

## THE CESAM2K20 STELLAR EVOLUTION CODE

L. Manchon<sup>1</sup>, M. Deal<sup>2</sup>, Y. Lebreton<sup>1,3</sup> and J. P. C. Marques<sup>4</sup>

**Abstract.** Cesam2k20 is a community stellar evolution code publicly available. It is able to compute standard as well as non standard models, including transport of chemical elements and angular momentum, mass loss, planetoide accretion, entropy calibrated convection, radiative acceleration, etc. It is the latest public version of the CESAM (Code d'Évolution Stellaire, Adaptatif et Modulaire) stellar evolution code, originally developed by Pierre Morel. We describe bellow the main numerical and input physics aspects of the code, as well as future plans for its development.

Keywords: methods: numerical - stars: evolution - stars: fundamental parameters - stars: interiors - rotation - convection - diffusion

### 1 Introduction

Cesam2k20 is the latest version of the CESAM (Code d'Évolution Stellaire, Adaptatif et Modulaire) stellar evolution code originally developed by Pierre Morel. This code was first used to produce standard solar models (Berthomieu et al. 1993) and numerical details were presented in Morel (1997) (hereafter M97). CESAM was originally written in FORTRAN 77 for its first four versions and later translated into Fortran 90 for the fifth. In the early 2000's, CESAM was renamed CESAM2k. Morel & Lebreton (2008) (hereafter ML08) described this new version in the context of the scientific preparation of the CoRoT mission (Convection Rotation and Transits; Catala et al. 1995; Baglin et al. 2006; Auvergne et al. 2009).

The availability of precise asteroseismic data on the stellar interior and the first measurements of internal stellar rotation motivated major efforts to test models of angular momentum transport processes. Such models were implemented in CESAM2k, then renamed Cesam2k20 (where the "T" stands for "Transport"), and using this code, Marques et al. (2013) (hereafter M13) and Goupil et al. (2013a) showed that our understanding of angular momentum transport is far from being complete (see Goupil et al. 2013b; Aerts et al. 2019, for a review). Much work has also been done to improve the usability of the code, in particular with an automated installation process, and a new Python (graphical) interface.

The developments of Cesam2k20, the result of an intense collaborative effort, were important in the following years and led to its selection for the computation of the 1<sup>st</sup> generation grid of stellar models for the PLATO mission (PLANetary Transits and Oscillations of stars; Rauer et al. 2024). These new developments are implemented in the new CESAM code for the decade 2020: Cesam2k20. A regularly updated version is available at [https://git.ias.u-psud.fr/joao.marques/cesam2k20\\_releases](https://git.ias.u-psud.fr/joao.marques/cesam2k20_releases).

The Cesam2k20 code is divided into a numerical layer, described in Sect. 2 and a physical layer, described in Sect. 3. Our plan for the future of the code is explained in the conclusion in Sect. 4.

<sup>1</sup> LESIA, Observatoire de Paris, PSL Research University, CNRS, Université Pierre et Marie Curie, Université Denis Diderot, 92195 Meudon, France. email: [louis.manchon@obspm.fr](mailto:louis.manchon@obspm.fr)

<sup>2</sup> LUPM, Université de Montpellier, CNRS, place Eugène Bataillon, 34095 Montpellier, France

<sup>3</sup> Université de Rennes, CNRS, IPR (Institut de Physique de Rennes) – UMR 6251, 35000 Rennes, France

<sup>4</sup> Institut d'Astrophysique Spatiale, Université Paris-Saclay, Orsay, France

## 2 Numerical methods in a nutshell: the spline-collocation method

In the stellar evolution code *Cesam2k20*, the subroutines that implement the numerical methods are separated from the one that implement the physical processes. This makes it easy to modify the code without a deep knowledge of the numerical aspects. Nevertheless, in this section we will describe the peculiarities of *Cesam2k20* in terms of numerical procedures.

All the physical quantities are represented as a sum piecewise polynomial (PP) functions of order  $m$ . These functions have compact support, i.e. they are always positive or zero. Each of the PP functions is projected onto a local basis of normalised PP, called B-Splines. These have several advantages. First, derivatives of B-Splines are also linear combinations of B-Splines of order  $m - 1$ . Second, at each point of the radial mesh, only  $m$  B-Splines are non-zero, which allows the various systems of equations to be rewritten as systems of equations for the coefficients of the linear combinations of B-Splines. Thirdly, discontinuities can be easily dealt with by placing two (or more) successive knots at the same location. In this way, the B-Spline loses the continuity of its first derivative. Finally, there are already many efficient and stable algorithms for handling B-Splines (De Boor 2001; Schumaker 2007). Unlike finite difference or finite volume schemes, if a quantity between two points of the mesh needs to be known, one need only evaluate the B-Splines, or its derivative, rather than relying on complex combinations of the quantities at different points.

