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SECULAR STABILITY OF THE DIPOLE-DIPOLE INTERACTION IN MAGNETIC BINARIES

C. Aykroyd[1], A. Bourgoin[1, 2], C. Le Poncin-Lafitte[1], S. Mathis[2] and M.-C. Angonin[1]

Abstract. The existence of robust, extensive, and stable magnetic fields in a substantial fraction of early-type stars (mainly Ap, Bp, and O spectral types), white dwarfs, and neutron stars, has been firmly established. However, the mechanisms that give rise to these fields continue to be actively explored, with proposed explanations ranging from fossil fields to mergers and shear-driven dynamos. It is possible that the interplay between magnetism and binarity could provide insights into the origin of these fields, given the significant impact of magnetic fields on the long-term dynamics of binary systems. The purpose of this investigation is to analyze the secular spin precession behavior of binary systems under the influence of purely magnetic dipole-dipole interactions, focusing on stars with predominantly dipolar, strong, and stable magnetic fields. We utilise the Lie series approach to develop an effective secular model for the spin precession equations. The spin equilibrium configurations and their associated stabilities are obtained by minimizing the magnetic interaction energy of the system. Our findings demonstrate the existence of a single globally stable secular state among the four possible equilibrium states. This state corresponds to the configuration where the magnetic axes of one star are reversed relative to those of the companion, and are orthogonal to the orbital plane. We compare our results to conventional methods of determining instantaneous equilibrium states, which typically neglect the effects of orbital motion.

Keywords: magnetic fields, celestial mechanics, massive stars, white dwarfs, neutron stars

1 Introduction

Strong magnetic fields are found across a range of massive main-sequence stars (MMSs), white dwarfs (WDs), and neutron stars (NSs), and play a role in their evolution. Yet, understanding the origins of these fields remains a debated topic. Pure dynamo hypotheses are discouraged, since the transport of magnetic fields from the convective core across radiative zones of massive stars is a formidable challenge, with diffusivity timescales estimated to be longer than the main-sequence lifetime of the star (Moss, D. 2003). On the converse side, spectropolarimetry observations support the presence of strong and stable fields, more famously in the case of Ap and Bp stars (Ferrario et al. 2015). The so-called fossil mechanisms propose magnetic fields that are relics of a previous stage of a star's evolution and preserved by flux conservation. In the most basic scenario, this would be flux trapped in an interstellar cloud at the time of the star's birth. Nevertheless, for WDs and NSs, fossil magnetic fields are thought to be inherited from their progenitor stars, often A, B, or O-type stars. While this hypothesis seems to account for the observed strong magnetic fields, it raises intriguing questions. Notably, observations have revealed a dearth of weakly magnetic MMSs, indicating that these stars typically possess either strong fields or none at all. To potentially explain such bimodal distribution, studies have suggested the existence of thresholds to field strength, below which shear or convection instabilities develop (see e.g. Aurière et al. 2007); or that, in some stars, the time needed to reach an equilibrium becomes longer than the age in the main sequence, due to the Coriolis force produced by rapid rotation (Braithwaite & Cantiello 2013). Observations seem to indicate significantly lower magnetic incidence in binary systems, which may point towards other field formation mechanisms, specifically those linked to binarity. Commerçon et al. (2010) argues that interstellar clouds with strong magnetic fields are harder to fragment, yielding selection biases towards

[1] SYRTE, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, LNE, 61 avenue de l'Observatoire, 75014 Paris, France
[2] Université Paris-Saclay, Université Paris Cité, CEA, CNRS, AIM, F-91191, Gif-sur-Yvette, France

less magnetic binary systems. Vidal et al. (2019) suggests instead that tidal instabilities in binary pairs can disrupt the magnetic fields via turbulent Joule diffusion within a few millions years, potentially explaining the scarcity of strong-field binaries. The merging scenario posits an alternative explanation: that the shear produced during the coalescence of protostars leads to magnetic field generation that subsequently settles into stable configurations. For WDs, this process is thought to occur during the common envelope phase. Such fields would only be present on the merged product, potentially explaining why single magnetic stars outnumber their binary counterparts. Many further routes for magnetic field generation have been proposed. For example, in the case of WDs, dynamos previously operating at the core of progenitor main-sequence stars may be compressed during core collapse (Stello et al. 2016).

