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APPROACH TO KEPLER'S LAWS BY DIRECT OBSERVATION OF THE MOON

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Abstract. Kepler's Laws are accessible at several levels from high school to engineering school. Approaches exist, but generally based on data provided to students, such as apparent diameters of the Moon or ecliptic longitudes, in the form of tables of numbers. By making direct observations within their reach (photos, webcam) with modest equipment, students process their own data. The programs are written in Python, the official programming language of the Ministry of National Education, and are adaptable according to the level.

Thus, to determine the distance of the Moon from a snapshot, the apparent diameter is obtained by fitting a circle to the lunar outline with a least squares algorithm. In high school, the code for this function will be provided, but in engineering school an implementation will be expected. Current results provide at little cost an excellent precision of 0.2 % on the diameter, therefore on the distance, and roughly on the eccentricity of the orbit, which is suitable for characterizing the latter around 5 %.

The second Law, which involves the angular rate, is tested by timestamping the crossing of a reference position by the Moon (the meridian line). As these passages rotate over an entire day in one month, a computer is fully assigned this task for a 24/24 recording, to be then analyzed by students. In the end, the results obtained over the past school year are worth the job, but considered incomplete, mainly because of the weather. This presents an opportunity to bring together several sites during a future season, to exchange images between establishments, and to add to the project a network dimension, facilitated by modern digital environments.

Keywords: Education, Kepler Laws, Moon, observation

1 Introduction

Johannes Kepler (1571 - 1630), established his laws of planetary motion by analyzing the abundant and meticulous observations collected by his predecessor, the Danish astronomer Tycho Brahe (1546 - 1601). Kepler thus stated a new property in 1609: *The planets move in elliptical orbits with the Sun at one focus.*

This was a breaktrough for that time! It shattered the dogma of circular orbits and explained with greater precision the deviations from the circles that Ptolemy sought to model in his Almagest, painstakingly combining deferents and epicycles. This law is also valid for the Earth-Moon pair. When moving through an ellipse with one focus being the Earth, its distance must therefore vary. But by how much ? Is it perceptible ? Is a measurement possible ? With what equipment ?

This is the starting point of a measurement campaign carried out by high school seniors at Lycée Rosa Parks in spring 2020, and running again today. Naturally we know the orbital elements of the Moon much more precisely today than at the time of Kepler and we also know since Newton that Kepler's Laws are only the consequences of a more fundamental Law : the Universal Gravitation. The measurement will not provide any additional astronomical knowledge, but the Moon, which completes its orbit around the Earth in just one month, with a notable eccentricity of 5 %, offers students the opportunity to tackle celestial mechanics and obtain a convincing result in a time compatible with their schooling. In addition, Kepler's laws are still in the curriculum of the new baccalaureate in Physics and Chemistry, which also focuses on Galileo's astronomical telescope.

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2 Ellipse eccentricity

2.1 Apparent diameter and distance

The Moon is 356700 km away from Earth at the closest distance (perigee), while it is 406300 km away at apogee. The ratio is the same for the apparent diameters, and the difference is twice the ellipse eccentricity (Eq. 2.1). This latter approximation is valid for small eccentricities. So in order to measure the eccentricity of the lunar orbit with a significant precision the need on the apparent diameter measurement is in the range of 10^{-3}.

$$r = \frac{D_{E-Mapogee}}{D_{E-Mperigee}} = \frac{\varnothing_{E-Mperigee}}{\varnothing_{E-Mapogee}} = \frac{1+e}{1-e} \approx 1+2e \quad (2.1)$$

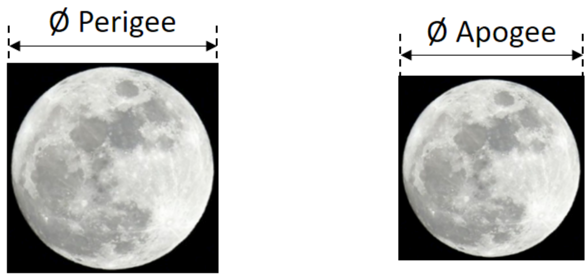

Fig. 1. The difference between perigee and apogee leads to 14% on the Moon apparent diameter, which is easy to determine accurately.

This is achieved by taking pictures of the Moon time after time. The resolution of common modern cameras is several thousands of pixels wide. A zoom lens is still necessary, or the camera is installed behind a small telescope (a focal length of 60 cm is enough). Of course, pictures are not feasible close to the New Moon, but a few days are concerned in a lunar month. Weather conditions are another matter !

