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MODELING THE LUNAR TIDES IN INPOP

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Abstract. The Moon is affected by tides, mainly exerted by the Earth and the Sun, that deforms its shape. Studies of its tide through its rotation allows to probe its inner structure. In current planetary and lunar ephemerides, the lunar visco-elastic tides are computed with a time delay model in order to take into account the phase lag of the tidal bulge. The time-delay is introduced in the lunar moment of inertia through the distortion coefficients of the harmonics of degree two of the gravitational potential. We use an alternative method in the INPOP ephemerides which consists in formulating the distortion coefficients with the Fourier series introduced by Williams & Boggs (2015). This formulation allows to introduce any variation of the Love numbers with respect to the period of the tidal forcing. We show that we need to have around five hundred terms in the Fourier series in order to reach an amplitude of the residual caused by the truncation of the series at the level of the observational accuracy of Lunar Laser Ranging data.

Keywords: Moon, tides, ephemerides, INPOP

1 Introduction

Since the start of the Artemis program, interest in lunar studies has been renewed. The Lunar Laser Ranging (LLR) experiment measures the Earth-Moon distance with an accuracy of around two centimeters and the Moon's librations at a one milliarcsecond accuracy (Viswanathan et al. 2018). This allows to provide a refined description of the tidal deformation of the Moon. The ephemeris INPOP of the Paris Observatory is a joint numerical integration of the orbits of the Moon and the planets as well as the lunar rotation, which is fitted to the LLR data. Studying the lunar tides allows us to probe the internal composition of the Moon. For example, recent results from tidal constraints highlight the presence of a solid inner core (Briaud et al. 2023b). The tidal response depends on the density and the rheology of the layers, and on the dissipation due to the viscosity of the lunar interior (e.g. Williams & Boggs (2015), Briaud et al. (2023a)). The tidal Love number k_2 and the dissipation inside the Moon depend on the period of the tidal forcing.

In INPOP the tidal deformation accounts for a k_2 independent of the excitation frequency and a unique time delay (Viswanathan et al. 2018). The formulation in Fourier series of the distortion coefficients (also called variation of the Stokes coefficients) by Williams & Boggs (2015) allows us to describe the tidal gravitational variation by taking into account the frequency dependency. We introduce the distortion coefficients as Fourier series in order to test the impact of the variation of the Love number and of the dissipation on libration measurements.

2 Method

In the case of an elastic lunar tides, the deformation of the Moon is instantaneous and the tidal bulge is oriented in the Earth-Moon direction. However, in the case of the visco-elastic deformation, the tidal bulge is shifted with respect to the Earth-Moon direction. In order to take into account this shift, in current ephemerides, the distortion coefficients are computed with the time-delay model. The tidal Love number k_2 quantifies the tidal deformation of the gravitational potential at harmonics of degree 2. In the visco-elastic case, the tidal Love number is complex and noted k_2^*. Its real and imaginary parts can be written

$$Re(k_2^*) = |k_2^*|\sqrt{1 - 1/Q^2}, \tag{2.1}$$

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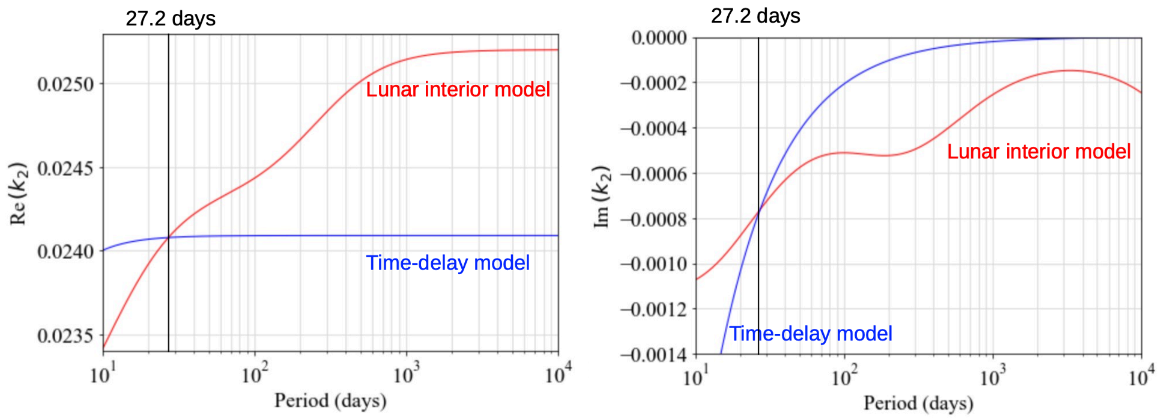

Fig. 1. Left: Variation of the real part of the tidal Love number with respect to the period of the tidal forcing. **Right:** Variation of the imaginary part of the tidal Love number with respect to the period of the tidal forcing..

$$Im(k_2^*) = -\frac{|k_2^*|}{Q}, \tag{2.2}$$

where Q is a dissipation factor. In the time-delay model, the dissipation factor is linked to the time-delay parameter through the relation

$$Q = \frac{1}{\sin(f\tau)}, \tag{2.3}$$

where f is the frequency of the tidal forcing.

