SF2A 2010
S. Boissier, M. Heydari-Malayeri, R. Samadi and D. Valls-Gabaud (eds)

SIMULATION OF BLACK HOLE FORMATION IN STELLAR COLLAPSE

J. Novak[1], M. Oertel[1] and B. Péres[1]

Abstract. The collapse of massive stars leads in principle to the formation of a black hole with possible observables, from gamma-rays to neutrinos and gravitational waves. The complex physics involved in this phenomenon require the use of numerical models. We here present a starting project, based on the code CoCoNuT, to perform realistic studies in full general relativity and with detailed microphysics, of the collapse of a rotating stellar core to a black hole.

Keywords: supernovae: general, methods: numerical, black hole physics

1 Introduction

In the last years many efforts have been devoted to the simulations of gravitational supernova (stellar core collapse) and, in particular, to the mechanism which leads to the explosion of the outer layers of the star (Buras et al. 2003). In connection with these simulations, much insight has been gained on the neutron star formation process, with important informations on their rotation frequencies and kick velocities at birth. On the other hand, much fewer investigations have been undertaken concerning the birth of black holes from the collapse of massive main-sequence stars. However, there are several highly interesting questions that are still unanswered at present time: what is the threshold mass of the main-sequence progenitor star that differentiates between the formation of a neutron star and a black hole? What is the general scenario: direct or delayed collapse? In particular, what happened to SN87A? How can long-gamma-ray-bursts be explained by the collapsar model? What are the neutrinos and gravitational wave signals coming from the stellar collapse to a black hole? What can be the kick velocity and spin of a newly born stellar-mass black hole? This short presentation is not intended to answer these questions, but to report on an emerging project, which aim is to study some of these aspects. Nevertheless, let us here mention that recent studies (e.g. O'Connor & Ott 2010) seem to point toward a global picture where the collapse would always first form a proto-neutron star, which would in turn collapse to a black hole when cooling down and accreting more mass from outer stellar layers.

Difficulties of such models reside in the complexity of involved physics and the many ingredients one must take into account to build a realistic model. First, initial data of massive main-sequence stars at the end of their evolution are poorly known (in particular their rotation state and magnetic field). Then models require relativistic approaches: for the hydrodynamics (during the collapse typical velocities can reach up to $0.3c$) and for the gravitation (a black hole is a general-relativistic object). More elaborated magneto-hydrodynamic studies show that there can be several instabilities occurring: the magneto-rotational instability (MRI) or the standing accretion shock instability (SASI), see (Foglizzo 2009). Finally, microphysical aspects are very important, with the appearance of additional particles (to standard neutrons, protons, electrons an neutrinos) which must be taken into account in the equation of state (EOS) and the neutrino transfer which implies the solution of the Boltzmann equation, which is in $6 + 1$ dimensions.

2 Numerical approach

The building-up of so complex models requires numerical tools. In our project, we use the `CoCoNuT`* code (Dimmelmeier et al. 2005) which solves relativistic magneto-hydrodynamic equations with Godunov-type methods

---

[1] Laboratoire Univers et Théories, Observatoire de Paris, CNRS, Université Paris Diderot, 5 place Jules Janssen, 92190 Meudon, France

*`http://www.mpa-garching.mpg.de/hydro/COCONUT`

and Einstein equations in conformally-flat condition or in the fully-constrained scheme (see Sec. 3) with spectral methods (Grandclément & Novak 2009). This conformal-flatness condition (CFC) has been introduced by Isenberg (2008), as an approximation to general relativity, where gravitational waves are discarded. It is a very good approximation for the case of stellar-core collapse because it is an exact formulation of Einstein equations in the case of spherical symmetry; therefore, the Schwarzschild black hole solution can be properly described within CFC. As gravitational waves are not present in CFC computational spacetimes, they are extracted from the `CoCoNuT` simulation with the standard quadrupole formula, using thus only information coming from the hydrodynamic quantities.

The equations for the gravitational potential are solved using the LORENE[†] library developed at LUTh since several years, and the transmission of informations from the hydrodynamic grid (finite volume methods) to the gravitational field one (spectral methods) is performed through the "*mariage des maillages*" technique, as described by Dimmelmeier et al. (2005). The system of equations for this relativistic model is closed by an EOS from Lattimer & Swesty (1991). Electron capture and neutrino escape during the collapse phase are modeled with a simple parameterized scheme defined by Liebendörfer (2005). Finally, the appearance of the black hole is tracked with an apparent horizon finder, as described by Lin & Novak (2007). This code is fully three-dimensional and uses spherical coordinate system and grid in order to better describe the geometry of the collapse.

