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RECONCILING PTA AND JWST AND PREPARING FOR LISA. A PARAMETRIC MODEL TO DESCRIBE THE FORMATION AND EVOLUTION OF MASSIVE BLACK HOLES.

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Abstract. We develop a parametrised model to describe the formation and evolution of massive black holes, suitable for comparing with both electromagnetic and gravitational wave observations. We find that observations of the black hole luminosity function are compatible with the nHz gravitational wave signal measured by pulsar timing arrays, provided we allow for an increased luminosity function at high redshift ($4-7$), as previously suggested by the outcomes of the Spitzer AGN survey, and more recently by JWST.

Keywords: Gravitational wave, black hole

1 Introduction

Massive black holes (MBHs), with masses of 10^5–$10^9 M_\odot$, reside at the centre of most galaxies in our local Universe (Gehren et al. 1984; Kormendy & Richstone 1995; Reines et al. 2011; Reines et al. 2013; Baldassare et al. 2020), including our own Milky Way. Scaling relations between the properties of galaxies and their MBHs point to the fact that MBHs track the hierarchical formation of galaxies.

One of our best probes of the population of MBHs is the luminosity function (LF) of accreting MBHs: quasars and active galactic nuclei (AGNs) more generally. This has been the target of surveys in the X-ray, UV, optical, infrared, and radio bands (see Hopkins et al. 2007; Shen et al. 2020, and references therein). These surveys, covering out to $z \lesssim 7$, have revealed that the LF evolves strongly – in normalisation and shape – with redshift. Interestingly, mid-infrared surveys appeared in tension with the aforementioned results. Using observations from the Spitzer Space Telescope survey (Werner et al. 2004), Lacy et al. (2015) find an increased LF up to $z \sim 3$. The higher redshift mid-infrared Universe is now being probed by the James Webb Space Telescope (JWST). The first results for the LF derived from these observations (Greene et al. 2024; Matthee et al. 2024) also point to an increased LF compared to previous expectations based on a compilation of data from mid-IR to X-ray Shen et al. (2020).

These electromagnetic observations are now being complemented with gravitational wave (GW) observations, as Pulsar Timing Array (PTA) collaborations are likely close to confirm the detection of a stochastic background consistent with merging MBH binaries with masses $\gtrsim 10^8 M_\odot$ (Antoniadis et al. 2023c; Agazie et al. 2023; Tarafdar et al. 2022; Reardon et al. 2023; Xu et al. 2023). In the next decade, the Laser Interferometer Space Antenna (LISA) will further extend our understanding of merging MBHs by detecting GWs from binaries with lower masses (10^4–$10^8 M_\odot$) and up to very high redshifts ($z \sim 20$) (Amaro-Seoane et al. 2017; Colpi et al. 2024).

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The astrophysical interpretation of these observations is achieved by comparing to theoretical models for the formation and evolution of MBHs. However, cosmological simulations (Volonteri et al. 2016; Springel et al. 2017; Kannan et al. 2021; Ni et al. 2022; Bhowmick et al. 2024) and semi-analytical models (SAMs) (SAMs; e.g., Cole et al. 2002; Volonteri et al. 2003; Monaco et al. 2007; Somerville et al. 2008; Benson 2012; Barausse 2012; Ricarte & Natarajan 2018; Bonetti et al. 2019; Dayal et al. 2019; Izquierdo-Villalba et al. 2020; Barausse et al. 2020; Trinca et al. 2022) are too computationally expensive to compare the full range of alternative models and parameter space with observations, resulting in a set of discrete models. This is a major obstacle for astrophysical inference within a Bayesian framework. In Toubiana et al. (2021), some of the authors carried a study with mock LISA data and showed that this discretisation in the description of the population could lead to severe biases in the astrophysical inference. Empirical models (Soltan 1982; Small & Blandford 1992; Tucci & Volonteri 2017; Conroy & White 2012; Allevato et al. 2021), while sufficiently fast for Bayesian parameter estimation (see, e.g., Zhang et al. 2023; Boettner et al. 2023), focus on statistical and empirical properties, rather than the underlying physics.

