*SF2A 2024*
*M. Béthermin, K. Baillié, N. Lagarde, J. Malzac, R.-M. Ouazzani, J. Richard, O. Venot, A. Siebert (eds)*

# ANALYTIC SOLUTION OF CHEMICAL EVOLUTION MODELS WITH TYPE IA SUPERNOVAE

P. A. Palicio[1]

**Abstract.** Stellar chemistry provides important information about how the composition of the interstellar medium evolves with time. In this work, we introduce an analytic solution for a chemical evolution model that accounts for the contribution from Type Ia supernovae, extending the Instantaneous Recycling Approximation through the use of the DTD formalism. Using this analytic solution, we explore how different DTDs affect the evolution of [Si/Fe] and [O/Fe] relative to [Fe/H], using both one- and two-infall models for the solar neighborhood. In the two-infall model, we introduce a delayed gas infall event to reproduce the chemical pattern observed in APOGEE DR17 data, including the "loop" feature that explains the thin disc chemistry. We also show how our solution can be applied to mimic the chemical evolution of simulated galaxies like GALACTICA from their star formation history. In order to make our solution easy to use, we developed the PYTHON package `ChEAP`, which is publicly available and includes documentation and examples to help researchers apply it in their studies.

Keywords: Galaxy: disc, Galaxy: abundances, Galaxy: evolution, Galaxy: solar neighborhood, ISM: general, ISM: evolution

## 1 Introduction

The analysis of the chemical abundances of stars provides unique information about the composition of the interstellar medium (ISM) in their birthplace. This composition does not remain unchanged over time but evolves, driven by both local (e.g., material ejected by supernova explosions) and external (e.g., gas infall and mergers) factors. In this context, chemical evolution models are valuable tools for understanding the contributions of these mechanisms, as well as for interpreting the growing number of observations from large ground-based surveys and the *Gaia* space mission (Gaia Collaboration et al. 2016; Recio-Blanco et al. 2023).

Historically, solving chemical evolution models analytically was possible only in a few special cases, where it was assumed that the pollution of the ISM by a new generation of stars occurs immediately after their formation, a scenario known as the Instantaneous Recycling Approximation (IRA). This assumption, however, limits the modeling of elements produced on longer timescales, such as iron in Type Ia supernovae (SNe). Unlike core-collapse supernovae, the binary system progenitor of Type Ia SN requires a minimum delay of around 30-40 Myr to explode (Belczynski et al. 2005). This delay is not fixed but ranges from the mentioned 30-40 Myr to over 10 Gyr, with its distribution described by the delay time distribution (DTD).

Typically, including the term associated with the DTD in model equations prevents analytical solutions, with the exceptions of studies by Weinberg et al. (2017), Pantoni et al. (2019), and Lapi et al. (2020), who considered an exponential DTD. In Palicio et al. (2023), we presented a more general solution to a chemical evolution model with Type Ia contributions, assuming a wider variety of DTDs.

This proceeding is organised as follows. In Section 2 we introduce the model equations and the different formulations for the DTD considered in Palicio et al. (2023, 2024). Section 3 illustrates some scientific applications of our solution. The conclusions are presented in Section 4.

[1] Université Côte d'Azur, Observatoire de la Côte d'Azur, CNRS, Laboratoire Lagrange, Bd de l'Observatoire, CS 34229, 06304 Nice cedex 4, France

## 2 Model equations and DTD formalism

We adopt the model presented in Vincenzo et al. (2017), which describes the change in the surface density, $\sigma_X(t)$, of a generic element $X$ as a function of time. More precisely, this model accounts for gas returned to the ISM, the reduction of gas due to star formation, the production of new elements through stellar nucleosynthesis, the loss of chemical elements via galactic winds, and the inflow of pristine gas onto the system. The combined effects of these mechanisms can be mathematically expressed by the equation

$$\frac{d\sigma_X(t)}{dt} = -\alpha\,\sigma_X(t) + \langle y_X \rangle \left(1 - R\right) \psi(t) + \langle m_{X,\mathrm{Ia}} \rangle\, \mathcal{R}_{\mathrm{Ia}}(t), \tag{2.1}$$

