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PROPERTIES OF PHONONS IN THE NEUTRON STAR CRUST

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Abstract. Neutron stars are compact objects, created in supernova explosions at the end of the life of massive stars. They contain matter under extreme conditions, in particular concerning the density : starting from a lattice of (neutron rich) nuclei in the crust one reaches nuclear matter at several times the density of atomic nuclei in the center. One way to understand this object is to confront theoretical modelisation with observations. Among observations of pulsars there is the thermal emission of its surface. This observable, which depend on the heat transport properties, is very sensitive to the superfluid and superconducting character of the different sutructures inside the star. The presentation is focussed on the inner crust, where we can find an interesting nuclear structure called the "Pasta Phase". Its excitation spectrum within a superfluid hydrodynamics approach will be discussed in view of calculating the contribution to the heat capacity.

Keywords: neutron star crust, nuclear matter, superfluidity, hydrodynamic modes

1 Introduction

Neutron stars are fascinating objects, containing matter under extreme conditions of temperature, density and magnetic field. In order to study these celestial bodies, theoretical modelisation has to be confronted with observations. A prominent observable is the thermal evolution of isolated neutron stars. The surface temperature can be deduced from the thermal emission. The age, if the neutron star cannot be associated to a known supernova event, can be obtained measuring the ratio $P/2\dot{P}$, where P is the pulse period. From this ratio, assumming the pulsar to be a rotating magnetic dipole, the age can be estimated. The observation of the thermal evolution puts stringent constraints on cooling models. Since the surface temperature is the result of the interplay between thermal radiation and heat transport from inside the star, it is sensitive to the microscopic processes occuring in the different parts of the star at different evolution stages determining heat transport properties of neutron star matter. These are, as neutrinos play an important role for neutron star cooling, neutrino emissivities, and in addition heat capacity and thermal conductivity. Here, we will concentrate on the heat capacity. There are contributions to the heat capacity from all the different possible excitations at the given temperature, such that it is important to consider the entire excitation spectrum. More details about the evaluation of the specific heat and a discussion of the usually considered contributions can be found in Gnedin (2001). In what follows, we will concentrate on the inner crust.

The inner crust of neutron stars is characterised by a transition from homogeneous matter in the core to a lattice of atomic nuclei in the outer crust. Ravenhall & al. (1983) predicted that this transition passes via more and more deformed nuclei. Starting from an almost spherical shape, they could form tubes or slabs immersed in homogeneous neutron rich matter at the different densities. These phases are commonly called the nuclear pasta.

In neutron stars older than several minutes, matter becomes superfluid with energy gaps of the order of 1 MeV in the inner crust, see Chamel (2008). This means, that individual nuclei cannot easily be excited and that their contribution to the specific heat is thus strongly suppressed. The main contributions to the heat capacity considered so far in the crust are thus electrons and lattice vibrations as well as collective excitations of nuclei. However, the superfluid character of neutron star matter induces collective excitations, not considered before, which can give an important contribution to the heat capacity in certain regions, see Aguilera (2009). The aim of this paper is to study these collective excitations in the inner crust employing a superfluid hydrodynamics approach.

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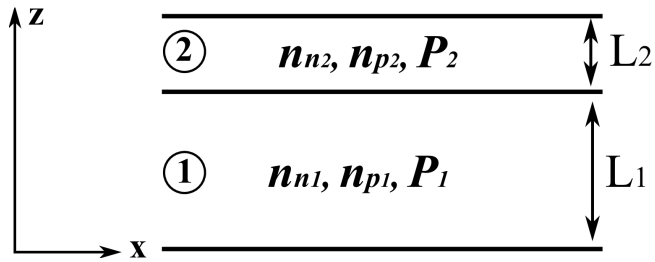

Fig. 1. Representation of "lasagna" structure

2 Superfluid hydrodynamics for the "Lasagna" phase

2.1 Characteristics of "Lasagna"

As mentioned above, the geometrical structure of the different phases in the inner crust can be very different. For the moment we limit our approach to one-dimensional inhomogeneities, i.e. to the phase called "Lasagna". It appears close to the core and it is characterised by a periodic alternance of slabs with different proton and neutron densities as illustrated in Fig. 1. As can be explained from the origin of this phase as deformed nuclei immersed in neutron rich matter, one slab contains nuclear matter with proton and neutron densities of the same order, whereas the other one, having a lower total baryon density, is largely dominated by neutrons. The latter will be labelled by an index "1", the former by "2". The width of a slab (L_1, L_2 in Fig. 1) is typically of the order of 5 fm and the overall baryon density is approximately half nuclear matter saturation density, i.e. 0.08 fm^{-3}.