The goal of this work is to provide a *fast* parametric approach to infer the physics behind the evolution of MBHs from current observations, and prepare for the LISA mission. We adopt an intermediate approach between SAMs and empirical models, proposing an effective description of the formation and evolution of MBHs within their host halos. The backbone of our model is a dark matter (DM) halo merger tree, which we generate and then populate with MBHs which are seeded – starting from high redshifts $z \geq 10$ – accrete and merge according to parametric prescriptions. Replacing the physics-informed semi-analytic prescriptions entering SAMs with this effective description allows for a significant computational speed-up. Thus, we can assign the 12 parameters entering our model large priors, rather than making narrow or discrete choices based on (uncertain) physical models, as in traditional SAMs. We then explore the parameter space using a Markov-Chain-Monte-Carlo (MCMC) algorithm to find the sets of parameters that are compatible with the desired datasets.

2 Description of the model

We generate DM halo merger trees down to $z_{\max} = 20$ using the extended Press-Schechter formalism (Press & Schechter 1974; Bond et al. 1991; Bower 1991; Lacey & Cole 1993; Parkinson et al. 2008; Benson 2017).

We populate leaf halos – the last halos along the branches of the merger trees[*] – with seed BHs. We only seed halos at $z \geq 10$ and with mass above a threshold $M_{h,\mathrm{seed}}$, with probability f_{seed}. We draw the mass of the seed BH from a truncated log-normal distribution, with mean μ_{seed} and standard deviation σ_{seed}. At the high end, we limit the mass of the BH seed to 10% of the baryonic mass of the halo $M_h\Omega_b/(\Omega_m - \Omega_b)$. At the low end, we take the minimum seed mass to be $100\,M_\odot$.

Following the merger of two halos, we track the evolution of the BHs they contain, distinguishing between *major* and *minor* halo mergers, based on the halo mass ratio $q_h = M_{h,2}/M_{h,1} \leq 1$. Based on the results of our simulations, we set the threshold for major mergers to $q_{h,\mathrm{major}} = 0.13$, which is similar to the value used in Ricarte & Natarajan (2018). Following a major merger where each halo contains one BH, these BHs from a binary that merges in a time given by the dynamical friction timescale, computed as in Volonteri et al. (2003), plus an additional delay t_{delay}. Folllowing a minor merger, the BH coming from the lighter halo has first to sink into the heavier BH within a dynamical friction timescale before forming a binary that merges within t_{delay}. We use the results of Bonetti et al. (2018) to handle interactions between more than two BHs. Once the total time delay has elapsed, binaries merge and form remnants with mass given by the sum of the masses at the time of merger.

We parametrise the growth of the mass, m_{BH}, of a BH through accretion by the Eddington ratio, f_{Edd},

$$\dot{m}_{\mathrm{acc}} = f_{\mathrm{Edd}}\dot{m}_{\mathrm{Edd}}, \tag{2.1}$$

$$\dot{m}_{\mathrm{BH}} = (1-\epsilon)\dot{m}_{\mathrm{acc}}, \tag{2.2}$$

$$\dot{m}_{\mathrm{Edd}} = \frac{m_{\mathrm{BH}}}{\epsilon t_{\mathrm{Edd}}} = \frac{L_{\mathrm{Edd}}}{\epsilon c^2}, \tag{2.3}$$

where $\dot{m}_{\mathrm{Edd}}$ and L_{Edd} are the Eddington accretion rate and luminosity, respectively, ϵ is the radiative efficiency, and $t_{\mathrm{Edd}} = 450$ Myr defines the accretion timescale. We consider two accretion modes: *burst*, which is triggered

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[*] Leaf halos can be at $z < z_{\max}$, if their mass drops below the mass resolution.