where $\alpha$ is a constant that encapsulates the effects of star formation, returned gas, and galactic winds on the mass of the element $X$. The factors $\langle y_X \rangle$ and $\langle m_{X,\mathrm{Ia}} \rangle$ are the yields from Type II and Type Ia SNe contributions to $\sigma_X(t)$, respectively. The term $R$ denotes the returned mass fraction, and $\psi(t)$ represents the star formation rate. Finally, $\mathcal{R}_{\mathrm{Ia}}(t)$ is the Type Ia SN rate at time $t$, which depends on the star formation rate $\psi(t)$ and the DTD as:

$$\mathcal{R}_{\mathrm{Ia}}(t) = C_{\mathrm{Ia}} \int_{\tau_1}^{\min(t,\tau_2)} \mathrm{DTD}(\tau)\, \psi(t-\tau)\, d\tau, \tag{2.2}$$

where $C_{\mathrm{Ia}}$ can be considered a normalisation constant and $\tau_1$, $\tau_2$ are the minimum and maximum time delays for a Type Ia supernova, respectively. We refer to Vincenzo et al. (2017) and Palicio et al. (2023) for a more detailed explanation of the model equations.

Among the DTDs available in the literature, we have selected seven of them, including that proposed by Matteucci & Recchi (2001, hereafter MR01) for the single degenerate scenario, those computed by Greggio (2005, G05) for the wide and close double degenerate scenarios, as well as those derived empirically from observations by Strolger et al. (2004, 2005, S05), Mannucci et al. (2006, MVP06), Totani et al. (2008, T08), and Pritchet et al. (2008, P08). As Figures 1 and 2 in Palicio et al. (2023) illustrate, this set of DTDs spans a wide range of shapes, minimum time delays, and fractions of prompt explosions. The S05 DTD is a Gaussian distribution centered at $t = 3.4$ Gyr with a dispersion of 0.68 Gyr. The MVP06 DTD combines a Gaussian and an exponential distribution to account for prompt and late Type Ia supernova explosions, respectively. Both the T08 and P08 DTDs are described by power laws, with indices of $-1$ and $-1/2$, respectively. The analytic formulations for the MR01 and G05 DTDs are more complex, so we refer to their corresponding works for a detailed explanation.

## 3 Results

We tested the performance of our analytic solution by exploring the single and double infall scenarios, as well as the potential application of modelling the chemical evolution of numerically simulated galaxies generated without chemical information. The main results are summarised in the following subsections.

### *3.1 One infall model*

We assumed a chemical evolution model for the solar neighborhood driven by a primordial exponential infall of pristine gas with a timescale of 7 Gyr. The amplitude of this infall was calibrated to match the present-day total surface density of $54\,\mathrm{M}_\odot\,\mathrm{pc}^{-2}$ (Spitoni & Matteucci 2011; Spitoni et al. 2015). For each DTD, the normalization constant $C_{\mathrm{Ia}}$ (see Eq. 2.2) was set to achieve a present-day Type Ia SN rate of 0.54 century$^{-1}$ (Li et al. 2011), while all other parameters followed the values adopted by Vincenzo et al. (2017).

We assessed the impact of the DTD on the evolution of silicon and oxygen abundances relative to iron. In the [Si/Fe] vs. [Fe/H] and [O/Fe] vs. [Fe/H] diagrams (see Fig. 4 of Palicio et al. 2023), we observe the high [Si/Fe] and [O/Fe] plateau at low [Fe/H] characteristic of the thick disc, followed by decreasing [Si/Fe] and [O/Fe] abundances at higher [Fe/H] that define the thin disc. While this plateau is consistent across all DTDs–forming at early times when the contribution from Type Ia SNe is negligible–the evolutionary track to the present-day abundances is influenced by the choice of DTD. On the one hand, the MVP06 DTD, which starts producing iron earlier, leads to a decrease in [Si/Fe] and [O/Fe] at lower [Fe/H] compared to the other DTDs. It shows a change in slope at [Fe/H]$\approx -1$ dex due to the contribution from late Type Ia SNe modeled by this DTD. On the other hand, the S05 DTD, lacking a prompt explosion regime, predicts a plateau extending to [Fe/H]$\approx -0.5$ dex, followed by a sharp decline in [Si/Fe] and [O/Fe] as [Fe/H] increases.

