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PLANE-PARALLEL NUMERICAL STUDY OF THE VISHNIAC INSTABILITY IN SUPERNOVA REMNANTS

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Abstract. In this work we study the Vishniac instability with the HYDRO-MUSCL2D code. In the framework of supernova remnants, we realize numerical simulations of a shock front perturbed by a sinusoidal disturbance in the plane-parallel geometry. We vary the wavenumber of the perturbation to understand its effect on the evolution of the instability. For the perturbative wavenumber range of this geometry, we observe an oscillation of increasing amplitude like the Vishniac overstability. Furthermore we obtain the similar dependence of the numerical and theoretical models between the growth rate of the instability and the wavenumber.

1 Introduction

The Vishniac instability discovered analytically in the context of radiative shocks is not well known nowadays. With numerical simulations, we want to explore the parameters allowing the triggering and the growth of the instability and we finally want to go deeper in the understanding of this process. In the astrophysical context, the Vishniac instability is invoked to explain the fragmentation of the shock front of supernova remnants (SNRs) when the blast wave resulting from a star explosion evolves in the radiative phase and particularly in the Pressure Driven Thin Shell (PDTS) stage. The linear regime of the instability was theoretically studied by Vishniac in 1983 (see also Cavet et al. 2008) and was continued to a less constraining approximation by Ryu & Vishniac (1987) and Vishniac & Ryu (1989). In SNRs, the action of instabilities on the resulting morphology of these objects is confirmed by observations (Mac Low & Norman 1999; Raymond 2003). But the role of the Vishniac instability is not yet proved in these astrophysical objects. In laboratory astrophysics, several experiments based on laser facilities have been achieved on this subject (Grun et al. 1991; Edens et al. 2007) and they have enabled to produce a hydrodynamic instability on the radiative shock front. But the discrepancy between the experimental and the analytical growth rate of the instability does not allow the probate of the existence of the instability in the laboratory. To improve the understanding of the Vishniac instability, we perform a numerical study of a perturbed thin shell of shocked matter propagating under the strong shock regime in plane-parallel geometry ((y, x) inverse coordinates). The instability starts when a small perturbation appears on the thin shell creating a mismatch between the ram pressure and the thermal pressure which push on both sides of the thin shell. The consequence of this mismatch is the establishment of opposite matter flows along the thin shell *i.e.* in the transverse direction of the shock front propagation. A crucial point of numerical simulations of the instability is to correctly introduce the perturbation on thin shell. In a previous paper (Cavet et al. 2009) we have introduced perturbative spots on the density to trigger the instability but this method does not allow to easily control the wavenumber of the perturbation. In the present approach, we introduce a sinusoidal perturbation on the thin shell defined by a wavenumber l and an amplitude A. To explore a part of the parameters leading the instability, we set the shock front velocity V_s (along the x axis) and A and we vary l to analyze the effect of the perturbative wavenumbers on the growth of the instability. Furthermore, we study the Vishniac overstability on short time evolution of the simulations. The overstability, predicted by Vishniac (1983), consists in obtaining an oscillation of increasing amplitude on the fluid parameters (density ρ, velocity

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v, and pressure p) and on the spatial parameter x. We observe this process on the thin shell displacement until unfortunately the numerical carbuncle instability (Quirk 1994) perturbs the system. Finally, we present the numerical results of the growth of the shock front density and we compare the growth rate of the instability to the analytical one.

2 Numerical simulations

To realize numerical simulations of a perturbed shock front we use the HYDRO-MUSCL2D code developed by our team (Cavet et al. 2009). We realize the simulations of the instability in two steps. Firstly we create an unperturbed isothermal (radiative approximation) shock provided by the strong explosion model adapted to the plane-parallel geometry. In order to simulate the strong and punctual explosion, we introduce an energy $E_1 = 10^{44}$ J in a strip of cells. The simulation produces three different media: the internal region included between the cell strip of the explosion and the internal side of the evolving thin shell (medium 1), the thin shell itself included between its internal side and the shock front (medium 2), and finally the region representing the interstellar medium (medium 3) where the density is $\rho_3 = 10^{-20}$ kg.m^{-3}. The adiabatic indices are set for the three media at $\gamma_1 = \gamma_3 = 5/3$ and $\gamma_2 = 1.1$ where the value of γ_2 reports the energy loss by radiation in an optically thin radiative approximation. We let evolve the self-similar shock front until it reaches a preselected shock front velocity V_s. This settled parameter have an effect on the growth of the instability but its effect is not predicted by the theory and then V_s can take values on a large domain. The only constraint is that the velocity has to be sufficient to preserve the strong shock regime (*i.e.* the shock front compression equal to $(\gamma_2 + 1)/(\gamma_2 - 1)$). Nevertheless the choice of the value of V_s is not easy. The self-similar law for the PDTS phase gives a velocity range $V_s \sim 150 - 200$ km s^{-1}. But observations of the radiative Crab Nebula SNR give a higher velocity $V_s \sim 300$ km.s^{-1} (Sankrit & Hester 1997). Considering different reasons, we have chosen in this study $V_s = 470$ km.s^{-1}.