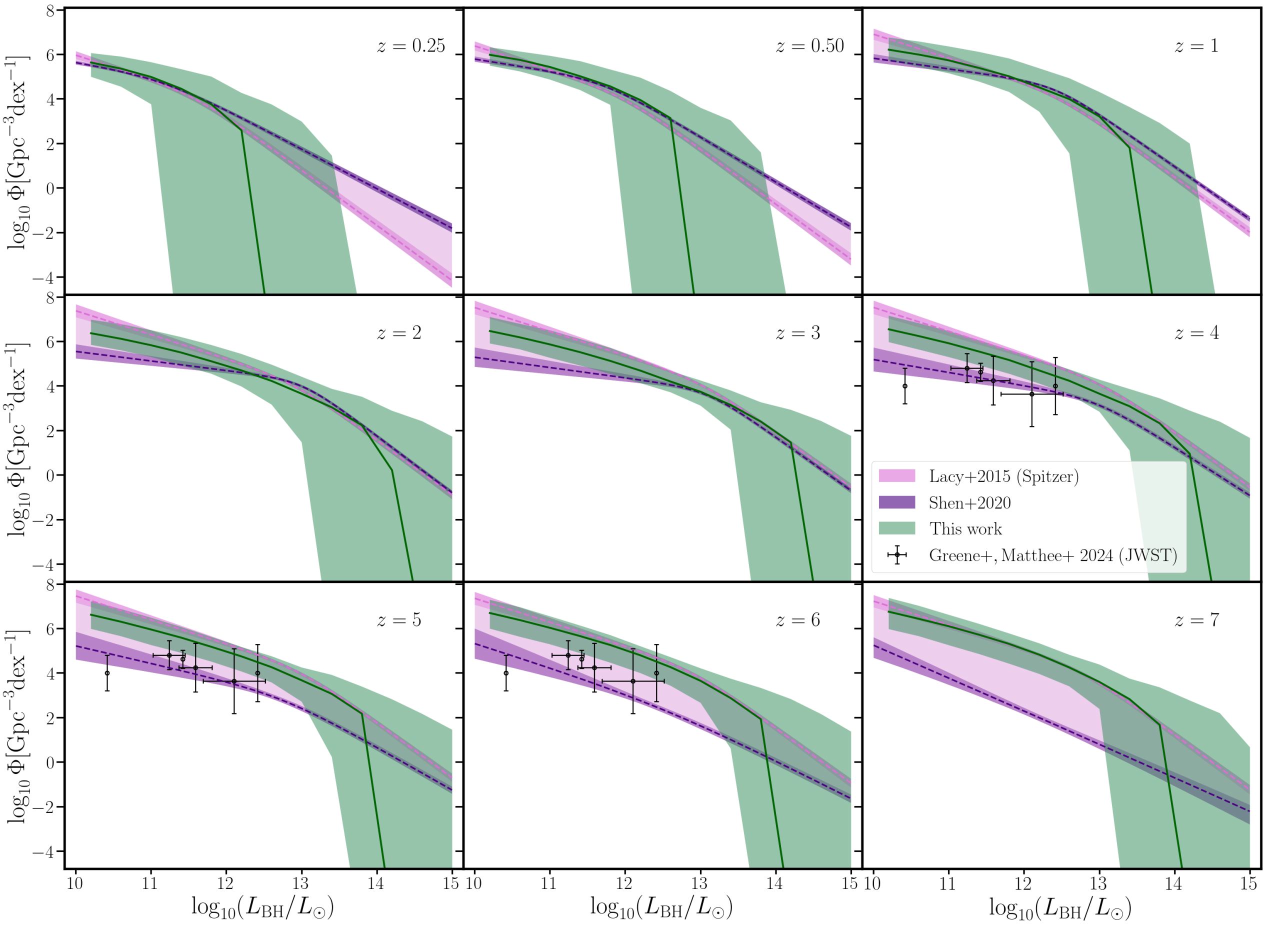

Fig. 1. The evolution of the LF. In green, we show the prediction of our model when fitting for the LF itself at redshifts 0.25, 0.5, 3 and 6, as well as for the GW spectrum measured by EPTA, displaying the median (green line) and 90% confidence interval (green band). This should be compared with the fits to the observed LF from Lacy et al. (2015) (pink) and Shen et al. (2020) (purple) and the region in between (light pink), where we allow the observed LF to lie in our likelihood. At redshifts $z = 4$, 5 and 6 we also plot constraints from JWST (black points) from Greene et al. (2024) and Matthee et al. (2024). The recovered LF is remarkably consistent with observations, also at redshifts where we do not explicitly fit the data.

by major halo mergers and allows for super-Eddington accretion, and *steady*, with small Eddington ratios that vary in time to capture AGN variability.

3 Fitting to observations

Given the current uncertainty on the LF, we wish to explore a scenario where the true LF lies somewhere between the fits proposed in Shen et al. (2020) and Lacy et al. (2015). Thus, when fitting the data with our simulations, we allow the LF computed from our simulations to lie anywhere in the region between these two fits. Additionally, we fit for the spectrum measured by EPTA (Antoniadis et al. 2023a,b,c, 2024, 2023d) in the frequency bin at $f_0 = 1/(10\ \mathrm{yr})$. Finally, we assign zero likelihood to simulations that produce BHs at $z = 0$ with masses exceeding $10^{12} M_\odot$, as these are unrealistically massive.