### 3.2 Two infall model

Inspired by previous works (Chiappini et al. 1997; Spitoni et al. 2019, 2021), we extended our chemical evolution model for the solar neighborhood by introducing an additional infall of gas, delayed by 3-4 Gyr relative to the primordial infall. The inclusion of this second infall requires reducing the timescale of the primordial infall to $\lesssim 1$ Gyr. With the aim at reproducing chemistry of the solar neighbourhood, we tested multiple combinations of the model parameters to match the [Si/Fe] versus [Fe/H] distribution observed in APOGEE DR17 data (Abdurro'uf et al. 2022). The resulting model of the APOGEE data assuming the T08 DTD is shown in Fig. 1, while Figs. 8, 10, and C.1 in Palicio et al. (2023) illustrate the scenarios with the MR01, wide G05, and close G05 DTDs, respectively. As can be seen, we found a suitable combination of parameters that reproduces the "ribbon" feature identified in other studies (Calura & Menci 2009; Spitoni et al. 2023), although we had to reduce silicon yields by 18% to match the observations.

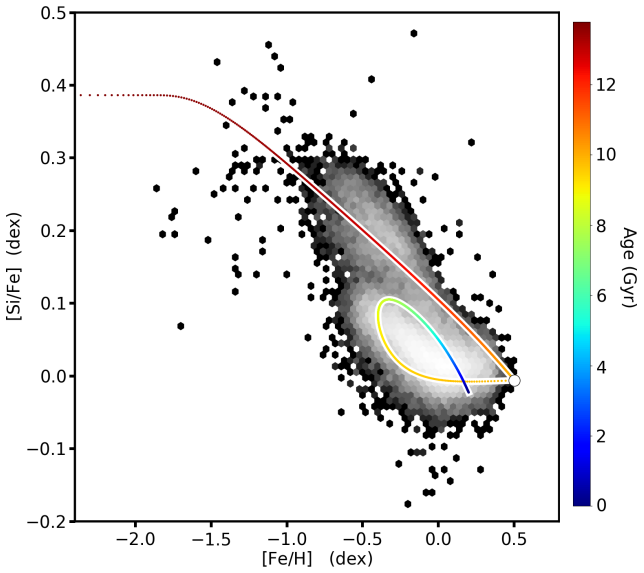


**Fig. 1.** Predictions for the evolution of silicon and iron assuming the two infall model with T08 DTD. The coloured map in the background indicates the distribution of APOGEE stars, while the solid line represents the solution to the model equations, color-coded with the stellar age. The start of the second infall is denoted by the open circle.

### 3.3 Modelling the chemistry of simulated galaxies

We explored a potential application of our analytic solution by modeling the chemical evolution of a numerically simulated Milky Way-like galaxy, GALACTICA (Park et al. 2021). Specifically, we aim to reproduce the [Si/Fe] vs. [Fe/H] diagram using the star formation rate (SFR) of GALACTICA as input. Initially, we fit the SFR as a sequence of exponential gas infall events, which are then applied in the analytic solution (see Section 5 in Palicio et al. 2023 for details of this fit). As a result, we obtained a plateau of [Si/Fe]≈0.5 dex up to [Fe/H]≈-1 dex, followed by a decreasing trend with metallicity*, which shows mild dilution effects associated with peaks in the SFR (see Fig. 12 of Palicio et al. 2023). Finally, one infall episode at $t \approx 6$ Gyr is strong enough to cause significant dilution, producing the characteristic loop attributed to the thin disc.

### 3.4 The ChEAP package

To facilitate the distribution of our analytic solution, we have implemented it in the PYTHON package `ChEAP` (`Chemical Evolution Analytic Package`), which is freely available in the repository `https://bitbucket.org/pedroap/cheap/src/master/`. The repository contains documentation and usage examples of the `ChEAP` code, which only requires standard PYTHON libraries to run.

*Here, [Fe/H] is used as a proxy for the overall metallicity.

## 4 Conclusions

In this work, we presented an analytic solution to a chemical evolution model that accounts for the delayed contribution of Type Ia SNe, using a variety of DTDs available in the literature. This solution provides an efficient approach to modeling the chemical abundances in galaxies, as demonstrated by its application to both observational and simulated data.