While each hypothesis offers valuable insights, current observational data are insufficient to definitively favor one over another, and it is plausible that multiple mechanisms are at work simultaneously. Future observations of binary systems may offer a pathway to distinguish between scenarios. In particular, the next generation of gravitational wave detectors such as the Laser Interferometer Space Antenna (LISA; see Amaro-Seoane et al. 2017) or the Einstein Telescope (ET; see Maggiore et al. 2020) can provide enough sensitivity to characterise the magnetic properties of compact systems of WDs and NSs (Bourgoin et al. 2022; Carvalho et al. 2022; Lira et al. 2022).

In this work, we study the binary interaction of the fields themselves. Following Paper I (Aykroyd et al. 2023), we consider stars that have strong and stable magnetic fields, which are modulated by rotation. We investigate the dynamics of the interacting magnetic fields, concerning ourselves with the stability of the different configurations.

Notation and conventions. For each vector $\vec{u} = (u^1, u^2, u^3) \in \mathbb{R}^3$, we represent its norm without arrows $u = \|\vec{u}\|$ and its direction by a hat $\hat{u} = \vec{u}/u$. We adopt the summation convention on repeated indices, for example $a_i b^i = a_1 b^1 + a_2 b^2 + a_3 b^3$. We shall adopt the following constants: G the gravitational constant, μ_0 the permeability of free space.

2 Binary dynamics

We shall be considering a pair of stars in closed orbit: either a MMS binary or WD / NS binary. We assign an index $a \in \{1, 2\}$ to each star, and define a mass m_a, a spin vector $\vec{s}_a$, and a magnetic moment $\vec{\mu}_a$. We assume a stable magnetostatic field that is rigidly frozen into each star, compatible with general observations in massive stars and compact objects. We consider the topology to be predominantly dipolar (see e.g. Borra et al. 1982), aligned with the stellar spin. The aim is to study systems under strong magnetic interaction, where magnetism drives the stellar orientation. For this we also assume that the timescales of magnetic dynamics are much larger than the orbital period.

2.1 Hamiltonian formulation

We begin by formulating the dynamics of the system under a Hamiltonian formalism. We introduce a two-body Hamiltonian in the centre-of-mass frame:

$$\mathcal{H}(\vec{r}, \vec{p}\,; \vec{s}_1, \vec{s}_2) = \mathcal{H}_{\rm kep}(\vec{r}, \vec{p}) + \varepsilon \mathcal{H}_{\rm mag}(\vec{r}\,; \vec{s}_1, \vec{s}_2), \tag{2.1}$$

with $\vec{r}$ the separation between the two stars and corresponding phase-space momentum $\vec{p}$. The spin degrees-of-freedom $\vec{s}_1$ and $\vec{s}_2$ are non-canonical and satisfy the Poisson bracket relations $\{s_a^i, s_a^j\} = \epsilon_{ijk} s_a^k$, where ϵ_{ijk} is the Levi-Civita symbol. The constant ε is for bookkeeping and we shall later take $\varepsilon = 1$. The first term $\mathcal{H}_{\rm kep}$ represents the Keplerian gravitational interaction between two point-particles and is fully-integrable:

$$\mathcal{H}_{\rm kep}(\vec{r}, \vec{p}) = \frac{p^2}{2\beta} - \frac{G m_1 m_2}{r}, \tag{2.2}$$

with $\beta = m_1 m_2/(m_1 + m_2)$ the reduced mass. The second term $\mathcal{H}_{\rm mag}$ describes the magnetic interaction between two dipolar fields (Pablo et al. 2019), which we express in tensor form:

$$\mathcal{H}_{\rm mag} = -\mathcal{B}_{ij} \mu_1^i \mu_2^j, \qquad \text{with } \mathcal{B}_{ij} = \frac{\mu_0}{4\pi r^3}(3\,\hat{r}^i \hat{r}^j - \delta_{ij}), \tag{2.3}$$