3 General relativity issues

One of the main recent features of our models is the use of original and efficient formulation of Einstein equations. Indeed, within standard approach to general relativity (so-called "3+1 formalism"), equations for the relativistic gravitational field are cast into evolution equations and constraints, as it is the case for Maxwell equations. Thus, there are two possibilities to solve for the Einstein equations. *Free schemes*: one does not solve the constraints as they are in principle satisfied during the integration, if they were satisfied initially. *Constrained schemes*: one solves all the constraints at every time-step, but only some of the evolution equations. On the one hand, free schemes can suffer from the appearance of constraint-violating modes, on the other hand the constrained scheme used in `CoCoNuT`, as devised by Bonazzola et al. (2004) exhibits problems of non-uniqueness of the solution of the system of non-linear elliptic equations, when approaching the formation of the black hole.

This non-uniqueness problem has been cured making use of local stability theorems for non-linear elliptic partial differential equations (PDE). Technically, the extrinsic curvature needs to be rescaled, together with a hierarchical integration of the PDE system, as explained in Cordero-Carrión et al. (2009). In this study, the authors also used the code `CoCoNuT`, with the new constrained formulation of the Einstein equation and showed that the code was then able to follow the collapse of a rotating neutron star to a black hole, with no more uniqueness problems for the system of elliptic PDEs arising from the constraints. Initial data are that of a rotating neutron star in Dirac gauge (Lin & Novak 2006), situated on the unstable branch (with the gravitational mass decreasing with increasing central density), for which pressure is initially lowered by about 1%. The collapse has been studied even after the formation of an apparent horizon and the numerical solution has been compared to other studies, in particular that of Baiotti et al. (2005) who used completely different settings: free evolution scheme, Cartesian grid, finite volume and difference numerical techniques . . . Among other points, the gauges used in each study are different and it is therefore quite difficult to compare directly gauge-dependent quantities. However, all qualitative behaviors are the same, as well as some of gauge-independent quantities. This is a good indication that both codes actually give reasonable results and that the new constrained scheme used in `CoCoNuT` works well.

The simulations leading to a black hole are stopped rather shortly after the apparent horizon forms. This is due to the stretching of the coordinate system, which is defined to avoid the singularity which is forming at a finite time inside the apparent horizon. The stretching induces strong gradients on various quantities and the collapse itself make central density grow to very high values. This gives very severe limitations on the time-step. From all these problems, one must deal with the black hole using some particular setting. In the last years, the simulations of black holes have made great progress using two techniques: so-called "punctures" or "excision". In our project, we plan to use excision technique, where a neighborhood of the central singularity is removed from the computational domain and replaced by boundary conditions, particularly for elliptic PDEs. In most common approaches, it is the apparent horizon that is chosen to be the excision surface. In particular it is

---

[†] `http://www.lorene.obspm.fr`

much better behaved than the event horizon, because it is a quasi-local concept, which makes the black hole to be described as a causal object. Recently, with this approach Vasset et al. (2009) were able to recover the rotating black hole (Kerr) spacetime only from such boundary conditions and Einstein equations. This results gives good hope that excision techniques can be implemented within the `CoCoNuT` code to simulate a rotating black hole in a stable and accurate manner. The goal here is also to be able to follow the interaction between the black hole and its surrounding (mass accretion, spin-up) on longer timescales.

4 Microphysics

Many of the improvements in the `CoCoNuT` code reside in the implementation of better matter models –or *microphysics*– as neutrino transport or better EOS (Oertel & Fantina 2010). As mentioned in the introduction, the treatment of the neutrino transport is a very demanding task. Approximate schemes are, of course, possible to model the impact of neutrinos onto the collapse. Among very simplified approaches one has the *leakage scheme* (Ruffert et al. 1995), where the neutrino mean free path is computed to determine the time decrease of neutrino density. The energy taken away by neutrinos gives a cooling term in hydrodynamic equations, but there is little possibility to obtain an energy spectrum, nor to use the escaping neutrinos to deposit energy behind the shock (neutrino heating). This leakage scheme is currently being implemented in `CoCoNuT`. More elaborated approximate schemes include *flux-limited diffusion scheme*, where the neutrino distribution function obeys a diffusion equation, for each energy group; isotropic diffusion source approximation (IDSA), where the distribution function is split into trapped and streaming neutrino parts and finally, full Boltzmann transport, with approximations on the dimensionality (e.g. “ray-by-ray” method).

Many numerical studies that use general relativity to model the gravitational field take into account over-simplified microphysics: polytropic EOS, or Γ-law in which the adiabatic index depends in a piecewise linear way on the density to mimic the stiffening of matter around nuclear density. There are therefore many vital improvements to be made on our models. First, the computation of a realistic EOS, based on the Lattimer & Swesty (1991) one, but taking into account the appearance of additional particles as muons, pions or hyperons, see Oertel & Fantina (2010). This is a particular feature of the core-collapse scenario forming black holes because, contrary to usual core-collapse supernovae giving birth to neutron stars, the temperatures reached in the central regions of the collapsing star are much higher and therefore, new particles can be formed. Another important point is the computation of neutrino-nucleon interaction terms that are made in a way that is consistent with the EOS. This point is a collaborative effort with the Institut de Physique Nucléaire (IPN) at Orsay, to build-up complete tables of all the interaction terms, from same principles as the EOS.