We first tested the performance of our analytic model using single and double infall scenarios for the solar neighborhood. In the single infall model, we found that different DTDs significantly affect the evolution of [Si/Fe] and [O/Fe] abundances with respect to [Fe/H], showing the MVP06 and S05 DTDs notable differences in their predicted abundance tracks. Furthermore, with the two infall model we were able to fit the APOGEE DR17 data, satisfactorily reproducing the loop in the [Si/Fe] vs. [Fe/H] diagram, though a modification of the silicon yields was necessary to match the observations.

We also explored the potential of our analytic solution to model the chemical evolution of numerically simulated galaxies. Using the SFR from the GALACTICA simulation, we modeled the [Si/Fe] vs. [Fe/H] diagram and found that the model reproduced the chemical pattern expected for a Milky-Way like galaxy.

To facilitate further applications of our chemical evolution model, we implemented our analytic solution in the PYTHON package `ChEAP`. This package is freely available and provides an accessible tool for chemical evolution modeling, requiring only standard PYTHON libraries. In conclusion, our analytic approach constitutes a versatile tool for understanding the galactic chemical evolution, with potential applications in both observational and simulated datasets.

P. A. Palicio acknowledges the financial support from the Centre national d'études spatiales (CNES). This work has benefited from the European Union's Horizon 2020 research and innovation program under SPACE-H2020 grant agreement number 101004214 (EXPLORE project).

## References

Abdurro'uf, Accetta, K., Aerts, C., et al. 2022, ApJS, 259, 35
Belczynski, K., Bulik, T., & Ruiter, A. J. 2005, ApJ, 629, 915
Calura, F. & Menci, N. 2009, MNRAS, 400, 1347
Chiappini, C., Matteucci, F., & Gratton, R. 1997, ApJ, 477, 765
Gaia Collaboration, Prusti, T., de Bruijne, J. H. J., et al. 2016, A&A, 595, A1
Greggio, L. 2005, A&A, 441, 1055
Lapi, A., Pantoni, L., Boco, L., & Danese, L. 2020, ApJ, 897, 81
Li, W., Chornock, R., Leaman, J., et al. 2011, MNRAS, 412, 1473
Mannucci, F., Della Valle, M., & Panagia, N. 2006, MNRAS, 370, 773
Matteucci, F. & Recchi, S. 2001, ApJ, 558, 351
Palicio, P. A., Matteucci, F., Della Valle, M., & Spitoni, E. 2024, A&A, 689, A203
Palicio, P. A., Spitoni, E., Recio-Blanco, A., et al. 2023, A&A, 678, A61
Pantoni, L., Lapi, A., Massardi, M., Goswami, S., & Danese, L. 2019, ApJ, 880, 129
Park, M. J., Yi, S. K., Peirani, S., et al. 2021, ApJS, 254, 2
Pritchet, C. J., Howell, D. A., & Sullivan, M. 2008, ApJ, 683, L25
Recio-Blanco, A., de Laverny, P., Palicio, P. A., et al. 2023, A&A, 674, A29
Spitoni, E. & Matteucci, F. 2011, A&A, 531, A72
Spitoni, E., Recio-Blanco, A., de Laverny, P., et al. 2023, A&A, 670, A109
Spitoni, E., Romano, D., Matteucci, F., & Ciotti, L. 2015, ApJ, 802, 129
Spitoni, E., Silva Aguirre, V., Matteucci, F., Calura, F., & Grisoni, V. 2019, A&A, 623, A60
Spitoni, E., Verma, K., Silva Aguirre, V., et al. 2021, A&A, 647, A73
Strolger, L.-G., Riess, A. G., Dahlen, T., et al. 2004, ApJ, 613, 200
Strolger, L.-G., Riess, A. G., Dahlen, T., et al. 2005, ApJ, 635, 1370
Totani, T., Morokuma, T., Oda, T., Doi, M., & Yasuda, N. 2008, PASJ, 60, 1327
Vincenzo, F., Matteucci, F., & Spitoni, E. 2017, MNRAS, 466, 2939
Weinberg, D. H., Andrews, B. H., & Freudenburg, J. 2017, ApJ, 837, 183