SF2A 2010
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AN EXTENDED EQUATION OF STATE FOR SIMULATIONS OF STELLAR COLLAPSE

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Abstract. In core-collapse events matter is heated and compressed to densities above nuclear matter saturation density. For progenitors with masses above about 25 solar masses, which eventually form a black hole, the temperatures and densities reached during the collapse are so high that a traditional description in terms of electrons, nuclei, and nucleons is no longer adequate. We will present here an improved equation of state which contains in addition pions and hyperons. They become abundant in the high temperature and density regime, and we will discuss the effect on the thermodynamic properties.

Keywords: core collapse, equation of state, dense matter

1 Introduction

Supernovae and hypernovae figure among the most spectacular events observed in the universe because of the immense amount of energy involved. In general, one can distinguish between thermonuclear and core-collapse events. Here we will be interested in the latter. These occur at the end of the life of massive ($M \gtrsim 8M_\odot$) stars: if the iron core exceeds the Chandrasekar mass a gravitational collapse is induced. At the center a compact star is formed, which is a neutron star in the classical gravitational supernova. Depending among others on the progenitor mass, its metallicity, and rotation, as well a black hole can be formed. These events are known as hypernovae or collapsars. For about thirty years, simulations are performed in order to explore these events and to answer related questions, for example on the precise conditions for forming a neutron star or a black hole. The simulations are extremely complex, since they involve many different ingredients: multi-dimensional hydrodynamics, neutrino transport, general relativity and complicated microphysics. Despite all the effort, many unknowns remain in the simulations, in particular on the engine driving a successful supernova explosion. Apart from the observations via electromagnetic radiation, the neutrino and gravitational wave signal could give interesting information on the models. More details about the state-of-the-art of the simulations and in particular on neutrino transport can be found in Novak et al. (2010).

The microphysics input for the simulations concerns essentially two domains, the interaction rates for neutrino-matter interaction and deleptonization, i.e. electron capture, and the equation of state (EOS). In this talk we will discuss the latter. It is not an obvious task to construct an EOS. The main difficulty arises from the fact that a very large range of (baryon number) densities ($10^{-10}\text{fm}^{-3} \lesssim n_B \lesssim 1\text{fm}^{-3}$), temperatures ($0 < T \lesssim 150$ MeV) and proton fractions ($0 < Y_p = n_p/n_B < 0.6$) has to be covered. Within this range the characteristics of nuclear matter change dramatically, from an ideal gas of different nuclei up to uniform strongly interacting matter, containing in the most simple case just free nucleons but potentially more exotic components such as hyperons or mesons. Even a transition to deconfined quark matter cannot be excluded. Although there is a large variety of EOSs available for cold dense matter relevant for the description of neutron stars, at present, only two hadronic EOSs exist which are commonly used in core collapse simulations, where temperature effects play a crucial role, that by Lattimer and Swesty (1991) and by Shen et al. (1998). These two equations of state use different nuclear interactions, but are based on the same limiting assumptions: they take into account non-interacting α-particles, a single heavy nucleus and free nucleons in addition to the electron, positron and photon gas. In particular at low densities the composition of matter is much more complicated,

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with a large number of different nuclei. Although this should not have a large impact on the purely thermodynamical properties, it is important to correctly describe the composition of matter in order to determine the electron capture rates and neutrino interactions. Therefore, in the last years, several groups have started to build EOSs using mainly statistical approaches to improve the low density part of the EOS (see e.g. Hempel and Schaffner-Bielich (2010)). This part of the EOS of state can be confronted with the knowledge obtained from multi-fragmentation models used to analyze data from low energy heavy-ion collisions. Up to know, there are less attempts to improve the high energy part of the EOS, although there are many indications that probably the physics of the standard EOS is too poor in this regime, too. First of all, our knowledge about the QCD phase diagram suggests a transition to the QGP plasma within the range of densities and temperatures reachable in core collapse events, i.e. within the range of our tables. Of course, there are lots of uncertainties about this phase transition, such that its occurence cannot be affirmed, but the possibility has to be kept in mind when employing a purely nuclear EOS such as the Lattimer and Swesty (1991) or Shen et al. (1998) one up to densities well above nuclear matter saturation density and temperatures as high as several tens of MeV. There is some first work including this phase transition, see Sagert et al. (2009). Secondly, even without thinking about a QCD phase transition, other forms of (exotic) matter could appear at high densities and temperatures. Already for a long time for cold EOS used for neutron star models, hyperons, pions and kaons have been considered. At temperatures above about 20 MeV, this point becomes even more crucial. This has been confirmed by the first attempts including hyperons and pions in the EOS by Shen et al. (1998) for simulations, see Sumiyoshi et al. (2009). We will discuss here the construction of a new EOS including hyperons, pions and muons based on the Lattimer and Swesty (1991) one and the effects on some thermodynamic quantities important for the simulations.

