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A MODEL OF MAGNETISED AND ROTATING CONVECTION FOR STELLAR AND PLANETARY INTERIORS

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Abstract. Convection transports energy in stars and planets. It is often modelled in stellar structure and evolution codes with the Mixing-Length Theory, a monomodal approach. However, the standard Mixing-Length Theory ignores rotation and magnetic fields. We generalise it by including both effects through a heat-flux maximisation principle. We find that rotation and magnetic fields each tend to inhibit convection, but combined, they can enhance it in some regimes. We derive expressions for convective velocity, length scale, and superadiabaticity modulations as functions of the rotation rate and the magnetic field strength. These results improve the modelling of convection and of its impact on stellar and planetary structure and evolution in both 1D and future multi-dimensional evolution models.

Keywords: convection, magnetic field, rotation

1 Introduction

Convection is omnipresent in stars and many planets during different evolutionary stages (e.g. Busse 1976; Kippenhahn et al. 2012; Guervilly et al. 2019). It is essential to model it to understand stellar and planetary structure and evolution. However, convection is turbulent, spanning wide spatial and temporal scales, which makes direct treatment on evolutionary timescales challenging (Kupka & Muthsam 2017). Semi-analytical models like the Mixing-Length Theory (hereafter MLT) (Böhm-Vitense 1958; Gough & Weiss 1976; Kippenhahn et al. 2012) provide simplified prescriptions widely used in stellar evolution codes (Paxton et al. 2011, 2019; Jermyn et al. 2023; Dotter et al. 2008; Eggenberger et al. 2008; Amard et al. 2019). MLT treats convection as fluid parcels travelling a mixing length before dissipating, yielding approximate gradients, velocities, and scales that have successfully reproduced many stellar and planetary observable properties (e.g. Baraffe et al. 2015). However, the standard MLT neglects the influence of rotation and magnetic fields. Rotation and/or magnetism are, however, known to modify the convective properties by shifting the onset of the instability (e.g. Chandrasekhar 1961; Gough & Tayler 1966; Eltayeb & Kumar 1977; Horn & Aurnou 2022) and modifying the structure of the convective flow (e.g. Julien et al. 2012; Aurnou et al. 2020; Ecke et al. 1992; Pothérat & Klein 2014). From a theoretical point of view, Stevenson (1979) and Flasar & Gierasch (1978), followed by Augustson & Mathis (2019), reformulated MLT for rapid rotation using the heat-flux maximisation principle of Malkus (1954). Direct Numerical Simulations in a Cartesian frame of reference have shown good agreement with these prescriptions (e.g. Barker et al. 2014; Currie et al. 2020). In this work, we extend the models of Stevenson (1979) and Augustson & Mathis (2019) to derive the prescriptions for the rotating and magnetised Mixing-Length Theory for any rotation rate and magnetic field strength.

2 Heat-flux maximised convection: theoretical model

The model assumes that the nonlinear convective state is dominated by the mode carrying the most heat, yielding characteristic velocities, length scales, and superadiabaticity. We consider an infinite fluid layer subject to vertical rotation $\boldsymbol{\Omega}$ and a vertical imposed magnetic field $\boldsymbol{B}$. The fluid is treated under the Boussinesq approximation, i.e. density fluctuations are neglected except in the buoyancy term, which is valid when convective length scales are much smaller than those of density variations. The thermal expansion coefficient is

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$\alpha = -\partial \ln \rho / \partial T|_P$, with ρ the density, T the temperature, and P the pressure. The layer is bounded by two infinite, impenetrable plates held at different temperatures and separated by a distance ℓ_0. We neglect thermal, viscous, and magnetic diffusivities as a first stem. We note s the growth rate of the convective instability and $\boldsymbol{k}$ its wavevector. We also introduce the characteristic pulsations $\omega_B^2 \equiv \frac{(\mathbf{k}\cdot\mathbf{B})^2}{\mu_0\rho}$, $N^2 \equiv \frac{g\alpha\beta k_\perp^2}{k^2}$, $\omega_\Omega^2 \equiv \frac{(2\Omega\cdot\mathbf{k})^2}{k^2}$., as well as the reduced quantities : $N_*^2 = |g\alpha\beta|$, $\hat{s} = \frac{s}{N_*}$ and $z^3 = 1 + a^2 = \frac{k^2}{k_z^2}$, where $a^2 = \frac{k_x^2}{k_z^2} + \frac{k_y^2}{k_z^2} = a_x^2 + a_y^2$., where $\boldsymbol{g}$ is the gravity and β is the average deviation of the thermal gradient from the adiabatic value