Secondly we introduce a sinusoidal perturbation on the previous unperturbed thin shell to trigger the instability as Blondin & Marks (1996) in their study of the Non linear Thin Shell Instability. We chose to perturb the shape of the shock front and not the fluid parameters differently of Mac Low & Norman (1993). The interest of our approach is its linkage property with the analytical model of Ryu & Vishniac (1987), thus we can retrieve the same optimal wavenumber. We can directly control the sinusoidal perturbation by two parameters: the wavenumber l and the amplitude A. In this study, we vary only l. The theoretical analysis gives the instable wavenumber range for the plane-parallel case (Ryu & Vishniac 1987): $l = [4 - 26]$ with a maximal growth rate for $l = 14$. Then we set the amplitude $A \sim 8\%$ of the wavelength λ (where $\lambda = 2\pi x_s/l$) to initialize the linear regime of the instability but in a future work we will study the forced nonlinear regime where $A > 10\%$ of λ. The results of simulation shown in Fig. 1 are realized with the more perturbative wavenumber $l = 14$ (*i.e.* the

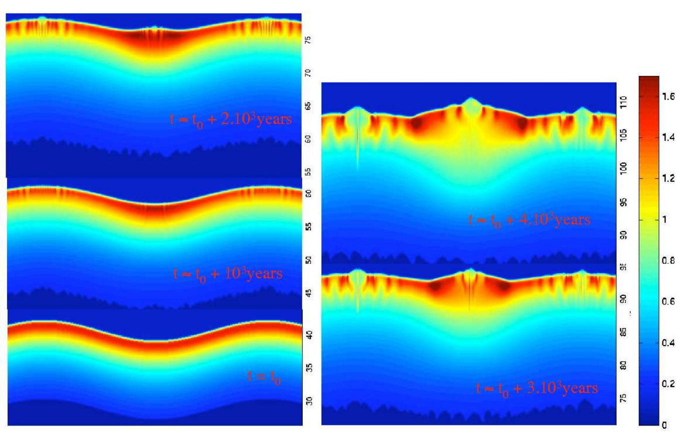

Fig. 1. Evolution of the Vishniac instability in plane-parallel geometry for $l = 14$, $A = 8\%$ of λ and $V_s = 470$ km.s^{-1}: snapshots of a zoom of the density map in 10^{-19} kg.m^{-3}. The shock front propagates according to the x axis (vertical direction) and the origin of this axis corresponds to $x_0 = 42 \times 10^{15}$ m. On the zoom, the evolution of one valley and two hills is observable. The shock front matter moves from the hills to the valley and *vice-versa*. The empty bubble appearing at $t = t_0 + 10^3$ years are due to the numerical carbuncle instability.

optimal wavenumber). In this simulation we have introduced the perturbation at the SNR age $t_0 \sim 4 \times 10^3$ years and we have let evolve the instability during $\Delta t = 10^4$ years. At the first step of the evolution ($t = t_0 + 10^3$ years), the process predicted by the theoretical model is acting on the thin shell: diminution of the density on the hills (orange in the density scale) and growth of density in the valleys (dark red) due to the action of a transversal flow moving the matter from the hills to the valleys. This flow of shoked gaz is the strongest at the maximum deflection point *i.e.* the middle point between a hill and a valley. At this moment, the maximum of density in the valley is $\rho_{valley} = 1.08\,\rho_{s,init}$ where $\rho_{s,init}$ is the initial density on the shock front *i.e.* the unperturbed shock front density. The spatial perturbation triggers the density variation. Then at $t = t_0 + 2 \times 10^3$ years, we already observe the deformation of the valley structure on the x axis indicating a change of the matter motion in the thin shell. At this time we remark a second linked effect which is the taking up of the lagging of the valleys. Indeed the positions of the valleys are close to the position of the hill. This evolution of the initial sinusoidal perturbation of the thin shell is the Vishniac instability. We understand better this phenomenon at $t = t_0 + 3 \times 10^3$ years when the transversal flow changes of direction. Indeed the valley matter is divided in two clumps, then the valleys lose their matter and become hills and *vice-versa*. This oscillating process of the thin shell displacement and the fluid parameters is the overstability predicted by Vishniac (1983). More latter and until the end of the simulation, we observe a numerically perturbed phase with transformation of numerical oscillations in empty bubbles with triangular structures due to the numerical carbuncle instability acting on perturbed shock front in this geometry (Quirk 1994).