2.2 Superfluid hydrodynamics approach

We consider a model with two (super)-fluids, one for protons and one for neutrons. Since the considered temperatures are much lower than the pairing gap, we are working in the zero temperature approximation. We are studying local microscopic effects, and the fluid velocities involved are low, such that relativistic effects are probably very small and we can use the non-relativstic approximation. In order to derive the superfluid hydrodynamics equations, we start from energy-momentum conservation and particle number conservation for each species*. We then expand the equations to first order in density perturbations around stationary equilibrium leading to wave equations for the sound waves. Since superfluids have no viscosity, naively we would end up with two sets of independent equations per fluid. There is, however, a non-dissipative interaction between the two fluids called entrainement. The entrainement effect coupling the two superfluids has been first discussed by Andreev & al. (1975) and is nowadays a well known ingredient for superfluid hydrodynamics in homogeneous neutron star matter, see e.g. Prix (2004). Because of entrainement, the two resulting sound modes with corresponding sound speeds are not pure proton or neutron modes, but they are coupled. The parameters of our model are calculated within a Landau-Fermi liquid approach using a relativistic mean field type effective nuclear interaction, see Avancini (2009).

In order to describe the propagation of waves through the slabs, we have to specify boundary conditions at each interface. Our basic assumption is here that contact between the fluids is maintained at all times. This is a standard assumption in problems of wave propagation in inviscid fluids. This assumption implies continuity of fluid velocities perpendicular to the interface and continuity of chemical potentials across the interface. The periodicity is taken into account using the Floquet-Bloch theorem. This means that the wave function $U(z)$ has to satisfy the following condition:

$$U(z+L) = e^{iqL}\ U(z)\ , \tag{2.1}$$

where $L = L_1 + L_2$ is the periodicity (see Fig. 1) and q the Bloch momentum. The Bloch condition, Eq. (2.1) closes our system of equations such that we can now proceed in computing the dispersion relation of propagating waves in the "Lasagna" phase.

*Stricly speaking it is baryon number and charge conservation

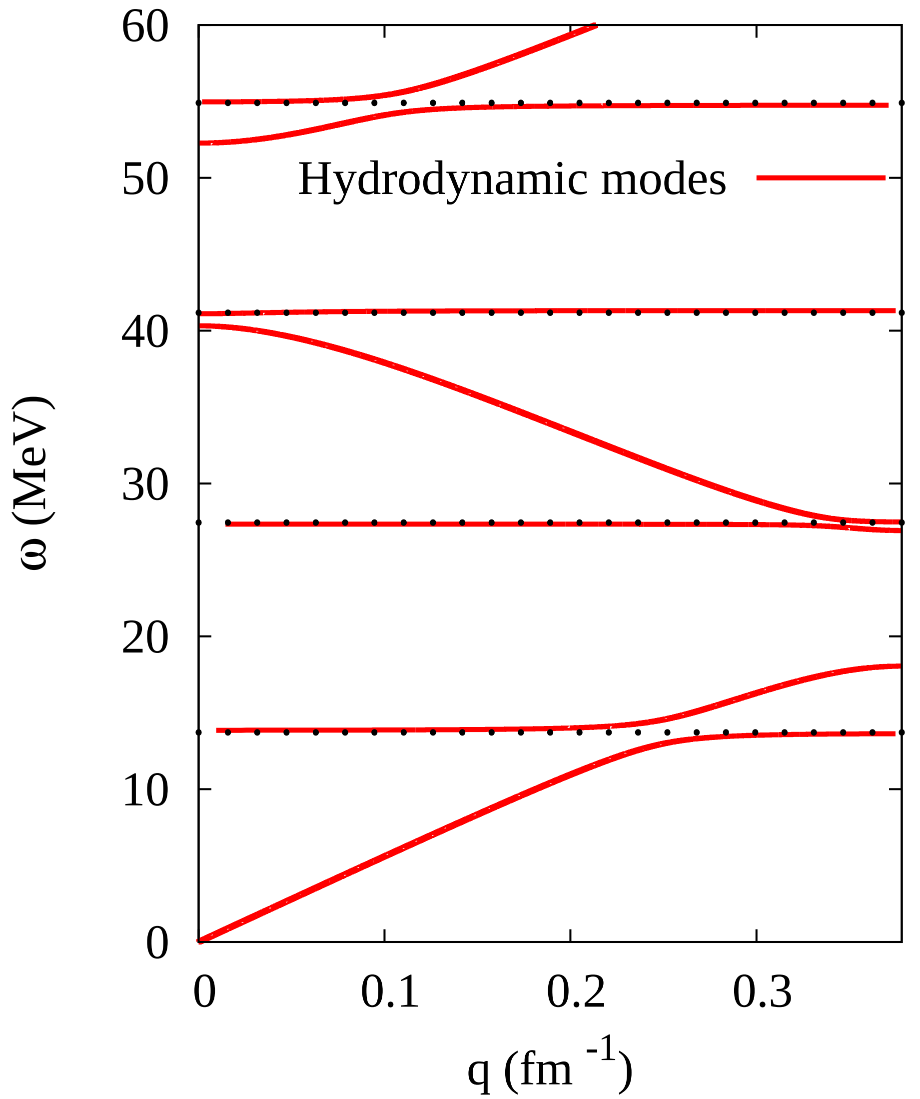

Fig. 2. Dispersion relation (energy as a function of the momentum) for the "Lasagna" phase at baryonic density $n_b = 0.0804\ \ \mathrm{fm}^{-3}$.

