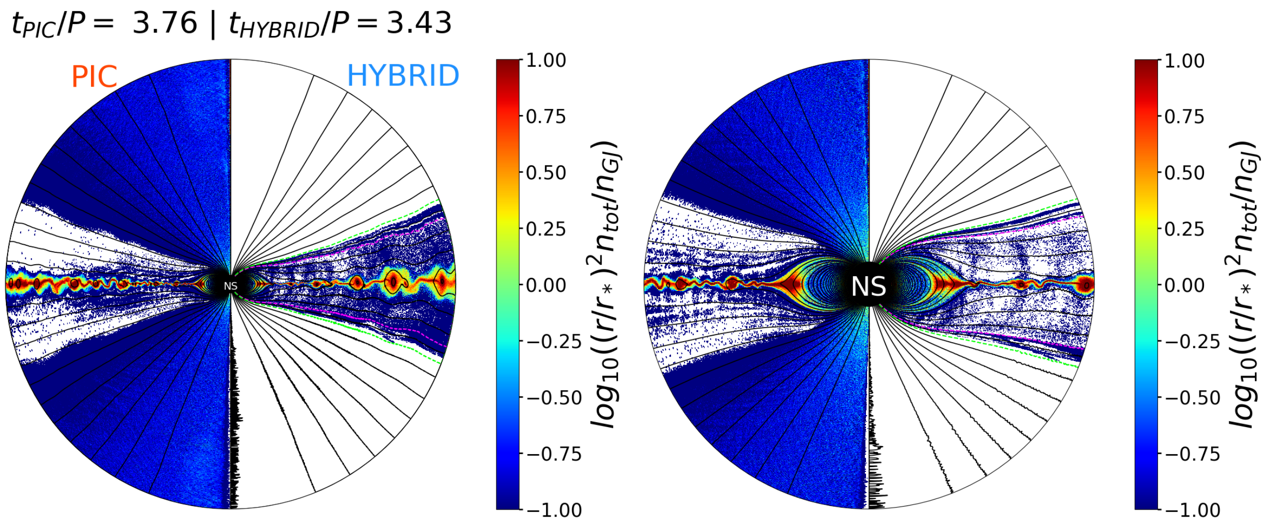

Fig. 3. Left: Map of the total plasma density. **Right:** Zoom-in view on the inner regions (within $2R_{\rm LC}$). Black lines are the magnetic field lines, green and magenta lines correspond to the transition layer as seen in Fig. 2. Densities are normalized by the Goldreich-Julian density $n_{\rm GJ}$ (Goldreich & Julian 1969)

To check whether the hybrid method works, we compare the results of our simulation to a full PIC simulation with the same input parameters. In Fig. 3, we compare side by side the total density maps of pairs (protons are not included in these simulations). The overall magnetospheric shape is consistent with the pure PIC and FF results (Cerutti et al. 2015; Spitkovsky 2006). Our analysis of the fields shows that we have a smooth transition between both domains. One of the most visual differences between the simulations is regarding the poles. With the hybrid method, we remove the flow of electrons from both poles by switching to the FF method. We retrieve in our hybrid simulation the large-scale current sheet within the PIC domain. The tearing instability leads to the fragmentation of the sheet into plasmoids. We have vigorous pair creation in both cases. We have also checked that a steady-state has been achieved in the hybrid simulation after a few rotation periods.

Fig. 4 shows the Poynting flux which carries away the pulsar spindown power in the form of an electromagnetic power as a function of radius. A fraction of this flux is dissipated and channeled into non-thermal particle acceleration. There is about a 10% difference in total Poynting flux between the hybrid and the full PIC solutions, but the amount of dissipation is compatible (about 20% at $r = 5R_{\rm LC}$).

4 Conclusions

We introduced a new hybrid method combining the FF and the PIC approaches in the same numerical framework. Preliminary results are encouraging: hybrid simulations retrieve the overall behavior of the magnetosphere, the current sheet and the formation of plasmoids. This method does not provide additional dissipation and we recover the one from the reference PIC simulation. The next steps will be to scale simulations up to unprecedented systems sizes to sharpen the predictive power of simulations, in particular regarding the particle and radiation spectra. This method will also be adapted for full general relativistic simulations and applied to black hole magnetospheres.

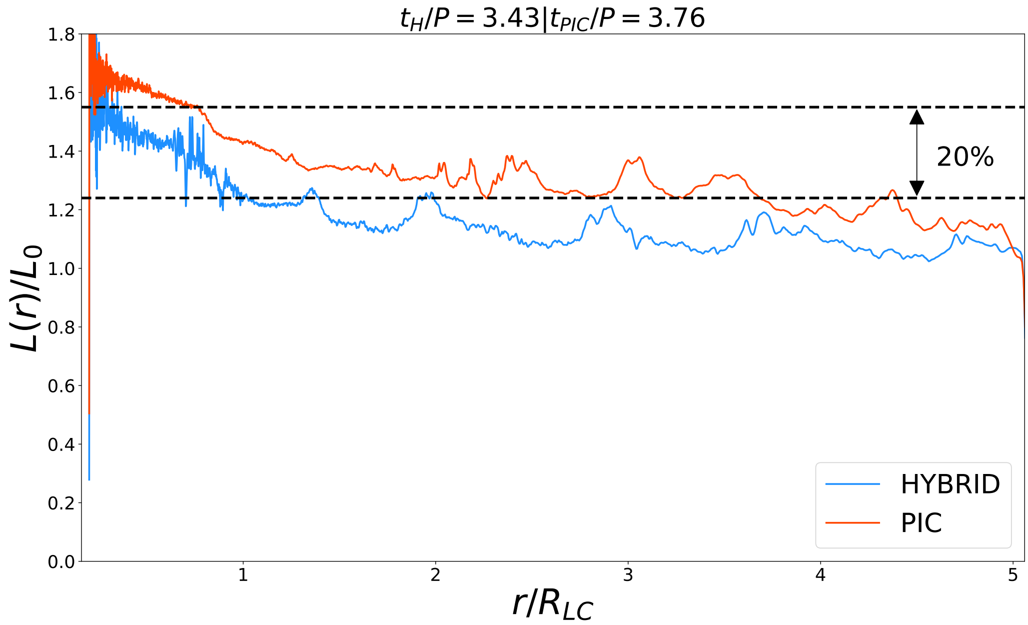

Fig. 4. Poynting flux at the end of the simulation ($t/P = 3.76$). The orange line corresponds to the Poynting flux of the reference PIC simulation, the blue line corresponds to the hybrid case.

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