SF2A 2024
M. Béthermin, K. Baillié, N. Lagarde, J. Malzac, R.-M. Ouazzani, J. Richard, O. Venot, A. Siebert (eds)

HYDRODYNAMICAL STELLAR TACHOCLINES IN LOW-MASS STARS: THE EFFECT OF THE EVOLUTION OF THE DIFFERENTIAL ROTATION IN THEIR CONVECTIVE ENVELOPE

C. Moisset[1], S. Mathis[1] and C. Dunand[2]

Abstract. Stellar tachoclines are thin sheared regions that make the transition between the differentially rotating convective envelope of low-mass stars and their radiative core which rotates almost as a solid body. Because of the latitudinal shear imposed by the surrounding convective envelope, they can be the seat of shear-induced turbulence that triggers momentum transport and chemicals mixing. This latitudinal differential rotation varies along the evolution of stars because the braking by stellar winds modifies their global rotation, consequently altering their convection. It is first cylindrical during the early phases when the star rotates rapidly. Then, it becomes conical as in the Sun with an equatorial acceleration, before becoming possibly anti-solar with poles rotating faster than the equator. In this work, we examine the impact of this evolution on the dynamics of hydrodynamical tachoclines, which is currently ignored in stellar evolution codes. We demonstrate that Mathis & Zahn (2004)'s formalism coherently treats hydrodynamical stellar tachoclines. We then characterise the differential rotation, meridional circulation and turbulent diffusion coefficient inside the tachocline as a function of the varying imposed shear. We show that their geometrical behaviors are similar for simple profiles of conical and cylindrical differential rotation.

Keywords: stars: solar-type, stars: rotation, stars: evolution

1 The model of hydrodynamical tachoclines

Solar-type stars from 0.45 to 1.5 $M_\odot$ present during their main sequence a uniformly rotating radiative core and a differentially rotating convective envelope (e.g. García et al. 2007; Brun et al. 2017; Benomar et al. 2018). The tachocline is the thin layer at the convection/radiation interface where the transition between these two rotation regimes occurs (Spiegel & Zahn 1992). As such, it is a region of strong shear and important mixing (Brun et al. 1999). This mixing is taken into account in some stellar evolution codes but the prescriptions only consider a forcing that corresponds to the current latitudinal differential rotation of the Sun. We investigate here the impact of other regimes of differential rotation in the convective envelope that develop along the evolution of stars. We use the following simplified rotation profiles as boundary conditions that the rotation in the tachocline must verify at its frontier with the convective zone: i) a conical differential rotation $\Omega = \Omega_0 + \Delta\Omega \sin^2\theta$ with Ω_0 the angular velocity at the poles, $\Delta\Omega$ the global shear between the equator and the poles and θ the co-latitude ($\Delta\Omega > 0\,(< 0)$ corresponds to the (anti-)solar regime); ii) a cylindrical differential rotation: $\Omega = \Omega_0 + \epsilon s^2$ with $s = r\sin\theta$ the distance to the axis, r the radius, and $\epsilon r^2 = \Delta\Omega$ the shear in the cylindrical formalism.

We first demonstrate that the formalism of Mathis & Zahn (2004) developed to treat simultaneously the bulk of radiative regions and hydrodynamical tachoclines leads to the Spiegel & Zahn (1992) formalism in the thin layer approximation. We use the reduced radius $\zeta = \mu_2\,(r_0 - r)\,/d$ with $d = r_0\left(2\overline{\Omega}/N_T\right)^{\frac{1}{2}}(4\kappa/\nu_h)^{\frac{1}{4}}$, defined in Spiegel & Zahn (1992) to express the differential rotation and meridionnal circulation. We introduce r_0 the radius at the top of the tachocline, $\mu_2 = (6/1575)^{-1/4}$ the eigenvalue, $\overline{\Omega}$ the mean rotation rate, N_T the thermal Brunt-Väisälä frequency, κ the thermal diffusivity and ν_h the horizontal turbulent viscosity triggered by the horizontal shear instability. This radius (ζ) is centered on the tachocline and, using the Sun's typical values: $\Delta\zeta = 2 \Leftrightarrow \Delta r = 50000$km. The expression for the meridional circulation is proportional to the imposed shear $\Delta\Omega$, which will change along the evolution of the star (Astoul et al. 2021). We plot here the differential rotation and meridional circulation in the tachocline normalized by the shear (cf. fig. 1).

