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NON-LINEAR EVOLUTION OF THE HORIZONTAL SHEAR INSTABILITY: INFLUENCE OF THE STRATIFICATION AND FULL CORIOLIS ACCELERATION

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Abstract. The radiative region of stellar interiors is composed of rotating and stratified fluid in a similar fashion as planetary atmospheres and oceans. With the ongoing efforts of characterisation and modelling of these environments, it becomes necessary to build coherent models that treat dynamical processes in any regime of thickness, stratification and rotation. However, many stellar, atmospheric and oceanic models include the traditional Coriolis acceleration with only the local vertical projection of the rotation vector taken into account, also known as the Traditional Approximation of Rotation (TAR). This corresponds to the regime when the stratification dominates the rotation and they do not treat the opposite case, when the rotation dominates the stratification or when both have the same strength. In this contribution, we report the efforts developed to consider the full Coriolis acceleration, which also includes the horizontal projection of the rotation vector, in the perspective of improving the prescriptions for the transport of angular momentum and the mixing of chemical elements. In particular, here, we show how the horizontal local component of the rotation vector can deeply modify the instabilities of horizontal sheared flows and the turbulence they can trigger.

Keywords: hydrodynamics, instabilities, turbulence, Stars: interiors, Planets and satellites: atmospheres

1 Non-linear evolutions of the two-dimensional horizontal shear instability

Shear flows are ubiquitous in stratified and rotating environments like stellar interiors or planetary atmospheres and oceans. Because of their importance for geo- and astro-physical flows, both the horizontal and vertical shear instabilities have been extensively studied in the presence of stratification, viscosity, thermal diffusion and "traditional" rotation (i.e. only considering the local vertical Coriolis component) (e.g. Deloncle et al. 2007; Park et al. 2020). While these studies have been conducted in linear and non-linear regimes, the influence of the full Coriolis acceleration, which includes the "non-traditional" part of the rotation (i.e. the local horizontal Coriolis component), has only been studied in the linear regime (Park et al. 2021; Park & Mathis 2025).

In this work, we investigate the influence of the stratification and full Coriolis acceleration on the non-linear evolution of the horizontal shear instability using direct numerical simulation (see Deloncle 2014). The dynamics is studied in the horizontal plane (x, y) (with x and y the longitudinal and latitudinal coordinates, respectively), where a hyperbolic tangent base flow is maintained at all time. The non-traditional component of the rotation implies that the dynamics remains two-dimensional but with three velocity components (see also Toghraei & Billant 2022, for a similar configuration).

The dynamics depends on four non-dimensional control parameters:

$$N = \frac{N_0 L_0}{U_0}, \quad \tilde{f} = \frac{\tilde{f}_0 L_0}{U_0}, \quad Re = \frac{U_0 L_0}{\nu_0}, \quad Sc = \frac{\nu_0}{D_0}, \tag{1.1}$$

where N_0 is the Brunt-Väisälä frequency, L_0 and U_0 are typical length and velocity scales imposed by the base flow, $\tilde{f}_0$ is the non-traditional Coriolis parameter ($\tilde{f}_0 = 2\Omega_0 \sin\theta$, with Ω_0 the rotation rate and θ the colatitude),

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ν_0 is the kinematic viscosity and D_0 is the diffusivity of the stratifying agent, i.e. either the temperature or the chemical composition/salinity depending on the context.

Figure 1 shows snapshots of the total vertical vorticity for the four typical non-linear evolutions that have been identified. The case for $(N, \tilde{f}) = (5, 0)$ (left panel) shows the non-linear evolution of the instability with the traditional approximation. The vertical vorticity is constrained between 0 and -1 (minimum imposed by the base flow), and assembles into a stable, coherent vortex, following the classical Kelvin-Helmholtz (KH) billow formation. The configuration for $(N, \tilde{f}) = (2, 1.5)$ (middle left panel) only shows light non-traditional effects: a coherent vortex still forms but it is slightly elongated in the latitudinal direction compared to the traditional case. There is also a limited production of positive and negative vorticity, located along the braid and in the core of the vortex, respectively. The third non-linear evolution for $(N, \tilde{f}) = (1, 0.5)$ (middle left panel) has significant non-traditional effects: the vortex forms and secondary instabilities develop within its core, but it still remains somewhat coherent. Positive and negative vorticity are also produced, with the maxima located in the core. The last case for $(N, \tilde{f}) = (0.5, 0.5)$ has strong non-traditional effects: the vortex is destroyed by strong secondary instabilities that transition the flow into small-scale turbulence. These secondary instabilities are new shear instabilities that have developed at the new inflexion points created by stretched braids of positive and negative vorticity. They are also responsible for enstrophy production and enhanced turbulent dissipation, two elements suggesting that the turbulence (and the dynamics) could become three-dimensional when non-traditional effects are strong.

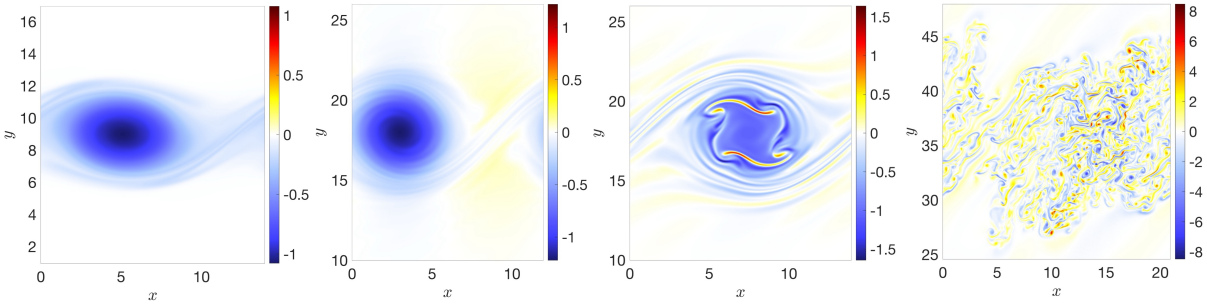

Fig. 1. Total vertical vorticity for different $(N, \tilde{f})$ with $Re = 2000$ and $Sc = 1$. **Left:** $(N, \tilde{f}) = (5, 0)$. **Middle left:** $(N, \tilde{f}) = (2, 1.5)$. **Middle right:** $(N, \tilde{f}) = (1, 0.5)$. **Right:** $(N, \tilde{f}) = (0.5, 0.5)$.

2 Conclusions

Our analysis has shown that non-traditional effects compete with the stratification to drive the non-linear evolution away from the traditional KH billow formation. In particular, for weakly stratified fluids, if $\tilde{f}/N > 0.05$, a transition to small-scale turbulence can occur. In stellar interiors, planetary atmospheres and oceans, this ratio is around $\tilde{f}/N \sim 0.01 - 0.1$, $\tilde{f}/N \sim 0.01$ and $\tilde{f}/N \sim 0.1 - 0.05$ respectively. As such, it is critical to include the full action of the rotation when studying the transport and mixing processes inside these environments.

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