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GRAVITATIONAL WAVEFORMS IN GENERAL RELATIVITY AND BEYOND

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Abstract. The analytical modelling of gravitational waves in general relativity has been one of the key elements to allow the numerous detections by the LIGO-Virgo-KAGRA collaboration. Extending those results to alternative theories of gravity is crucial to test our gravitational paradigm at an unprecedented precision with next generation detectors, such as the Einstein Telescope and the space-based interferometer LISA. In this talk, I will review the current status of analytical gravitational wave modelling both in general relativity and alternative theories of gravity.

Keywords: gravitational waves, general relativity, alternative theories of gravity

1 Introduction: the gravitational universe

For almost a decade, we have entered the era of gravitational wave (GW) astronomy. In three observing runs, the GW interferometers LIGO, Virgo and KAGRA have detected around ninety coalescences, witnesses to the dynamics of binary black holes and neutron stars in the most energetic cosmic encounters (Abbott et al. 2023, 2024). Our first look at gravitational wave astrophysics is becoming a mature field, fuelled by a continuous network of detectors as the future ground-based Einstein Telescope (ET) and Cosmic Explorer (CE) interferometers, and the LISA space telescope, with all of them that should be operational in the 2035's.

As gravitational waves' studies rapidly become data-driven, the challenges of characterizing the variety of noisy and possibly unknown signals and of interpreting the signature of new physics become even greater. In particular, it is crucial to develop ever more precise waveforms, both in general relativity (GR) and in modified theories of gravity, to control systematic effects and to disentangle astrophysical signatures of different origins. In summary, the transformation of gravitational astronomy into a precision science depends on our ability to map the tiny details of these observations into fundamental physics.

1.1 Going beyond general relativity

Alternative theories of gravity have a history almost as long as general relativity and are motivated by interrogations at the opposite sides of the energy spectrum. At cosmological scales, the accelerated expansion of the universe cries out for the presence of dark components in the Universe. Dark energy is incorporated in the cosmological concordance model of cosmology with an ad-hoc cosmological constant whose extremely small value does not have a satisfactory explanation. Regarding dark matter, usually described by a new particle that only (or mostly) interacts through gravitational interactions, it has not yet been observed despite many attempts from ground-based particle physics detectors or through astrophysical observations. On the other side, at very high energies and small scales, general relativity is not renormalizable and we expect a quantum theory of gravity to prevail.

An alternative to these unknowns consists in modifying the theory of gravity by relaxing one of the assumption that leads to general relativity (GR). For example, one could go to higher than four dimensions such as Kaluza-Klein models, or to relax diffeomorhism invariance such as in the Hořava-Lifshitz family of theory (Horava 2009). An other possibility is to modify the number of propagating gravitational degrees of freedom, by considering a massive graviton, such as in massive and bimetric gravity theories or by simply adding additional scalar or vector fields (de Rham 2014; Langlois 2019). All these possible modifications lead to a full zoo of

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theories that needs to be further constrained. It can be done in two ways: either relying on fundamental principles (such as stability, well-posedness, number of degrees of freedom, *etc.*), or by confronting predictions to observations (solar system tests, pulsar timing, gravitational waves, *etc.*) and experiments (table-top, free-fall, *etc.*).

1.2 Modelling gravitational waves

The coalescence of a compact binary system is described by three distinct phases. First, the initial spiraling phase, when the gravitational field and the relative velocities are still fairly weak, makes it possible to use the so-called post-Newtonian (PN) and post-Minkowskian (PM) expansions and to obtain analytical waveforms (Blanchet 2014). Then, when the two objects come closer, the gravitational field becomes too strong and we can no longer resort on a perturbative treatment. Einstein's equations have to be solved using very powerful numerical tools in order to obtain the waveform describing the merger (Pretorius 2005; Baker et al. 2006; Campanelli et al. 2006). Finally, the last relaxation phase after the fusion is described semi-analytically using black hole perturbation theory (Kokkotas & Schmidt 1999; Berti et al. 2009).