Another singularity of *Cesam2k20* resides in the use of a collocation method. For a given system of differential equations of order  $r$ , to be solved for unknowns  $f_j$ , and a given set of  $n$  grid points  $\{x_i\}_1^n$ , the unknowns are decomposed on a basis of  $M$  B-Splines  $\{N_i^{m+r}\}_1^M$  of order  $m+r$ . Exact solutions are found at  $M-r$  collocation points, and continuity is ensured at each grid points. In this case, the dimension  $M$  of the B-Spline basis is  $M = (n-1)m+r$ , and  $m$  collocation points must be found between two grid points. Collocation points are set at the roots of a Legendre polynomial of order  $m$  between two successive grid points. Such a method ensures so-called superconvergence, i.e. the solution is of order  $2m$  at grid points.

## 3 Physical processes implemented in *Cesam2k20*

### 3.1 Microphysics

*Cesam2k20* implements microphysical ingredients (nuclear reaction rates, equation of states, opacities) that are, for the most part, common to other stellar evolution codes, making comparisons easier. The more modern opacity tables are constructed using results from the OPAL team (Iglesias & Rogers 1996) or the Opacity Project (OP; Seaton 2005, 2007) and we provide them for several common solar chemical composition determinations. Thanks to the long history of *Cesam2k20*, many equations of state (EoS) are available, from the simplest analytical ones to much more complex ones such as the OPAL 2005 EoS, published by the OPAL team (Rogers & Nayfonov 2002). These EoS follow either the chemical or the physical picture. Finally, the nuclear reaction rates follow the compilations of NACRE or NACRE II (Xu et al. 2013), except for the  $^{14}\text{N}(p,\gamma)^{15}\text{O}$  reaction, or more recent measurements collected in the Reaclib database (Cyburt et al. 2010). Several nuclear reaction networks are also available, depending on which and how many chemical elements one wants to follow.

In the remainder of this section, we will focus on three important features of *Cesam2k20*. More details on these and on the rest of the codes' capabilities will be described in a forthcoming paper.

### 3.2 Convection

Convection is described in *Cesam2k20* by ad hoc parametric prescriptions. The most commonly used is the Mixing Length Theory (MLT), for which we have implemented different possible regimes by considering either an optically thick or thin convective bubble. The user can also use different formulations of Full Spectrum of Turbulence models (FST; Canuto & Mazzitelli 1992; Canuto et al. 1996). All these models have one free parameter in common:  $\alpha$ , which is usually poorly constrained, but which has a large influence on the structure and evolution. Recently, we have implemented in *Cesam2k20* a new method called the entropy calibration method, which allows  $\alpha$  to be tuned along the evolution, so that the entropy of the adiabat of the 1D models matches a value predicted by a prescription calibrated on 3D model atmospheres. This method links  $\alpha$  to a well-defined physical quantity (Spada et al. 2018; Manchon et al. 2024).

In addition to that, great progress have been made regarding the determination of the limit between radiative and convective zones. In previous version of the code, they were determined using a dichotomy method. It leads

to very erratic changes of the limits, in particular for evolved models where the convective core is receding and a gradient of mean molecular weight has build-up. To tackle this issue, we implemented the scheme proposed by Gabriel *et al.* (2014), who advocates that the convective boundaries should be found by extrapolating or interpolating the location of the boundaries between two points located in a convective zone. This results in a smoother evolution of the convective limit location in time, and shorter lived convective cores.

### 3.3 Transport of chemical elements

Cesam2k20 offers the possibility to consider atomic and turbulent diffusion of chemical elements. Atomic diffusion velocities can be calculated with the formalism of Burgers (1969) or with that of Michaud & Proffitt (1993), which considers all elements except hydrogen as test particles diffusing with respect to protons. We can also include in the calculation of the diffusion velocities the contribution of the radiative accelerations, following the Single Value Parameter (SVP) method (Alecian & LeBlanc 2020; Deal *et al.* 2018), which consists, under certain assumptions, in separating the properties of the plasma and of the element, leading to an expression for the radiative acceleration that depends only on six parameters, related to the element of the local properties of the medium.

