

LONG-TERM & LARGE-SCALE SIMULATIONS OF SATURN'S RINGS : VARIABLE VISCOSITY & SATELLITE INTERACTIONS

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Abstract. We use a 1-dimensional hydrodynamic code to simulate the global evolution of Saturn's Rings through viscous spreading, including satellites torques. While previous studies, using constant viscosities, suggest a rapid spread out of the ring system in a few hundred million years (Esposito 1986), we show that new viscosity prescriptions derived from N-body simulations such as the one of Daisaka et al. (2001) would dramatically affect the large scale evolution of the ring system, allowing for a survival of the rings over 5 billion years. We show also that transitions from self-gravitating to non self-gravitating regions would produce large scale structures. Surprisingly the final state of the ring system seems somewhat independent of the initial mass using the viscosity of Daisaka et al. (2001). The possibility of confinement by nearby satellites is still under investigation but first results suggest that they could significantly lengthen the rings viscous age.

1 Introduction

Saturn's rings have been studied for more than 400 years and still remain a puzzling object of our Solar System. While most popular scenarios for their formation are based on phenomena occurring in the early ages of our Solar System (Pollack et al. 1973; Dones 1991; Charnoz et al. 2009), several recent observations come to the conclusion that the rings must be quite young (Esposito 1986; Cuzzi & Estrada 1998).

The evolution of Saturn's rings share a lot of similarities with other disks in the universe, e.g. protoplanetary discs or accretion discs. They evolve through 3 main physical processes : viscous spreading, resonant interactions with nearby satellites, and meteoritic bombardment. In the present study we investigate the effects of non-constant viscosity that has been quantified recently in small scale N-body simulations.

The viscous spreading is responsible for the flattening and the widening of the rings (Linden-Bell & Pringle 1974). During this process, mass is lost when material falls onto the planet, or crosses the Roche limit and accretes in satellites. Constraining the timescales for this physical process is thus fundamental to determine the age of Saturn's rings.

2 Viscous spreading with variable viscosity

2.1 Our model

We study the evolution of the surface density of the disk Σ , considering only the radial component of the evolution. Using the formalism of Pringle (1981) we combine the equations of mass and angular momentum conservation to obtain the single equation

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[\sqrt{R} \frac{\partial}{\partial R} \left(\nu \Sigma \sqrt{R} \right) \right] \quad (2.1)$$

where R is the distance to Saturn, and ν is the kinematic viscosity.

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Several N-body simulations have been performed to determine the expression for the viscosity of a disk of particles (Salo 1992; Richardson 1994; Daisaka & Ida 1999; Ohtsuki & Emori 2000). In this work we use the results of Daisaka et al. (2001). They propose a 3-component model for the viscosity : 1) the translational (or "local") viscosity that accounts for the direct collisions between particles, 2) the collisional (or "non-local") viscosity that accounts for the finite size of particles, and 3) the gravitational viscosity that accounts for the self-gravitational torque between particles. This third component is the main improvement of this work. We use the analytical fit to their numerical results in our simulations.

We have developed a 1D hydrodynamical code to compute the variation of the surface density through time by numerically solving equation 2.1. We evaluate the variation of the surface density at each point of a grid by computing the material fluxes through the limits of each bin of the grid. Because of this approach we lose every perturbation smaller than the grid resolution, but it allows us to perform simulations over the age of the Solar System.

2.2 Viscous spreading over 5 billion years

We study the evolution of the disk considering only the viscous torque, using the variable viscosity model described in section 2. Our initial ring is a Gaussian distribution centred on $R = 110000$ km and 3000 km wide. The initial mass of the rings is about the mass of the satellite Mimas, which is the estimated value of today's rings' mass (Tiscareno et al. 2007). We use particles with $r_p = 1$ m, as suggested by Goldreich & Tremaine (1982).

The first hundred thousand years of evolution are plotted on the left graph of Fig. 1.