2.2 Image processing

First the raw image from the camera is plotted with enhanced contrast by adjusting the color palette, but no resampling is performed, to keep the resolution. Then the purpose of the algorithm is to identify a set of points on the lunar outline, and to run a least square circular fit to get the apparent diameter (Fig. 2). The trade-off between ergonomy and programming complexity has been to clic the points on the outline by hand, being careful not putting points on the terminator of a gibbous Moon for example.

The source code is based on the Coope algorithm (Coope 1993), and a Python implementation is available for download on the author's github : `https://github.com/jeanguerard/MoonCircle`

It is also important to produce all the image bank with the same camera, and the same zoom lens. If different cameras were to be used, a normalization of each scale factor should be performed on a reference object (the Pleiades cluster is a good candidate).

Difference between polar diameter and equatorial diameter is about one thousandth, which is negligible with respect to the accuracy of the pictures, as well as the impact of lunar mountains.

2.3 Results

In most cases the diameter estimator accuracy is in the expected range of 10^{-3}. Fig. 3 shows the result for the 2020 "Covid" campaign, with several contributions to the image processing, proving the robustness of the diameter estimator among several students. From this the ratio r can be evaluated, leading to the eccentricity :

$$e = 0.0648 \pm 0.0016 \quad (2.2)$$

Which is compatible with the real instantaneous eccentricity at the time of the measurement. It is even possible, with successive pictures of the Moon during the same night, to show that the observer moves closer and then further from the Moon... because the Earth is round and it rotates. The devil is in the details, and so the Truth is !

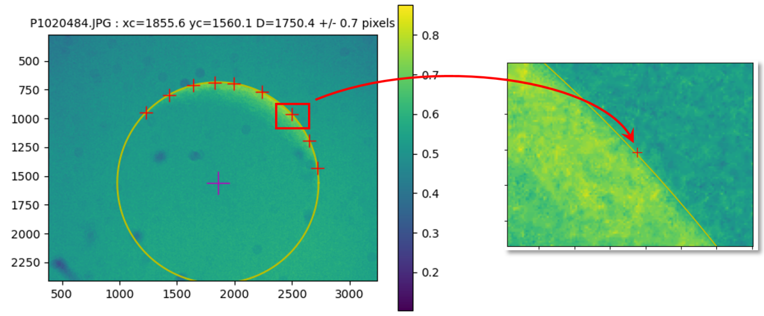

Fig. 2. Control points are placed by hand on the outline of this thin (and hardly visible) crescent Moon. The result of the circle fit is a diameter of 1750 pixels, with a quadratic error of 0.7 pixel (1 σ).

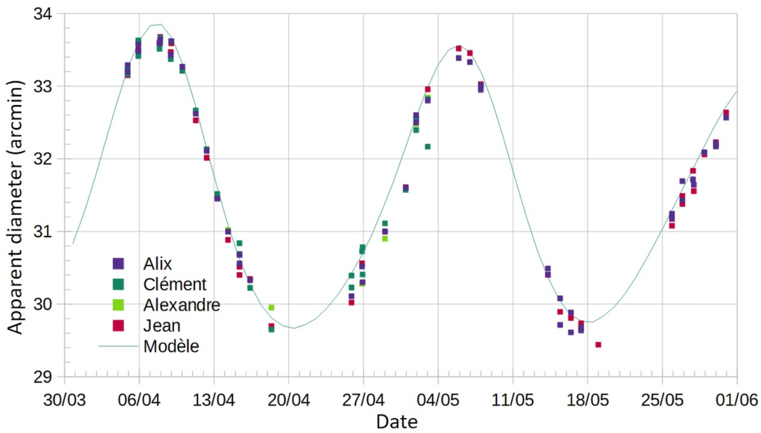

Fig. 3. Moon diameter estimation results. Although several students each clic different points (and points number), the estimator accuracy remains better than what is measured.

3 Angular Rate

3.1 Measurement

The question of the angular rate for the Second Law is more difficult. By completing its orbit in one month, the average angular rate of the Moon is $\approx$ 12 degrees per day. When observing the Moon in the range of an hour, the rate becomes 0.5 degree per hour, and we are looking for variations of this rate below 1 %. This is a one pixel shift in previous pictures with respect to an hypothetical background reference star. Very few stars can be shot close to the Moon.