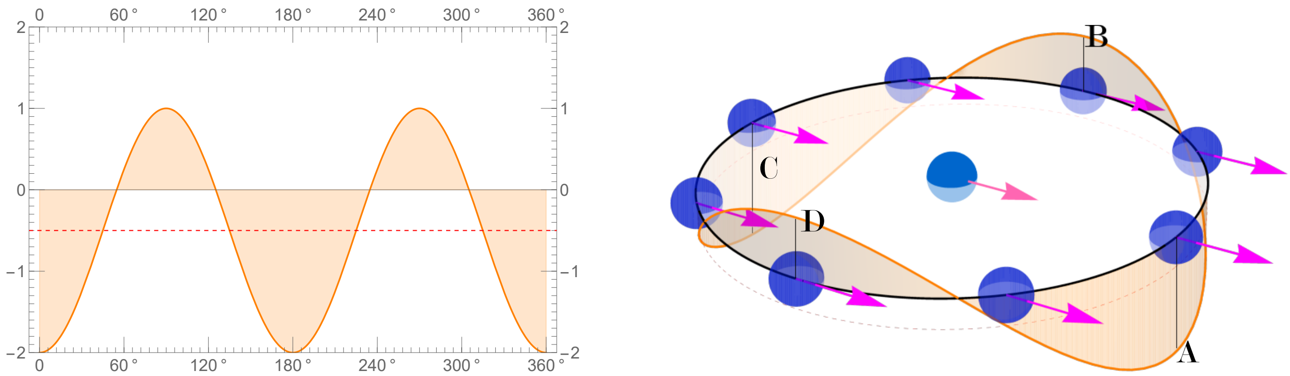

Fig. 1: Magnetic energy $\mathcal{H}_{\rm mag}$ (orange) as a function of the orbital phase, for a binary with two aligned dipole axes. Stationary points are labelled A through D. Energy oscillations emerge throughout the orbit. On longer timescales, such configuration is unstable to perturbations of the magnetic moments $\vec{\mu}_a$.

with δ_{ij} the Kronecker delta. We include a rigidbody constraint, enforcing alignment between the spin and moment, $\vec{\mu}_a = \mu_a \hat{s}_a$. Then, the evolution of the magnetic axes (or equivalently, of the spins) can be obtained from Hamilton's equations in Poisson bracket form:

$$\frac{\mathrm{d}\mu_a^i}{\mathrm{d}t} = \{\mu_a^i, \mathcal{H}\} = \epsilon_{ijk}\mu_a^j \left(\frac{\mu_a}{s_a}\mathcal{B}_{k\ell}\mu_b^\ell\right), \tag{2.4}$$

which corresponds to a precession of the vector $\vec{\mu}_a$. We remark that since the magnetic field tensor $\mathcal{B}_{ij}$ depends on the separation $\vec{r}$, the precession and orbital dynamics are coupled. We shall thus have two interacting timescales, the orbital period $P_{\rm orb}$ and the precession timescale $\tau \sim 8\pi a^3 s_a/(\mu_0\mu_1\mu_2)$ [see Eq. (2.4)]. As illustrated in Fig. 1, the (normalised) magnetic Hamiltonian exhibits oscillations in the orbital timescale. In the next step we search for magnetic configurations that are stable to perturbations. For this we employ an orbit-averaging procedure to eliminate small-timescale oscillations and obtain an effective description of the system at precession timescale τ.

2.2 *Secular stability*

We introduce a 'secular system', where the effects of orbital translation are averaged out from the potentials. In this new description, the precession dynamics and orbital motion will become naturally decoupled. The transformed Hamiltonian is computed by expanding Eq. (2.1) in a Lie series to first order in ε, yielding a new magnetic term which is integrable in the orbital variables:

$$\mathcal{H}^* = \mathcal{H}_{\rm kep} + \varepsilon\bar{\mathcal{H}}_{\rm mag} + O(\varepsilon^2). \tag{2.5}$$