---

[1] Université Paris-Saclay, Université Paris Cité, CEA, CNRS, 91191 Gif-sur-Yvette, France
[2] Lycée Paul Valéry, 75012 Paris, France

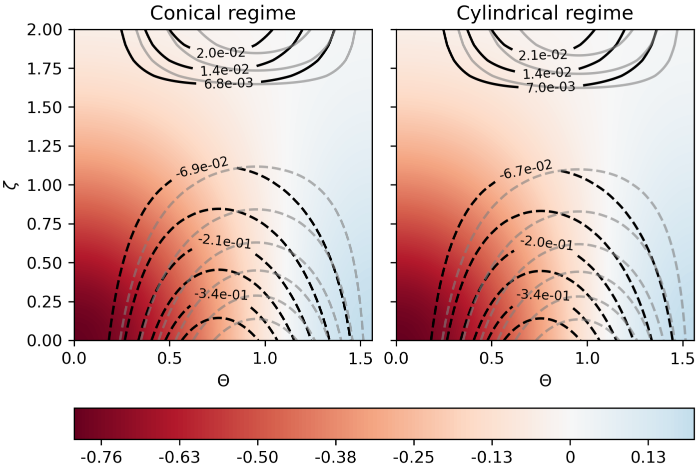

Fig. 1. Differential rotation (background color) and streamfunction for the meridional circulation expanded to $l = 2$ (gray contours) and $l = 2 + 4$ (black contours). **Left panel:** Conical forcing. **Right panel:** Cylindrical forcing.

The conical and cylindrical forcings in the convective region do not seem to have a significant impact on the geometrical behaviour of the rotation and circulation inside the tachocline. Developing the meridional circulation to higher latitudinal order (l) shows how the loops are shifted towards the poles by the extra positive and negative contributions at the order $l = 4$.

Following Mathis & Zahn (2004) and Chaboyer & Zahn (1992), the effect of the vertical advection of the chemical elements by the meridional circulation in presence of an horizontal turbulence is parametrised as an effective vertical turbulent diffusion (cf. fig. 2). As expected from the similarity of the loops of meridional circulation, the different forcings in the convection envelope do not seem to impact strongly the prescriptions for the effective mixing.

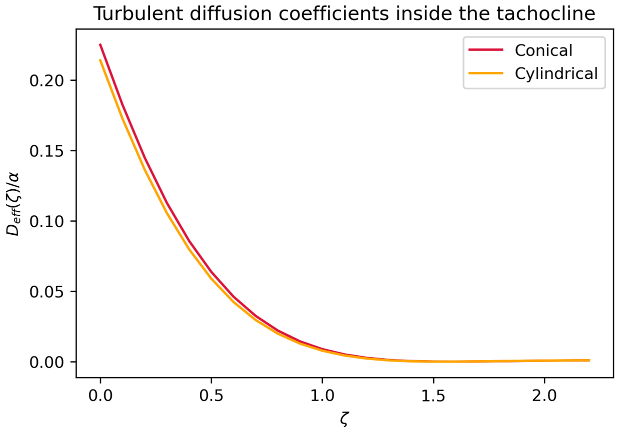

Fig. 2. Effective turbulent diffusion coefficient computed for the meridional circulation expanded up to $l = 4$ and normalized with $\alpha = (\Delta\Omega/\overline{\Omega})^2(d/r_0)^2\nu_h$.

2 Conclusion

We investigated how the evolution of the differential rotation profile in the convection envelope during stellar life modifies the differential rotation, meridional circulation and the chemical mixing in hydrodynamical tachoclines. All these quantities are a linear function of the imposed latitudinal shear and follow its evolution with a similar geometrical behaviour for the simplest models of conical and cylindrical differential rotation we examined in this first work. This opens the path to study more realistic differential rotation profiles in a near future.

C.M. and S.M. acknowledge support from the European Research Council (ERC) under the Horizon Europe program (Synergy Grant agreement 101071505: 4D-STAR) and from the CNES SOHO-GOLF & PLATO grants at CEA-DAp. Opinions expressed are however those of the authors only. Neither the European Union nor the granting authority can be held responsible for them.

References

Astoul, A., Park, J., Mathis, S., Baruteau, C., & Gallet, F. 2021, A&A, 647, A144
Benomar, O., Bazot, M., Nielsen, M. B., et al. 2018, Science, 361, 1231
Brun, A. S., Strugarek, A., Varela, J., et al. 2017, ApJ, 836, 192
Brun, A. S., Turck-Chièze, S., & Zahn, J. P. 1999, ApJ, 525, 1032
Chaboyer, B. & Zahn, J. P. 1992, A&A, 253, 173
García, R. A., Turck-Chièze, S., Jiménez-Reyes, S. J., et al. 2007, Science, 316, 1591
Mathis, S. & Zahn, J. P. 2004, A&A, 425, 229
Spiegel, E. A. & Zahn, J. P. 1992, A&A, 265, 106