3 Excitation spectrum of the "Lasagna"

At the present stage we consider only waves propagating perpendicularly to the interfaces, i.e. in z-direction (see Fig. 1). Then, for a typical baryonic density $n_b = 0.0804\ \mathrm{fm}^{-3}$ appearing in the model of Avancini (2009) for the "Lasagna" phase, we obtain the dispersion relation shown in Fig. 2. For that baryon density $L_1 = 4.535$ fm and $n_1 = 0.0705\ \mathrm{fm}^{-3}$. For this example, phase 1 contains only neutrons. Phase 2 is smaller in size $L_2 = 3.770$ fm with a density of $n_2 = 0.0923\ \mathrm{fm}^{-3}$ and a proton fraction $Y_p = n_p/n_b = 0.0436$.

The lowest branch is an "acoustic branch". It is called acoustic because it follows a linear dispersion law, $\omega = c_s\, q$ at low momenta, with c_s being the sound speed. For the example at hand, $c_s = 0.2852\, c$, where c denotes the speed of light. We are mostly interested in energies of the same order as the temperature, i.e. below

~ 1 MeV, since these give the main contribution to the thermal energy,

$$E_{th} = \int d^3q\, n(\vec{q})\, \omega(\vec{q}) \ , \tag{3.1}$$

and thus to the specific heat. $n(\vec{q})$ represents here the (bosonic) occupation number. We therefore conclude that for the present example, the only relevant contribution arises from the low momentum part of the acoustic branch with a linear dispersion law.

The other, higher lying branches are called "optical branches", known to appear in periodic structures like the "Lasagna" phase. At the density discussed for the present example, these branches play no role at temperatures below 1 MeV. It is, however, interesting to analyse their structure. The dotted lines in Fig. 2 represent a frequency

$$\omega_j = \frac{u_2}{L_2} j\pi \tag{3.2}$$

with an integer j. u_2 is the sound speed corresponding to the proton dominated mode in phase 2. Many of the optical branches follow well these frequencies. Remembering that phase 1 only contains neutrons, this suggest that these modes could be interpreted as (mainly) protons oscillating in a cavity given by the extension of phase 2. In less dense parts, where the sound speeds are smaller, these optical branches could well give a non-negligible contribution to the thermal properties, too.

4 Summary and outlook

We have presented a first calculation of wave propagation in the "Lasagna" phase in the inner crust of neutron stars. This phase is characterised by alternating slabs of nuclear matter with different densities and proton fractions. We have discussed the resolution of the superfluid hydrodynamics equations taking into account the periodic structure of the medium. The dispersion relation shows one acoustic and several optical branches. We have motivated that these modes, not considered before, can have an influence on the thermal properties of the matter, in particular the specific heat. For the example presented, the main contribution comes from the acoustic branch at low momenta, where the dispersion law is almost perfectly linear. For lower overall baryon densities, other branches could contribute, too.

These rather qualitative arguments should be confirmed by a computation of the contribution to the specific heat. For that purpose, wave propagation has to considered in all directions, not only perpendicular to the interfaces. Work in this direction is in progress.

It would in addition be interesting to extend the work to other geometrical structures like rods, tubes and spheres, as the contributions to the specific heat are expected to be more important for these less dense phases. Finally, the contributions of these collective excitations have to be added to that from electrons and lattice phonons as well as the nuclei in order to examine the influence on the overall cooling behavior of the star.

It is to be expected that the excitations considered here have an effect on neutrino propagation in matter, too, since they are susceptible to couple to neutrinos. It could therefore be interesting to investigate the influence on neutrinos, too.

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References

Aguilera, D. N., Cirigliano, V.,Pons, J. A., Reddy, S., and Sharma, R. 2009, Phys. Rev. Lett. , 102, 091101

Andreev, A. F. and Bashkin, E. P. 1975, JETP, 42, 164

Avancini, S. S., Brito, L., Marinelli, J. R. et al. 2009, Phys. Rev. C, 79, 035804

Chamel, N., and Haensel, P. 2008, Living Reviews in Relativity, 11, 10

Gnedin, O. Y., Yakovlev, D. G. and Potekhin, A. Y. 2001, MNRAS, 324, 725

Prix, R. 2004, Phys. Rev. D, 69, 043001

Ravenhall, D. G., Pethick, C. J. and Wilson, J. R. 1983, Phys. Rev. Lett., 50, 2066