In order to detect and determine the parameters of the observed system, it is necessary to have a database of complete waveforms, covering the widest possible parameter space (masses, spins, eccentricities, *etc.*). The latter is obtained using two distinct methods: the phenomenological class of models that simply connect the different waveforms (Khan et al. 2016), and the effective one-body (EOB) formalism that makes it possible to obtain, after re-summation of certain terms and calibration with numerical results, a complete waveform (Buonanno & Damour 1999). In addition, a lot of efforts is also being made to describe extreme mass ratio inspirals (EMRI) using black hole perturbation and self force techniques (Barack & Pound 2019). In the following, we will focus on the spiralling phase for which we use perturbative methods to obtain *precise analytical templates of gravitational wave signals both in general relativity and in alternative theories of gravity.*

2 Analytical modelling of gravitational waves

2.1 State-of-the-art

Analytical methods have been developped since several decades reaching a very high precision using different flavor of techniques, that all end up with similar results. The state-of-the-art of PN waveform modelling is the following:

- for the conservative dynamics, the equations of motion have been obtained at 4PN order using a Fokker Lagrangian (often called "traditional" PN in the following), an ADM-Hamiltonian and Effective Field Theory (EFT) techniques (Bernard et al. 2016; Marchand et al. 2017; Damour et al. 2014; Jaranowski & Schäfer 2015; Foffa et al. 2019; Foffa & Sturani 2019).
- for the dissipative dynamics, the current status is 3.5PN order using "traditional" PN with results at 4.5PN order that have also been obtained in the center-of-mass frame (Nissanke & Blanchet 2005; Gopakumar et al. 1997).
- for the gravitational flux and waveforms, the state-of-the-art is 4.5PN order using "traditional" PN and the gravitational modes have been obtained at 4PN order for quasi-circular orbits (Marchand et al. 2016; Blanchet et al. 2023b,a).

All these results were obtain by modelling the compact objects by point-particles, on top of which we can also add finite-size effects. The state-of-art for the tidal effects is next-to-next-to-leading (NNL) order, which corresponds to a 7PN contribution in the equations of motion and $N^{5/2}L$ (7.5PN) order in the gravitational flux and waveform modes for quasi-circular orbits (Henry et al. 2020a,b). Adding spin effects have also attracted a lot of efforts in the past years, using both "traditional" PN and EFT techniques. Current state-of-the-art for the spin part of the harmonic Lagrangian is the NNNL (5PN) order (Kim et al. 2022, 2023b,a; Levi et al. 2023; Levi & Yin 2023). All the cubic and higher spin interactions are also known at leading order (Levi & Steinhoff 2015b,a; Vines & Steinhoff 2018). Radiation-reaction forces were included in the dissipative dynamics for the NL SO and SS effects at 4PN and 4.5PN respectively in Refs. (Maia et al. 2017a,b), and to all orders in spin in Ref. (Siemonsen et al. 2018). Regarding the gravitational radiation, the GW flux has been obtained to NNL (4PN) order both for the SO and SS contributions (Bohé et al. 2015; Cho et al. 2022). The GW phase has been obtained within the multipolar-post Minkowskian (mPM) formalism at the NNL (4PN) order for the SO contributions and the waveform modes have been obtained to NNL (3.5PN) order in the quasi-circular orbit approximation (Bohé et al. 2013; Marsat et al. 2014; Henry et al. 2022).

In the following, we will present the *multipolar post-Minkoskian post-Newtonian (mPM–PN) formalism*, in which the binary problem is usually separated into two distinct ones that are intrinsically related: i) the conservative dynamics, which describes the motion of the binary system; ii) the dissipative flux, which describes the emission of gravitational waves.

2.2 The conservative dynamics

2.2.1 The point-particle solution

To derive the equations of motion for each object, we will follow a Fokker Lagrangian approach (Bernard et al. 2016). In the following, we remind the main building blocks for such a construction, taking the example of a simple scalar-tensor theory of gravity.

i) We start from the action,

$$S_{\mathrm{ST}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}g^{\alpha\beta}\partial_\alpha\phi\,\partial_\beta\phi\right] + S_{\mathrm{m}}\left(\mathfrak{M}, g_{\alpha\beta}\right)\,, \tag{2.1}$$

where ϕ is a single massless scalar field minimally couples to the metric $g_{\mu\nu}$, $\omega(\phi)$ is an arbitrary function. The last term in Eq .(2.1) describes the coupling to matter which is given, in the point-particle approximation, by

$$S_{\mathrm{pp}} = -c\sum_{a=1,2}\int \mathrm{d}\tau_a\, m_a\left(\phi\right)\,, \tag{2.2}$$

where $m_a(\phi)$ is the mass of object a that can vary with respect to the value of the scalar field. From this action, we derive the field equations for the metric and the scalar field.

ii) Next, we solve iteratively these equations up to a certain PN order *, determined by the Fokker approach. For example, if we are interested only in the correction due to tidal effects, it is sufficient to know the point-particle contributions to the metric and scalar field perturbations.

iii) We inject these solutions in the total action up to the required order. It results in a generalized (Fokker) Lagrangian that depends not only on the positions and velocities of the particles but also on their higher order derivatives.

iv) Varying the generalized action w.r.t. to the position of the particles, using the Eular-Lagrange equations, we obtain the equations of motion for each particle (Bernard 2018, 2019).