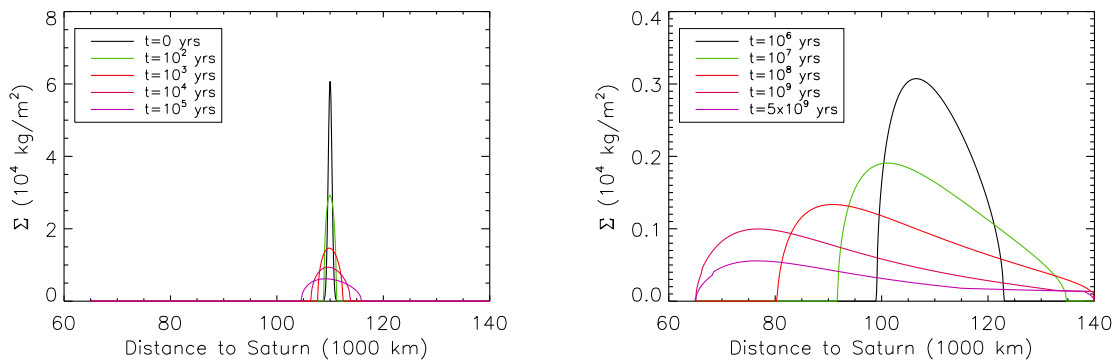


Fig. 1. Viscous evolution of the surface density over 10^5 years (left) and 5×10^9 years (right)

The first comment we can make is that the disk spreads with very stiff edges. This is very different from the spreading with constant viscosity (e.g. (Pringle 1981)) where the progressive edges of the gaussian are conserved through the spreading. This is due to the fact that the edges of the rings have very low surface density and so their viscosity is very low. Therefore, they spread less rapidly than the more dense and viscous core of the rings. Once the edges have been overwhelmed by the material from the core, their viscosity raises and the whole disk spreads.

The evolution up to 5 billion years is plotted on the right graph of Fig. 1. One can see that the spreading rate is quite slow, particularly for the inner edge. The disk is fully spread only after 1 billion years. It is also noticeable that after 5 billion years of evolution, the disk has not disappeared, its average surface density being of the order of 10^2 kg/m^2 which is close to the surface density of the middle of today's A ring (Tiscareno et al. 2007).

With variable viscosity, the survival of a disk of dense planetary rings in the dynamical environment of Saturn over the age of the Solar System seems then possible.

2.3 Evolution of the disk's mass

We have plotted in Fig. 2 the evolution of the mass of the disk, for different initial masses ranging from 1 to 10 Mimas' masses.

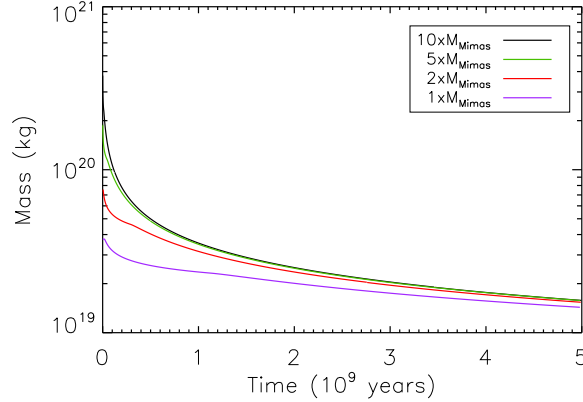


Fig. 2. Evolution of the disk's mass over 5 billion years.

Starting with 1 Mimas' mass (violet curve), the disk's mass is still about 10^{19} kg after 5 billion years of viscous evolution. To study the influence of the initial mass on this result, we performed the same simulations starting with different masses. It appears that whatever the initial disk's mass, the final mass is always about 1×10^{19} . Using the condition of self-gravity of the disk (Toomre 1964; Salo 1995), we computed that this mass corresponds to the maximal mass of a fully non self-gravitating disk. This disk evolves very slowly because in the viscosity model we use the viscosity drops importantly when the disk becomes non self-gravitating.