The figure 1 shows the variation of the Love number with respect to the period of the tidal forcing. The left and right figures show respectively the variation of the real part and of the imaginary part. The blue curves show the variation for the time-delay model and the red curves show the variation from a solution provided by a lunar interior model computed by Briaud et al. (2023a). The time delay model does not allow to reproduce a Love number variation similar as in a solution from a lunar interior model. The time-delay model allows to have a realistic value of the Love number only for the dominant terms near 27 days.

The tidal contribution in the moment of inertial I_{tides} is related to the gravitational potential through the relation :

$$I_{tides} = MR^2 \begin{pmatrix} \frac{1}{3}\Delta C_{20} - 2\Delta C_{22} & -2\Delta S_{22} & -\Delta C_{21} \\ -2\Delta S_{22} & \frac{1}{3}\Delta C_{20} + 2\Delta C_{22} & -\Delta S_{21} \\ -\Delta C_{21} & -\Delta S_{21} & -\frac{2}{3}\Delta C_{20} \end{pmatrix} \tag{2.4}$$

where ΔC_{2m} and ΔS_{2m} (with $m = 0$, 1 or 2), are the distortion coefficients of the gravitational potential for the harmonics of degree two, M is the mass of the Moon and R its radius. Williams & Boggs (2015) provided a Fourier series formulation of the distortion coefficients :

$$\Delta C_{2m}(t) = \sum_q C_{2mq} \left(Re\{k_2^*(P_q)\} \cos\zeta_q(t) - Im\{k_2^*(P_q)\} \sin\zeta_q(t) \right), \tag{2.5}$$

$$\Delta S_{2m}(t) = \sum_q S_{2mq} \left(Re\{k_2^*(P_q)\} \cos\zeta_q(t) - Im\{k_2^*(P_q)\} \sin\zeta_q(t) \right). \tag{2.6}$$

The advantage of such a formulation is that it allows to separate the different periods of the tidal forcing and to associate for each term its associated Love number.

3 Results

Figure 2 shows the residual for the distance to the retroreflector Apollo 11 (A11) between the solution of INPOP19a and the solution of the version of INPOP with the distortion coefficients computed in Fourier series. The Fourier series contain the terms presented in Williams & Boggs (2015). The amplitude of the residual reach two meters, which is well above the observational accuracy of two centimeters provided by the LLR data.

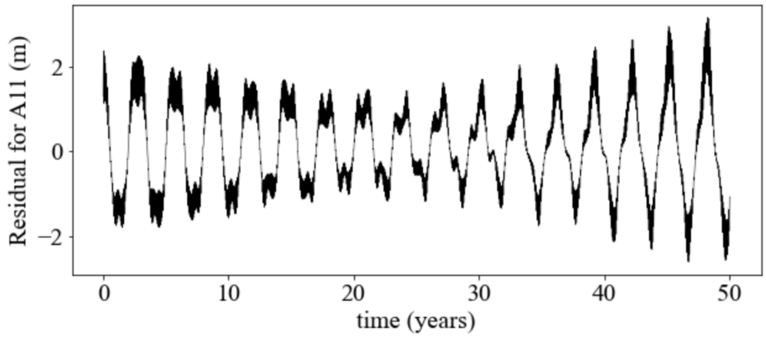

Fig. 2. Residual of the distance to the retroreflector A11 in the case where the Fourier series contain the termes of Williams & Boggs (2015).

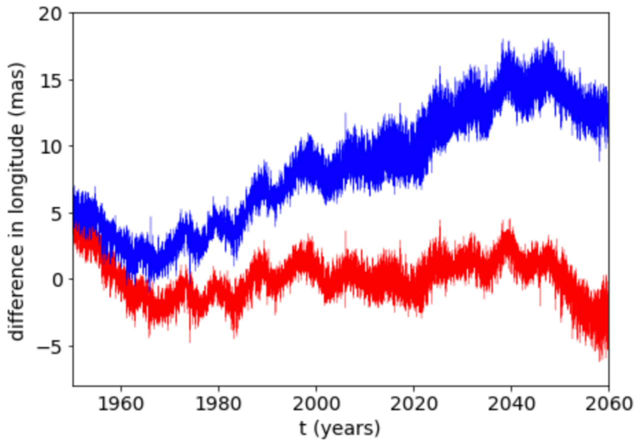

Fig. 3. Residual of the longitude of the Moon in spherical coordinates. The blue and red curves are for the cases with the secular parameters fitted respectively to DE405 and INPOP19a.