The new magnetic term $\bar{\mathcal{H}}_{\rm mag}$ is computed by averaging $\mathcal{H}_{\rm mag}$ over the flow of the zero order term,

$$\bar{\mathcal{H}}_{\rm mag} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\mathcal{H}_{\rm mag}\circ\phi_t^{\mathcal{H}_{\rm kep}}\,\mathrm{d}t, \tag{2.6}$$

where $\phi_t^{\mathcal{H}_{\rm kep}}$ is the flow along $\mathcal{H}_{\rm kep}$. By plugging the solutions for the Keplerian two-body problem we get:

$$\bar{\mathcal{H}}_{\rm mag} = \bar{\mathcal{B}}_{ij}\mu_a^i\mu_b^j, \qquad \text{with } \bar{\mathcal{B}}_{ij} = \frac{\mu_0}{8\pi a^3(1-e^2)^{3/2}}\left(3\,\hat{L}^i\hat{L}^j - \delta_{ij}\right), \tag{2.7}$$

where a is the semi-major axis, e the eccentricity, and $\vec{L} = \vec{r}\times\vec{p}$ is the conserved angular momentum of the orbit. Following Paper I, we search for the magnetic configurations $\vec{\mu}_a$ that are stationary for $\mathcal{H}^*$. These are 'secular' or long-term equilibrium states and are unaffected by the short-period orbital perturbations. Conversely, equilibrium states obtained from system (2.1) (see Pablo et al. 2019) would only be stationary at a given orbital phase (cf. Fig. 1). There are four secular equilibrium states satisfying $\dot{\vec{\mu}}_a = \{\vec{\mu}_a, \mathcal{H}^*\} = 0$, illustrated in Fig. 2. From the Hessian of $\bar{\mathcal{H}}_{\rm mag}$ (computed in Paper I), we show that only a single of these states is stable: the configuration in which the dipole axes are orthogonal to the orbital plane and anti-aligned. Binaries with orientations controlled by magnetic interactions are expected to converge to this state.

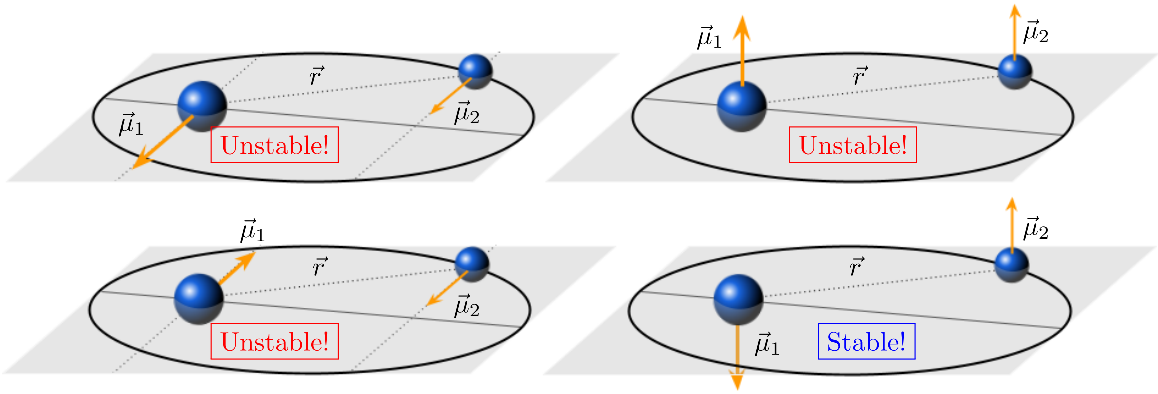

Fig. 2: Secular spin equilibrium states and their respective stabilities. These correspond to directions that are either inside the orbital plane, in some arbitrary direction (**left** column), or orthogonal to the orbital plane (**right** column). In each scenario, the two spins must be either parallel (**top** row) or anti-parallel (**bottom** row).

3 Conclusions

We have determined the secular dynamics and equilibrium of a pair of stars under the magnetic dipolar interaction. For that we have assumed that the fields are fixed with respect to the object's frame, and that the magnetic moments are aligned with the stellar spin. The method we have presented predicts a set of equilibrium states that are stationary throughout the whole orbit. We have found that there is a single state which is stable, where the stars have anti-aligned moments which are perpendicular to the orbital plane. This is a minimum of secular energy towards which we expect the system to converge. Our results are applicable for binaries of both compact objects (WD, NS) and massive stars. We remark that the ϵ-Lupi system, the only known doubly-magnetic MMSs binary system, is found in our stable state (cf. Pablo et al. 2019).

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