In this framework, the dispersion relation for the convective instability is:

$$\hat{s}^4 + \hat{s}^2 \left(2\mathcal{P}^2 + \mathcal{O}^2 - \frac{(z^3 - 1)}{z^3} \right) + \mathcal{P}^4 - \mathcal{P}^2 \frac{(z^3 - 1)}{z^3} = 0, \tag{2.1}$$

where we have introduced the rotation and magnetic parameters are: $\mathcal{O}^2 = \frac{4\Omega^2}{N_*^2}$ and $\mathcal{P}^2 = \frac{\omega_B^2}{N_*^2}$, respectively. Moreover, we maximise the heat flux, following Malkus (1954) principle:

$$\mathcal{F}_{\Omega,B} = \frac{\rho c_p}{\alpha g} \frac{N_*^3}{k^2} \hat{s} \left(\hat{s} + \frac{\mathcal{P}^2}{\hat{s}} \right). \tag{2.2}$$

Finally, we make the assumption that the heat flux with rotation and magnetism is equal to the maximum value of the heat flux without rotation or magnetism, because the luminosity due to the internal energy source is not strongly modified by these phenomena (see e.g. Augustson & Mathis 2019):

$$\mathcal{N}_0 q^3 z^3 = \hat{s}^2 \left(\hat{s} + \frac{\mathcal{P}^2}{\hat{s}} \right), \tag{2.3}$$

where $\mathcal{N}_0 = \left(\frac{3}{5}\right)^{3/2} \frac{2}{5}$, and $q = \frac{N_{*,0}}{N_*}$, where the index $\cdot_0$ denotes the non-rotating and non-magnetised value.

This leads to the modulation of superadiabaticity in the rotating and/or magnetised case, compared to the non-rotating and non-magnetised situation $\tilde{\epsilon} = \frac{\epsilon_{\Omega,B}}{\epsilon_0} = \frac{1}{q^2}$, as well as the velocity modulation $\tilde{U} = \frac{v_{\Omega,B}}{v_0} = z^{-3/2} \frac{5\hat{s}}{\sqrt{6}q}$. and the wavenumber modulation $\tilde{K} = \frac{k_{\Omega,B}}{k_0} = z^{3/2}\sqrt{\frac{2}{5}}$.

Moreover, we define the main dimensionless key control parameters of this study. The Rossby number is: $\mathcal{R}o = \frac{v_0}{2\ell_0\Omega} = \frac{\sqrt{6}q}{5\pi\mathcal{O}}$, and the inverse Alfvén number is: $A = \frac{v_A}{v_0} = \frac{5}{\sqrt{6}} \frac{\mathcal{P}}{q}$, where $v_A = B/\sqrt{\rho_0\mu_0}$ is the Alfvén velocity, μ_0 being the magnetic permeability in vacuum.

To compute the velocity, wavenumber and superadiabaticity modulations in the rotation and/or magnetised case, we seek the mode that maximises the flux using Eq. (2.2).

3 Rotating convection

The rotating MLT (R-MLT) model has been developed by Stevenson (1979) in the rapid rotation limit, and extended by Augustson & Mathis (2019), who treated all ranges of Rossby numbers, as well as the effects of heat and viscous diffusivities. As we solve the same set of equations as Augustson & Mathis (2019), we recover their results. Such R-MLT results have been compared with three-dimensional direct numerical simulations of Boussinesq convection in local Cartesian boxes, which hold for three decades in the Rossby number (e.g. Käpylä et al. 2005; Barker et al. 2014; Currie et al. 2020; de Vries et al. 2023).

4 Magnetised convection

We take the non-rotating limit of the previous set of equations (2.1), (2.2) and (2.3), i.e. $\mathcal{O} = 0$. The results for the modulation of the convective wavenumber, velocity and superadiabaticity are shown in Fig. 2 and compared with Stevenson (1979) asymptotic expressions for the low-A and the high-A regimes. As Stevenson (1979) does not account for the transition around $A \sim 1$, we represented this region with a hatched background. The tendencies for the magnetised case are the same for our study and the one from Stevenson (1979): the higher the magnetic field strength, the higher the Alfvén number and the lower the convective velocity and the higher the convective wavenumber. Moreover, the superadiabaticity also increases when the magnetic field is stronger, which is in line with the general idea that the magnetic field inhibits convection.

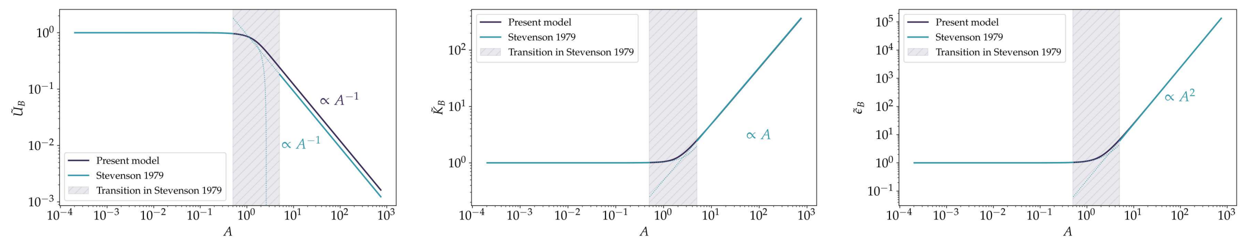

Fig. 1. Results for the Magnetised MLT model. Left: Convective velocity modulation, Centre: Convective wavenumber modulation, Right: Superadiabaticity modulation, as a function of the inverse Alfvén number A.