2.2.2 The tail effects

On top of the particular solution derived in point ii) above, we also have to include a homogeneous solution of the wave equations (Bernard et al. 2017; Marchand et al. 2017). It describes how the gravitational wave emmitted at one time can back-react on the curvature of space-time and affects the dynamics of space-time at a later time. It is known as the *tail effects*.

In GR, such a solution starts contributing at 4PN, corresponding to an interaction between the mass monopole and the quadrupole moment, $M \times I_{ij}$. In ST theories, things are different due to the presence of a scalar dipole moment. As a consequence the leading contribution will be at 3PN order, corresponding to an interaction between the mass monopole and a scalar dipole moment, $M \times I_i^{(s)}$ (Bernard 2018).

*We call nPN order the order $2n$ in the expansion in $\mathcal{O}\left(v/c\right)$.

2.2.3 The tidal effects

On top of the point-particle modelling, one can add finite-size effects using an effective action describing the tidal effects (Bernard 2020; Bernard et al. 2024):

$$\begin{aligned} S_{\rm fs}^{(s)} = -c \sum_{a=1,2} \int \mathrm{d}\tau_a \Bigg\{ & \sum_{l=1}^{\infty} \frac{1}{2l!}\ \lambda_a^l\,(\phi)\left(\nabla_L^{\perp}\varphi\right)_a\ \left(\nabla_{\perp}^{L}\varphi\right)_a \\ & - \frac{1}{c^2}\sum_{l=2}^{\infty}\frac{1}{2l!}\left[c_a^l(\phi)\,G_L^a\,G_a^L \ + \ \frac{l}{(l+1)c^2}\,d_a^l(\phi)\,H_L^a\,H_a^L\right] \\ & + \sum_{l=2}^{\infty}\frac{1}{l!}\nu_a^l(\phi)\,G_a^L\ \left(\nabla_L^{\perp}\varphi\right)_a \Bigg\}\,, \end{aligned} \tag{2.3}$$

where G_L and H_L are the electric and magnetic-type gravitational tidal moments, defined as

$$G^a_{\mu_1\ldots\mu_l} = -c^2\left[\nabla^{\perp}_{<\mu_1}\ldots\nabla^{\perp}_{\mu_l-2}C_{\mu_{l-1}\underline{\rho}\mu_l>\sigma}\right]\,u_a^{\rho}\,u_a^{\sigma}\,, \tag{2.4a}$$

$$H^a_{\mu_1\ldots\mu_l} = 2\,c^3\left[\nabla^{\perp}_{<\mu_1}\ldots\nabla^{\perp}_{\mu_l-2}C^{*}_{\mu_{l-1}\underline{\rho}\mu_l>\sigma}\right]\,u_a^{\rho}\,u_a^{\sigma}\,, \tag{2.4b}$$

with $C_{\mu\nu\rho\sigma}$ the Weyl tensor and $C^{*}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\kappa}C^{\lambda\kappa}{}_{\rho\sigma}$ and $\varepsilon_{\mu\nu\lambda\kappa}$ denotes the completely anti-symmetric Levi-Civita tensor.

In the tidal action (2.3), one can differentiate three types of tidal effects:

a) on the first line, the *scalar tidal effects*, parametrized by the scalar deformability parameters λ_a^l, are new to ST theories and start contributing at 3PN order.

b) on the second line, we recover the standard *gravitational tidal effects*, parametrized by the scalar deformability parameters c_a^l and d_a^l. They start contributing at 5PN order.

c) on the third line, the *gravito-scalar tidal effects*, parametrized by the scalar deformability parameters ν_a^l, are new to ST theories. They represent the interaction between the two previous type of tidal effects and they start contributing at 5PN order.

2.3 The dissipative flux

We now turn on to the computation of the flux and gravitational waves emitted by a compact binary system.