Finally, a crucial physical reaction for the core-collapse are the electron capture rates, which are responsible for the sudden decrease of pressure and neutrino production through inverse β-decay. Electrons are captured on free protons and heavy nuclei, but these capture rates can be rather sensitive to temperature effects, which have not been taken into account in any supernova simulation. The study of Fantina et al. (2009) looks at this temperature influence on capture rates and shows, using a one-zone, Newtonian code, that deleptonization is reduced when temperature effects are taken into account, which influences the overall dynamics, with an increase of the shock energy of about 0.4 FOE. As this is not negligible, it is of highest importance to incorporate such effects in the numerical model. Although this effect alone cannot solve the supernova puzzle (Buras et al. 2003), it can contribute, together with other realistic physical ingredients, to the modeling of successful explosions of core-collapse supernovae. It also shows the potential importance of microphysical realistic models for the simulation of stellar core-collapses.

5 Conclusions

At present time, the code `CoCoNuT` is able to successfully model the three-dimensional collapse of a rotating neutron star to a black hole. Nevertheless, much more effort is to be put in these models in order to get very interesting astrophysical informations on the birth of stellar-mass black holes. Most of all, initial data remain the main uncertainty in such models. In particular, the lack of reliable two-dimensional models of massive star evolution prevents us from having good models for rotation and magnetic field in evolved massive stars. Another issue with initial models is the little number of groups working on the subject: almost all the numerical simulations of stellar core-collapse rely on the models by Heger & Woosley. It is quite important to have some alternative computations of these very important models and we have now been in contact with Limongi & Chieffi who are developing such models.

As about 99% of the energy in a supernova is released via neutrino emission, it is vital to be able to correctly model this phenomenon. The incorporation of neutrino transport is just starting in `CoCoNuT`, but we already consider the necessity of having a neutrino-nucleon interaction, which is coherent with the EOS, in particular as we devise an EOS which is really adapted to the case of the collapse to a black hole (higher temperatures). The `CoCoNuT` code is able to take into account also the magnetic field, through ideal MHD equations, but because of too low resolution, it seems for the moment very difficult to look at the most interesting phenomenon, which is the magneto-rotational instability. On the other hand it may be possible, once the modeling of black hole is improved with excision techniques, to look on highly interesting phenomenon of SASI in the presence of black hole, and its influence on black hole kicks and spins.

Finally, gravitational waves should be extracted in a clean manner from our simulations thanks to the use of general relativistic models, using a fully constrained scheme, which is now accurate and stable. However, a major issue in the future for our project is the development of efficient parallel version of LORENE and `CoCoNuT`, to be able to combine all the above mentioned features with sufficient resolution. Since this a only a starting project, and since many important expertise is present, we have good hopes for the future.

References

Baiotti, L., Hawke, I., Montero, P.J et al. 2005, Phys. Rev. D, 71, 024035

Bonazzola, S., Gourgoulhon, E., Grandclément, P. & Novak, J. 2004, Phys. Rev. D, 70, 104007

Buras, R., Rampp, M., Janka, H.-T. & Kifonidis, K. 2003, Phys. Rev. Lett., 90, 241101

Cordero-Carrión, I., Cerdá-Durán, P., Dimmelmeier, H. et al. 2009, Phys. Rev. D, 79, 024017

Dimmelmeier, H., Novak, J., Font, J.A., Ibàñez, J.M. & Müller, E. 2005, Phys. Rev. D, 71, 064023

Fantina, A., Donati, P. & Pizzochero, P.M. 2009, Phys. Lett. B, 676, 140

Folglizzo, T. 2009, ApJ, 694, 820

Grandclément, P. & Novak, J. 2009, Living Rev. Relativity, 12, 1

Isenberg, J. 2008, Int. J. Mod. Phys. D, 17, 265 (*following a first unpublished version of 1978*)

Lattimer, J.M. & Swesty F.D. 1991, Nucl. Phys. A, 535, 331

Liebendörfer, M. 2005, Astrophys. J., 633, 1042

Lin, L.M. & Novak, J. 2006, Class. Quantum Grav., 23, 4545

Lin, L.M. & Novak, J. 2007, Class. Quantum Grav., 24, 2665

O'Connor, E. & Ott, C. 2010, Class. Quantum Grav., 27, 114103

Oertel, M. & Fantina, A. this volume, p. 155 (`http://proc.sf2a.asso.fr/2010/2010sf2a.conf..0155D.pdf`)

Ruffert, M., Janka, H.T. & Schäfer, G. 1995, Astrophys. & Space Sci., 231, 423

Vasset, N., Novak, J. & Jaramillo, J.L. 2009, Phys. Rev. D, 79, 124010