2 The model for the equation of state

Let us start the description of our model for the extended EOS with a description of the original EOS by Lattimer and Swesty (1991). As mentioned above, it contains electrons, positrons, photons, nucleons, α-particles, and one (average) heavy nucleus. Electrons and positrons are treated as noninteracting free Fermi gas in pair equilibrium, and the α-particles as free Boltzmann gas. The nuclear interaction is based on a liquid drop model. The phase transition between the phase containing nuclei and the nuclear matter phase is built via a Maxwell construction. More details about the underlying theory can be found in Lattimer and Swesty (1991) or under `http://www.astro.sunysb.edu/dswesty/lseos.html`. On the same web page, the original tables as well as the source code can be found. To obtain our EOS, we have corrected a small error for the value of the α-particle binding energy, which has to be measured with respect to the neutron mass, too, as all other energies.

We have added to the Lattimer and Swesty (1991) pions, muons and hyperons. For the first two, no interaction has been assumed and they have just been added as a free gas, satisfying the overall constraint on charge neutrality. Concerning the muons we have to mention the following point. Traditionally, the electron fraction $Y_e = (n_{e^-} - n_{e^+})/n_B$ is used in addition to the temperature and baryon number density n_B as parameter for the EOS. If the muon lepton number was conserved independently of the electron lepton number we should introduce an additional variable, the muon fraction and the simulation codes should evolve muon number, too. For the moment we have assumed that muons and electrons have the same chemical potential and have thus generalized the definition of the electron fraction to include muons as $Y_e = (n_{e^-} - n_{e^+} + n_{\mu^-} - n_{\mu^+})/n_B$. Hyperons are added by extending the model by Balberg and Gal (1997) to finite temperature. We have slightly adapted the parameters for the hyperon-nucleon (YN) and hyperon-hyperon (YY) interaction in order to be compatible with current experimental data on hypernuclei. This approach has the great advantage that hyperons are added on a nuclear interaction which is exactly the same as in the original EOS by Lattimer and Swesty (1991), such that an “artificial phase transition”, induced only by matching one nuclear model to another and thus completely unphysical, is avoided.

For comparison, we have kept for the nuclear part the three different parameter sets provided by the original code by Lattimer and Swesty (1991), characterised by the value of the compression modulus K of symmetric nuclear matter at saturation density. The values available are $K = 180, 220, 375$ MeV. Today, the preferred value for K from nuclear experiments is 240 ± 10 MeV Piekarewicz (2010). The obvious error is rather small, such that the two extreme values for K used in the Lattimer and Swesty (1991) EOS are in principle disfavored and the preferred parameter set for simulations should be that with $K = 220$ MeV. We will, however, keep the two other sets for two reasons. The first one is purely historical: in many simulations the parameter set with $K = 180$ MeV has been used, such that for comparison with the existing literature it is interesting to have this value at hand. The second one is that the experimental result of 240 ± 10MeV is not uncontested. In particular, the extraction of this value from data on isoscalar giant monopole resonances depends on the density dependence of the nuclear symmetry energy, a quantity under intensive debate in recent years. We therefore think that a wider range of values should be considered. In that sense, the range of values of the EOS represents an extreme variation of the nuclear parameter sets, i.e. it can give an indication about the uncertainties in the simulations coming from the uncertainties on the EOS.

There are even more uncertainties on the YY and YN interaction. In order to give an indication about these uncertainties, we compare the results using the model by Balberg and Gal (1997) with a completely different model, the G-matrix parametrization by Vidaña et al. (2010). Apart from the question of matching the latter model to another subsaturation EOS, its weak point is that it becomes very soft at high densities and low temperatures such that the predicted maximum mass for neutron stars is not easily compatible with observational data. We are using it here mainly for comparison to give an indication on the uncertainties coming from the underlying baryonic models. In any case, we are mainly interested in the high temperature region, where the interaction should become less important because the kinetic energy gains in importance with respect to the interaction energy.

3 Some thermodynamics

Within this section we will discuss the effects of our extension of the EOS on matter composition and thermodynamical quantities relevant for the simulations. Let us look as an example on the fractions of Λ and Σ^- hyperons at $T = 60$ MeV and $n_B = 0.32\text{fm}^{-3}$ which are shown in Fig. 1. Λ and Σ^- are the first hyperons to appear and at that density, which corresponds to roughly two times nuclear matter saturation density, and temperature they are the most abundant ones. Different simulations have shown that these conditions can well be reached in core-collapse events for progenitor masses above roughly 25 $M_\odot$. "LS+" thereby indicates our new EOS of state with the model by Balberg and Gal (1997) for the hyperons and "G-matrix" the hyperonic model by Vidaña et al. (2010). It can be seen that in both cases the abundancies of the additional particles in the EOS are of the order of 5%. This means that they can have a non-negligible influence on the thermodynamic properties. This can be seen from the results for pressure and sound speed displayed in Fig. 2. They are shown for the same density and temperature as the abundancies in Fig. 1. For comparison we have shown here the original EOS by Lattimer and Swesty (1991) with the $K = 180$ MeV parameter set. The results with the G-matrix approach differ considerably from the others, such that one remark is that it seems important to test a variety of baryonic models in order to get an idea on the uncertainties induced in the simulations. Only two different models, Lattimer and Swesty (1991) and Shen et al. (1998), are much too restrictive. The other remark is that including additional particles on the Lattimer and Swesty (1991) EOS induces a difference of the