3 Satellite interactions

We now include in our simulations a satellite with a mass close to Janus' mass. We consider only the first order Lindblad resonances of the disk, and use the formalism of Takeuchi et al. (1996) to compute the evolution of the disk and the migration of the satellite. We plot our results on Fig. 3.

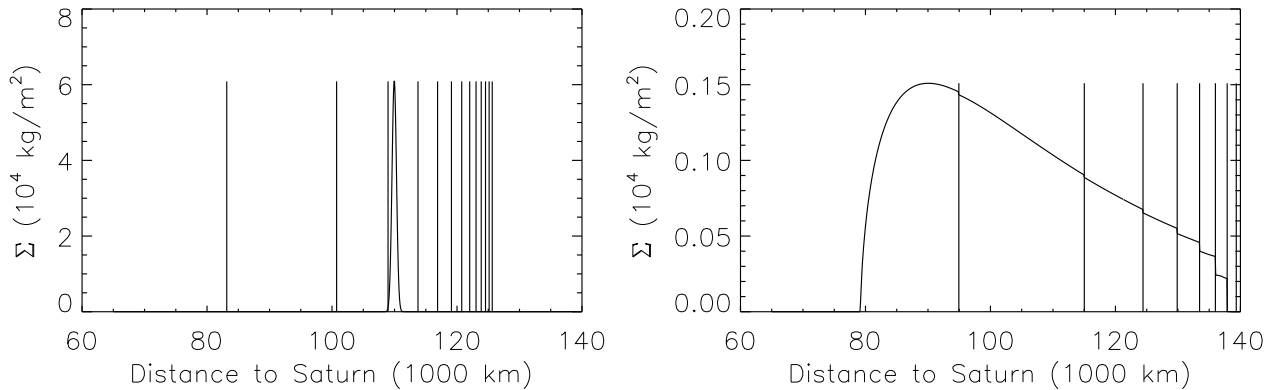


Fig. 3. Initial disk's surface density (left) and after 100 million years of evolution (right). The vertical lines correspond to the Lindblad resonances' locations.

The vertical lines on the graph are the positions of the 15 first first-order Inner Lindblad Resonances. At this locations, the satellite exerts a negative torque that flushes the material inward. The disk itself exerts a

retroactive torque on the satellite that starts to migrate outward. The initial configuration is plotted on the left graph of Fig. 3. The satellite's initial position is 132000 km.

The situation of the simulation after 100 million years of evolution is plotted on the right part of Fig. 3. The disk has spread because of the viscosity, but the shape of the surface density is altered at resonances positions : while the viscous torque tends to send material outward, the negative torques at resonances positions prevents the material from doing so. The material is then stuck at the resonance, and so the surface density increases locally, leading to this stairs-like shape.

It is also noticeable that the resonances have moved toward the right. This is due to the retroactive torque from the disk that makes the satellite migrate outward, which moves the resonances positions. After 100 million years of evolution, the satellite's semi-major axis is 169000 km.

Just like the surface density, the mass lost by the disk is also altered by the presence of the satellite. First it is significantly reduced, about two times less than without the satellite. Second, as material is stuck at resonances positions, there are abrupt releases of material when a resonance leaves the disk. The loss of mass by the disk in this configuration is then very "bumpy" and not continuous like in Fig. 2.

4 Conclusions

Using a model of variable viscosity we showed in our simulations that Saturn's rings may have survive against viscous spreading over 5 billion years. We found that the final mass of the disk seems somewhat independent of the initial mass, and that the disk seems to evolve toward a more stable state which is a situation where it becomes entirely non self-gravitating.

Contrarily to the spreading with constant viscosity, the shape of the disk's surface mass density is significantly modified, in particular in the formation of very stiff edges because of the viscosity dropping due to the low surface density values.

While only an early result, we showed that resonant interactions with outer satellites can significantly reduce the outward spreading of the rings, which would increases their survival time. Adding several other satellites is needed to constrain more precisely the effect of the resonant interactions on the spreading time scale of the rings.

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