2.3.1 The multipolar solution

First, we solve the ST *vacuum* field equations, $\Box h^{\mu\nu} = \tau^{\mu\nu}$, in the exterior region of the isolated matter system by means of a multipolar decomposition conjointly with a non-linear post-Minkowskian expansion (Blanchet & Damour 1986). The solution is written as

$$h^{\mu\nu}_{\rm ext} = G h_1^{\mu\nu} + G^2 h_2^{\mu\nu} + \mathcal{O}\left(G^3\right)\,. \tag{2.5a}$$

The multipolar expansion is entirely specified by the most general expressions for the multipolar decomposition of the linear coefficients $h_1^{\mu\nu}$, that depends on some moments I_L, J_L (and also W_L, X_L, Y_L and Z_L), expressed as functions of the source. The multipole expansion of $h^{\mu\nu}$ is obtained using a matching to the PN expansion in the near zone of the source as (Blanchet 1998), as obtained in the previous section. Defining $\Sigma \equiv (\bar{\tau}^{00} + \bar{\tau}^{ii})/c^2$, $\Sigma_i \equiv \bar{\tau}^{0i}/c$ and $\Sigma_{ij} \equiv \bar{\tau}^{ij}$, and taking into account the harmonic gauge condition, $\partial_\nu h^{\mu\nu} = 0$, we get

$$\begin{aligned} I_L(u) = \underset{B=0}{\mathrm{FP}}\int \mathrm{d}^3\mathbf{x}\,\tilde{r}^B\int_{-1}^{1}\mathrm{d}z\Bigg[&\delta_\ell(z)\hat{x}_L\Sigma - \frac{4(2\ell+1)}{c^2(\ell+1)(2\ell+3)}\delta_{\ell+1}(z)\hat{x}_{iL}\Sigma_i^{(1)} \\ & + \frac{2(2\ell+1)}{c^4(\ell+1)(\ell+2)(2\ell+5)}\delta_{\ell+2}(z)\hat{x}_{ijL}\Sigma_{ij}^{(2)}\Bigg](\mathbf{x}, u + zr/c)\,, \end{aligned} \tag{2.6a}$$

together with a similar expression for the current and scalar-type moments J_L and $I_l^{(s)}$.

Next, we introduce a radiative type coordinate system (T, R), with $U \equiv T - R/c$ being an asymptotically null coordinate such that

$$U = u - \frac{2GI}{c^3} \ln\left(\frac{r}{cb}\right) + \mathcal{O}\left(\frac{1}{r}\right) , \tag{2.7}$$

where I is the mass monopole moment and b is an arbitrary constant time-scale. Then, we denote $\mathcal{U}_L$, $\mathcal{V}_L$ and $\mathcal{U}^s_L$ the radiative moments and parametrize the asymptotic transverse – trace-free (TT) tensorial waveform and the scalar waveform in the radiative coordinate system at leading order $1/R$ in the distance. We have,

$$h^{\mathrm{TT}}_{ij} = -\frac{4G}{c^2 R} \perp^{\mathrm{TT}}_{ijab} \sum_{\ell=2}^{+\infty} \frac{1}{c^\ell \ell!}\Big(N_{L-2}\,\mathcal{U}_{abL-2}(U) - \frac{2\ell}{c(\ell+1)} N_{cL-2}\varepsilon_{cd(a}\mathcal{V}_{b)dL-2}(U)\Big) + \mathcal{O}\Big(\frac{1}{R^2}\Big) , \tag{2.8a}$$

(2.8b)

where $\perp^{\mathrm{TT}}_{ijab} \equiv \frac{1}{2}(\perp_{ia}\perp_{jb} + \perp_{ja}\perp_{ib} - \perp_{ij}\perp_{ab})$ with $\perp_{ij} = \delta_{ij} - N_i N_j$ being the TT projection operator (we remind that *e.g.* $N_{L-2} = N_{i_1} \cdots N_{i_{\ell-2}}$), and a similar expression for the scalar waveform. The tensorial and scalar fluxes $\mathcal{F}$ and $\mathcal{F}^s$ are deduced in terms of the radiative multipole moments directly from Eqs. (2.8) as, for example for the scalar flux,

$$\mathcal{F}^s = \sum_{\ell=0}^{+\infty} \frac{G\phi_0\,(3+2\omega_0)}{c^{2\ell+1}\ell!(2\ell+1)!!} \overset{(1)}{\mathcal{U}^s}_L \overset{(1)}{\mathcal{U}^s}_L .$$