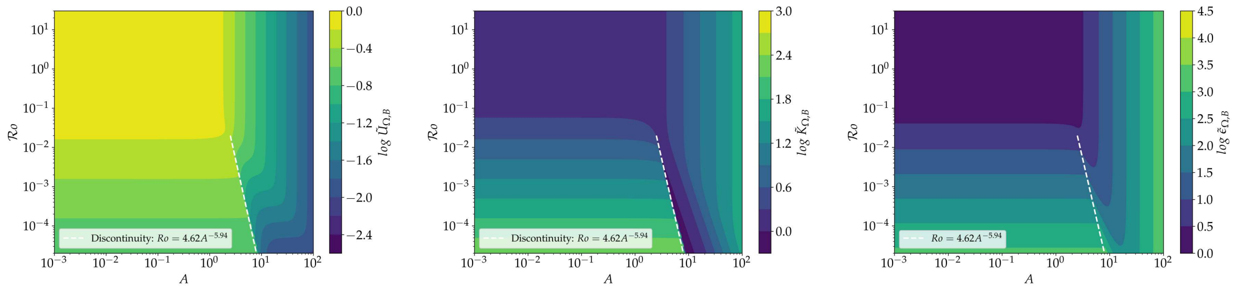

Fig. 2. Results for the Rotating and Magnetised MLT model. Left: Convective velocity modulation, Centre: Convective wavenumber modulation, Right: Superadiabaticity modulation, as a function of $\mathcal{R}o$ and A.

5 Rotating and magnetised convection

Finally, we consider the case with both non-zero rotation and non-zero magnetic field, i.e. $\mathcal{O} \neq 0$ and $\mathcal{P} \neq 0$. We use again the heat-flux maximisation principle, with the method described in Sec. 2. We plot the convective velocity $\tilde{U}$, wavenumber $\tilde{K}$, and superadiabaticity $\tilde{\epsilon}$ modulations in Fig. 2. All quantities show a discontinuity near $\mathcal{R}o \sim 2.62A^{-5.94}$, due to a change in the mode that carries the most heat. In the strong-field ($A \gg 1$) or rapid-rotation ($\mathcal{R}o \ll 1$) limits, convection weakens: $\tilde{U}$ decreases while $\tilde{K}$ and $\tilde{\epsilon}$ increase. Across the discontinuity ($A \sim 1-7$, $\mathcal{R}o \lesssim 5 \times 10^{-2}$), $\tilde{K}$ drops by two orders of magnitude, implying larger convective scales and reduced velocity. This scale enlargement, observed experimentally in liquid mercury (Nakagawa 1957), has not been found in recent simulations (Horn & Aurnou 2025), likely due to boundary effects. By contrast, $\tilde{\epsilon}$ remains continuous. Rotation and magnetism generally increase superadiabaticity, reducing the convection efficiency. Yet near the discontinuity ($A \sim 1-9$, $\mathcal{R}o \lesssim 10^{-1}$), stronger fields lower $\tilde{\epsilon}$ at fixed $\mathcal{R}o$, suggesting that rotation and magnetic fields together can enhance convective transport (e.g. Chandrasekhar 1961; Horn & Aurnou 2022).

6 Conclusions

We have developed a generalised Mixing-Length Theory (RM-MLT) including both rotation and magnetic fields, extending earlier (Stevenson 1979; Augustson & Mathis 2019) formalisms based on the heat-flux maximisation principle (Malkus 1954). The present model recovers known asymptotic behaviours for the convective wavenumber modulation, velocity modulation and superadiabaticity modulation (e.g. Stevenson 1979) for the rotating and the magnetised case, while allowing to treat any value of the rotation rate and the magnetic field amplitude. When rotation and magnetic field are put together, we predict a discontinuity linked to competing convective modes. Rotation and magnetism generally reduce convective efficiency, yet together may enhance transport in specific regimes (Horn & Aurnou 2022).

Perspectives include adding diffusive processes (Augustson & Mathis 2019), testing against DNS and experiments (e.g. Nakagawa 1957; Horn & Aurnou 2025), and implementing RM-MLT in stellar evolution codes (see e.g. Ireland & Browning 2018). This work is also relevant to studying specific physical mechanisms such as the overshoot (Zahn 1991), wave excitation (Mathis et al. 2014; Augustson et al. 2020), and tidal dissipation (de Vries et al. 2023). More generally, it offers new insights into the coupled role of rotation and magnetic fields in stellar and planetary evolution.

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