4 Combined fit to the luminosity function and the gravitational wave background

Our results for the LF and the GW background are shown in Fig. 1 and 2 respectively. In each case, we present in green our model's median (solid line) and 90% confidence interval (shaded region), as well as the observational data.

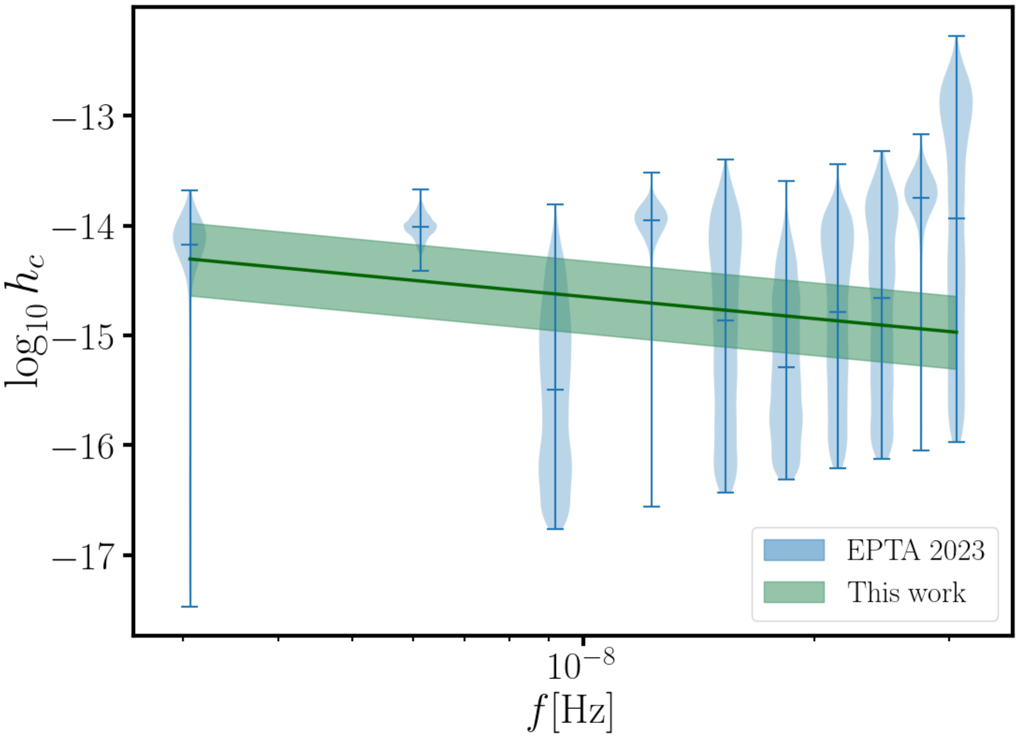

Fig. 2. The stochastic GW background predicted by our model, in green, and the free spectrum measured by EPTA, in blue. We recall that we fit only for the first frequency bin and that we assume MBH binaries to be circular, so that the slope of the GW background spectrum is fixed to be -2/3. Our result is compatible with the EPTA data, although it suggests a slightly lower median value.

5 Conclusions

In this study, we have developed a parametric model to describe the formation and evolution of massive black holes, and used it to demonstrate the consistency between the LF and the amplitude of the PTA GW background, particularly when allowing for an enhanced LF at high redshift, as suggested by extrapolating the results from Lacy et al. (2015) and by the preliminary findings from JWST (Greene et al. 2024; Matthee et al. 2024).

Looking ahead, we aim to introduce several improvements to our model. We plan to incorporate eccentricity, in order to assess the improvement in the fit to the GW background spectrum, and to include more formation channels for the MBH seeds. Additionally, we plan to model the spin evolution of MBHs, which is critically sensititve to the environment of merging MBH binaries (Sesana et al. 2014; Spadaro et al. 2024). Moreover, we intend to adopt a more physically motivated accretion model, such as one based on the mass reservoir available for accretion, which could constrain the parameter space and prevent the formation of unrealistically large MBHs. Addressing these issues will be crucial for fully extracting the astrophysical information encoded in LISA's observations and realising the observatory's immense potential (Gair et al. 2011; Klein et al. 2016; Bonetti et al. 2019; Toubiana et al. 2021; Fang & Yang 2023; Langen et al. 2024; Spadaro et al. 2024).

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