Finally, the spherical harmonic modes, that are useful for matching with numerical relativity results, are computed for quasi-circular orbits. We perform the standard computation for the tensor modes $h^{\ell m}$ and the scalar modes $\psi^{\ell m}$. The TT tensor h^{TT}_{ij} can be decomposed into two independent modes along the polarization vectors, h_+ and $h_\times$, which can be recast into a complex field that can itself be decomposed on the basis on spin-weighted spherical harmonics of weight -2,

$$h = h_+ - \mathrm{i}h_- = \sum_{\ell=2}^{+\infty} \sum_{m=-\ell}^{\ell} h^{\ell m}\ {}_{-2}Y^{\ell m} . \tag{2.10}$$

The pure spin-0 scalar field can be decomposed on standard (spin-0) spherical harmonics,

$$\psi = \sum_{\ell=0}^{+\infty} \sum_{m=-\ell}^{\ell} \psi^{\ell m}\ Y^{\ell m} . \tag{2.11}$$

Last, the spherical harmonic radiative modes can be linearly related to the STF multipole moments (Blanchet 2014; Bernard et al. 2022).

2.3.2 The hereditary effects

Once the vacuum linearized solutions are obtained, the non-linear contributions can be computed using the iteration of the multipolar post-Minkowskian (mPM) algorithm of Blanchet & Damour (1986).

The memory effects. In GR, the non-linear memory (Christodoulou 1991; Thorne 1992; Blanchet & Damour 1992; Wiseman & Will 1991) is a non-local effect due to the radiation of linear waves by the stress-energy tensor, dominantly associated with the mass quadrupole moment. Such an effect arises in the waveform at 2.5PN order. In ST theory, there is a new type of memory effect associated with the scalar dipole radiation that comes from the quadratic interaction between two scalar dipole moments, say $I^s_i \times I^s_j$. As we shall see, such an effect arises at 1.5PN order in the waveform.

As an example, the correction to the electtric-type radiative quadrupole moment due to the scalar dipolar memory effect as (Bernard et al. 2022),

$$\delta\mathcal{U}^{\mathrm{memory}}_{ij}\Big|_{I^s_i \times I^s_j} = \frac{3+2\omega_0}{3} \int_{-\infty}^{u} \mathrm{d}v\, \overset{(2)}{I^s}_{\langle i}(v) \overset{(2)}{I^s}_{j\rangle}(v) . \tag{2.12}$$

An important point to notice is that, although it is a scalar-origin memory effect, it gives a correction to the gravitational part of the energy flux.

The tail effects. The hereditary effects coming from the non-linear iteration of the Einstein field equations are the so-called tails. In GR, the leading effect arises from the quadratic interaction between the conserved gravitational monopole $I = M/\phi_0$ and the mass quadrupole moment I_{ij}, giving a leading contribution in the flux at 2.5PN order. In ST theories, the leading contribution comes from the interaction between the mass monopole moment and the scalar dipole moment $I_i^s(u)$ (Bernard et al. 2022).

To 1.5PN order in the gravitational flux, only the mass quadrupole radiative moment differs from its twice-differentiated source counterpart,

$$\mathcal{U}_{ij} = \overset{(2)}{I}_{ij} + \frac{2GM}{\phi_0 c^3} \int_{-\infty}^{U} \mathrm{d}V \overset{(4)}{I}_{ij}(V) \left[\ln\left(\frac{U-V}{2b}\right) + \frac{11}{12} \right] + \text{memory terms} \tag{2.13}$$

In addition, we find that for the 1.5PN ST waveform, the radiative-type scalar monopole, dipole and quadrupole moments acquire tail contributions, for instance for the dipole moment

$$\mathcal{U}_i^s = \overset{(1)}{I}{}_i^s + \frac{2GM}{\phi_0 c^3} \int_{-\infty}^{U} \mathrm{d}V \overset{(3)}{I}{}_i^s(V) \left[\ln\left(\frac{U-V}{2b}\right) + 1 \right] + \text{"inst. terms"} + \mathcal{O}\left(\frac{1}{c^7}\right) . \tag{2.14a}$$

3 Conclusions

In conlusion, we have reviewed the main steps that lead to the gravitational waveforms using the mPM-PN formalism. Such a formalism is applicable to GR but also to simple scalar-tensor theories as presented here. The results are one of the key building block to construct full waveform templates that are used for the detection and parameter estimation of gravitational wave events in GW experiments. In the future, the next generation of GW detectors, such as LISA and Einstein Telescope, makes it crucial to improve our models by including environmental effects in GR or by additional modelling in alternative theories, if we want to test fundamental physics with an unprecedented precision.

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