3 Discussion of the results

After this first morphological study, we focus ourselves on the first phase of the instability where we have only small numerical problems. We let evolve the perturbed shock front during $\Delta t = 2 \times 10^3$ years. During this period, we want to understand how the wavenumber l acts on the evolution of the fluid parameters by calculating the growth rate s of the instability and by comparing the numerical growth rate law $s(l)$ with the analytical one. First, we make a cut on the map density following the shock front to see the parameter evolution. We

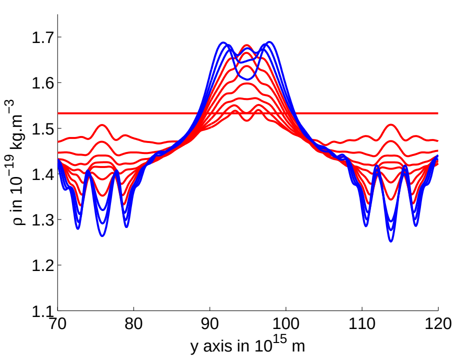

Fig. 2. Evolution of a density cut following the shock front (cut alongt the y axis, density in 10^{-19} kg.m^{-3}). Each curves correspond to one time and the time step is $dt = 0.2 \times 10^3$ years . The density is smoothed to remove the numerical oscillations present on hills and valleys.

visualize in Fig. 2 one part of this cut (two hills and one valley) to study the density variation at the center of the valley (at $y = 95 \times 10^{15} m$ in Fig. 2). At this position, we see the growth (in red) and the decrease (in blue) of the density due to the transversal motion of the matter in the thin shell and we can calculate the density perturbation $\delta\rho = |\rho_{pert} - \rho_{s,init}|$ where we choose $\rho_{s,init}$ constant and given by the straight line at $t = 0$, $\rho_{s,init} = 1.54 \times 10^{-19}$ kg.m^{-3}. The theoretical analysis gives the variation of the density perturbation with time t: $\delta\rho \propto K t^s$ where $s = s_r + i\,s_i$ is the complex growth rate and K a constant including the self-similar profile of the unperturbed density. With a χ^2 fitting, we find the growth rate of the instability for several wavenumbers visualized in Fig. 3. Comparing with the analytical model, we do not find the same value of the growth rate but we observe the same dependence between the growth rate and the wavenumber: the optimal wavenumber is $l = 14$. But we have large error bars in our data due to the small size of the sample and due to the numerical noise created by the carbuncle instability. We note that not all our simulations realized with a

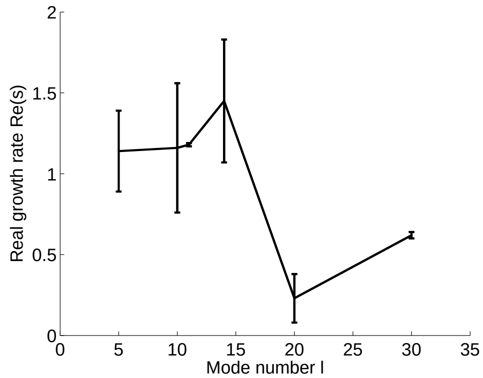

Fig. 3. Real part of the growth rate s_r versus the wavenumber of the perturbation l. We observe a similar law with the analytical model.

specific wavenumber are included in this plot due to some initialization problems in these simulations preventing a normal evolution of the density variation in our measurement point, the valley center. This fact points out that the measure of the density in only one point is not the better parameter to determine the growth rate. In future works we will use the mass of a hill and a valley, *i.e.* the density integrated over a half period of the wavelength λ, to find s.

4 Conclusion

In this numerical study, we have obtained the Vishniac overstability. We have observed the oscillation of the thin shell displacement and we have found numerically the same theoretical law between the growth rate and the wavenumber of the perturbation. But in this work we have tested only one part of the instable parameters and we need to purchase this study to understand the effect of the other variables on the growth of the instability, specially in cylindrical and spherical geometries where the action of the carbuncle instability is reduced on the axis of the thin shell. In an other domain, we have two experimental projects at short and long terms. In a close future, we propose in collaboration with Edens et al. on the Z-Beamlet laser of Sandia laboratory to prove the existence of the Vishniac instability in laboratory. In a later future, we want to explore the instability parameters on the LIL facility (high-power laser, 60 kJ) in Bordeaux. Then, by two different approaches, namely numerical analysis and laboratory experiments, we tend to a complete overview of the Vishniac instability.

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