In addition to atomic diffusion, there is the chemical transport induced by thermohaline convection and semi-convection. The first occurs when the medium is thermally stable but unstable due to the mean molecular weight gradient, and is modelled in Cesam2k20 with the formalism of Brown *et al.* (2013). The second occurs under the opposite conditions and we follow the formalism of Langer *et al.* (1983). Finally, the user can include an additional turbulent mixing, with a turbulent diffusion coefficient decreasing towards high density, as prescribed by Richer *et al.* (2000). We can also slightly tune this prescription to recover the abundances of a given element, i.e. the  $\text{Li}^7$  abundance of the Sun. We are also in the process of implementing the contribution to the chemical transport of (M)HD instabilities included in Cesam2k20, mainly with the intention of transporting more angular momentum.

### 3.4 Transport of angular momentum

Models of angular momentum transport (TAM) have seen many developments in Cesam2k20 in the recent years. Angular momentum is globally conserved except that it can be gained at the surface by planetoid infall, or lost by magnetised stellar winds, for which few scaling relations are available. To initialise the angular momentum distribution, we make use of the disc-locking model (Bouvier *et al.* 1997), where the initial assumed strong magnetic field locks the convective zone to the accretion disc and forces them to co-rotate.

Inside a convective zone, due to the efficient mixing, we assume a uniform distribution of angular velocity (solid body rotation) or of angular momentum (local conservation of angular momentum). Inside a radiative zone, we follow the picture of Zahn (1992) and Talon *et al.* (1997). First, the meridional circulation advects angular momentum. Second, shear-induced turbulence (Kelvin-Helmoltz instability) diffuses the angular velocity  $\Omega$ . Third, since this viscosity is much stronger horizontally than vertically, we assume the radiative zone is in shellular rotation, i.e.  $\Omega$  depends only on the radial coordinate. The TAM is then modelled in Cesam2k20 as an advecto-diffusive process. Many models have been proposed to provide a value for the diffusion coefficients of the Kelvin-Helmoltz instability, either from theoretical considerations or from experiments. Cesam2k20 implements three of them as proposed in Palacios *et al.* (2003); Mathis *et al.* (2004) and Mathis *et al.* (2018).

In improve on this this picture, which is known to be deficient in reproducing observed rotation rates, Cesam2k20 incorporates additional transport mechanisms such as the Goldreich-Schubert-Fricke (GSF) (Hirschi & Maeder 2010; Barker *et al.* 2019) and the Tayler-Spruit (Maeder & Meynet 2004) instabilities, as well as the transport induced by mixed-modes (Belkacem *et al.* 2015).

## 4 Conclusions

We have presented the main points of the numerical machinery of Cesam2k20, as well as some of the options available for modelling three types of physical processes which have known intense developments in recent years: the modelling of convection, chemical and angular momentum transport. Cesam2k20 has, of course, other capabilities that will be described in detail in a forthcoming paper (Manchon *et al.*, in prep.).

Recently, part of our efforts have been put into opening Cesam2k20 to the community. This is done by

creating a website\*, with improved documentation, gradually translated into English, a few models available, and links to the public repository†. The public version will be updated regularly, with new releases and bug fixes. The code is licensed under the GPLv3, which means it can be changed or shared, as long as the source is released under the same licence. We also plan to publish a grid of standard models and more realistic models, including transport processes. The repository also includes the Python package `pycesam` dedicated to post-processing, the optimisation code OSM‡ and two stellar oscillation codes coupled to CEsam2k20: ADIPLS (Christensen-Dalsgaard 2008) and ACOR (Ouazzani et al. 2012). Although we do not provide the source codes, it is also interfaced with the optimisation codes AIMS (Rendle et al. 2019) and SPInS (Lebreton & Reese 2020).

To help the user community with the code, and in addition to the session at the week of the SF2A 2024, we have also organised two-day workshops for two consecutive years, and we plan to continue this rhythm. We are also trying to structure a developer community around CEsam2k20. If you want to implement a new model or prescriptions in the code, please contact us and we will help you understand the details.

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\*<https://www.ias.u-psud.fr/cesam2k20/>

†[https://git.ias.u-psud.fr/joao.marques/cesam2k20\\_releases](https://git.ias.u-psud.fr/joao.marques/cesam2k20_releases)

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