We choose a more integral method. From day to day, the Moon crosses a defined reference line in the Earth sky, while moving its own way (so that $t_2 - t_1 > 24h$, see Fig. 4), and its angular rate Ω_M can be derived from the timestamp difference and the well-known Earth angular rate Ω_E, one turn in one sideral day of 23h56m04s (Eq. 3.1. Of course we choose the meridian line.

$$\begin{aligned} \theta &= \Omega_E\ (t_2 - t_1)\ -\ 360° \\ \Omega_M &= \frac{\theta}{t_2 - t_1} \end{aligned} \tag{3.1}$$

The meridian crossing can happen anytime, so that we use an automatic webcam driven by a pc, taking a snapshot every minute. Dating accuracy is satisfactory, but the application generates 2 GB of images per week, while only a few ones are useful every day. Here again, the trade-off between ease of use and program complexity has been to find the crossing image by scrolling manually the directory (yet with little help of the Moon ephemerides). Each photo is written with year-month-day hour:minute:second in file name (Fig. 5).

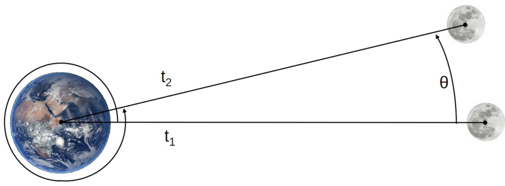

Fig. 4. From t_1 to t_2, the Moon crosses the meridian line in the Earth sky, but has also moved itself.

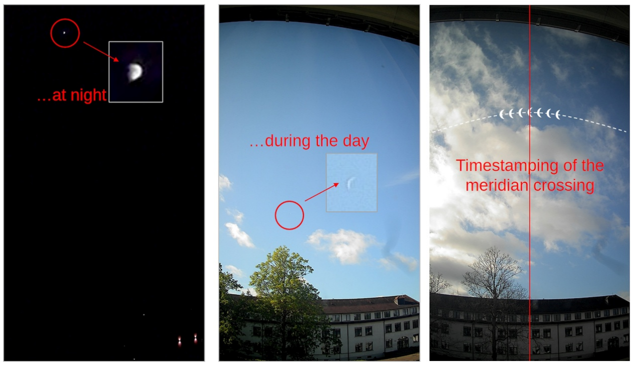

Fig. 5. The exact date and time of the meridian crossing is read by scrolling the snapshot directory.

3.2 Processing

The objective of the angular rate measurement was ideally to demonstrate the Kepler's second Law, which states that the area swept out by the Moon (or a planet) is constant with time. It is mathematically expressed by $r^2 \times \dot{\theta} = Constant$, with r coming from the direct images (see section 2), and $\Omega_M = \dot{\theta}$ coming from the meridian crossing. Actually the weather conditions in France last winter and spring were so bad that we have less than 10 % observable moons, considering the fact that to get one angular rate we need two consecutive clear days, and that distance images do not come alwayse the same date as the angular rate images.

But separate accuracies are compatible are excellent on both measurements, especially for angular rate, despite the fact that a parasitic harmonic has to be discarded before the $r^2 \times \dot{\theta}$ product (Fig. 6). The angle θ is not primarily the expected ecliptic longitude of the Moon, it is polluted by the inclination of Earth axis.

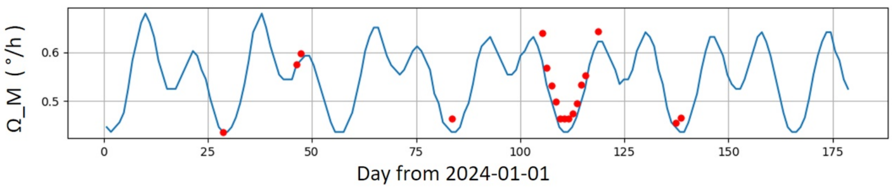

Fig. 6. Measured angular rate by meridian crossing (red), compared to Astropy model (blue).

4 Conclusions

This pedagogic activity can be carried out with modest means and reasonable investment of time, and is scalable to the level of students. The next campaign will try to federate several entities to increase the weather probabilities, and add a data share dimension. The project began at the initiative of the authors themselves, but is now included in the work programme of the SAF Education Commission.

References

Coope, I. 1993, Journal of Optimization Theory and Applications, 69