The terms of the series computed by Williams & Boggs (2015) are based on the ephemerides DE of JPL. We computed the terms of the series based on the solution of INPOP19a. In order to do so, we used the Paris Lunar Ephemeris (ELP) which is a semi-analytical representation of the lunar orbit. The last version of ELP is called ELP/MPP02 and has been fitted to DE405 (Chapront & Francou 2003). We fit the secular parameters of the Moon of ELP to the solution of INPOP19a. Figure 3 shows the residual of the longitude of the Moon in spherical coordinates between the solution of INPOP19a and the solution of ELP. The slope of the residual for the case of ELP fitted to DE405 is eliminated thanks to the fit of the secular parameters. However, the difference induced on the distance to the retroreflector by this latter solution with respect to the solution with the series of Williams & Boggs (2015) is only at the submillimeter level.

We shown that the amplitude of the residual in figure 2 is mainly due to the truncation of the series. Figure 4 shows the residual for different cases with different number of terms in the series. We need to include around five hundred terms in the Fourier series of each distortion coefficient in order to have a residual on the distance to the retroreflectors that is at the level of the accuracy of the Lunar Laser ranging data.

Now that we have reduced the residual due to the truncation of the series, we can introduce the frequency variation of the Love number according to a solution from a lunar interior model. Figure 5 shows that the effect on the proper rotation of the Moon introduced by the variation of the Love number according to a solution from a lunar interior model is much more important than the effect of the truncation of the Fourier series. The terms at 206 days, 365 days and 1095 days are in particular excited because of the difference of their associated Love number.

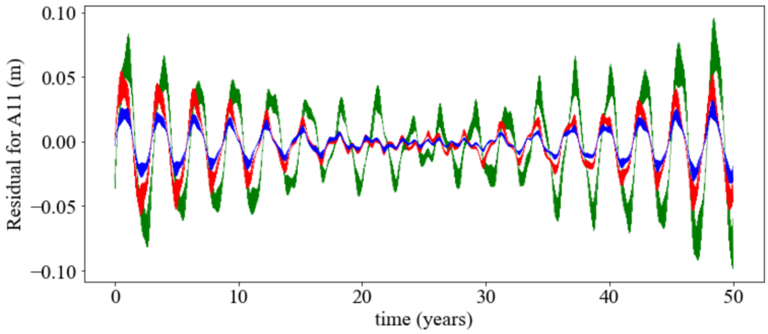

Fig. 4. Residual of the distance to the retroreflector A11. The green, red and blue curves are respectively for the cases with one hundred terms, two hundred terms and five hundred terms.

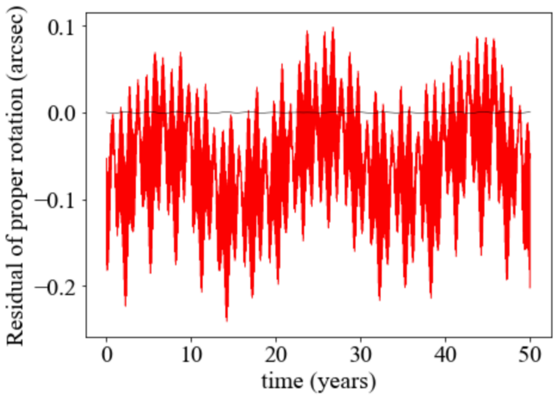

Fig. 5. The black curve shows the residual only due to the effect of the truncation of the Fourier series and the red curve shows the residual due to the effect of the variation of the complex tidal Love number according to a solution from a lunar interior model

4 Conclusions

We introduced in the ephemerides INPOP, the formulation in Fourier series of the distortion coefficients of Williams & Boggs (2015). We have fitted secular parameters of ELP to the solution of INPOP19a. From this solution of ELP fitted to INPOP19a, we have computed the terms of the Fourier series of the distortion coefficients. We find that the difference induced by this last solution of the series with respect to those of Williams & Boggs (2015) is at the submillimeter level and is negligible. We shown that we need to add up to around five hundred terms in the series in order to reduce the residual of the distance to the retroreflectors at the level of the observational accuracy. We are able to introduce the Love number variation with respect to the period of the tidal forcing according to lunar interior models. In future work we will test several solutions of Love number variation provided by interior models in order to test which models are abe to absorb the residuals.

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