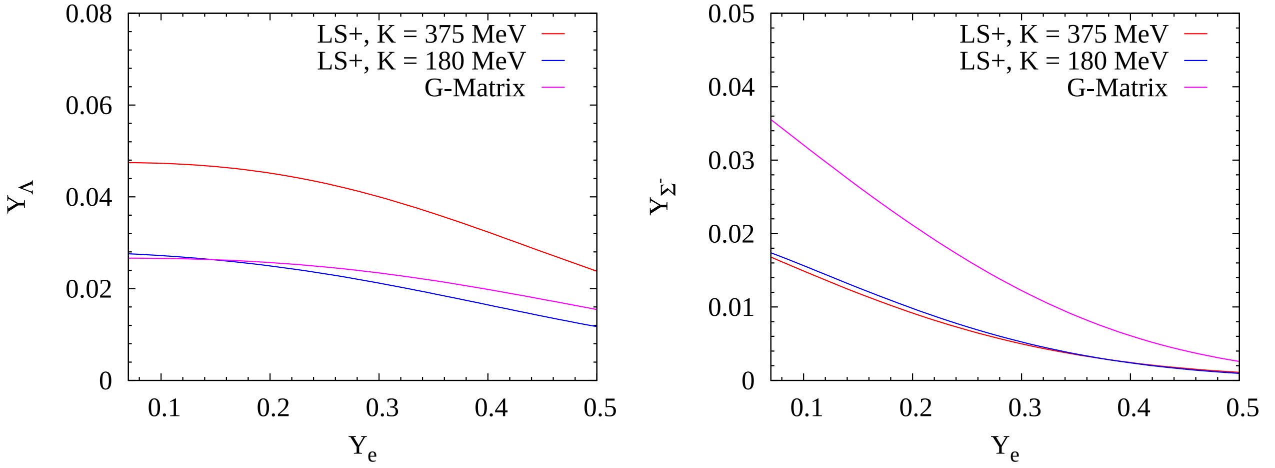

Fig. 1. The fraction of Λ and Σ^- hyperons at a baryon number density of $n_B = 0.32\text{fm}^{-3}$ and a temperature of $T = 60MeV$ as a function of the electron fraction.

order of the difference between the three parameter sets. That means that those particles should be included at high density and temperature.

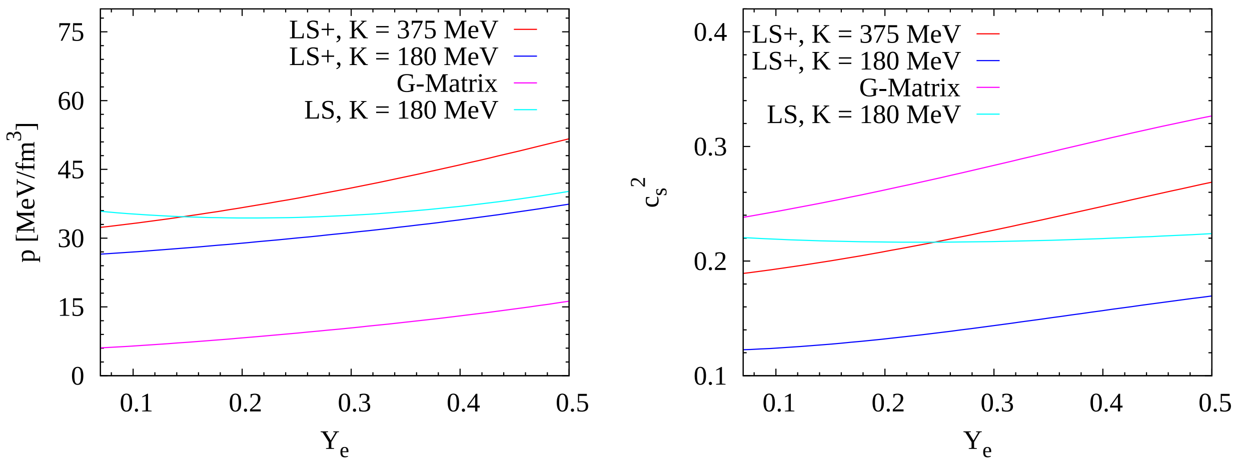

Fig. 2. Pressure and sound speed as a function of the electron fraction at the same density and temparture as in Fig 1.

4 Outlook

The ultimate answer on the importance of the additional particles can only come from realistic simulations. We have shown that at high density and temperature they become important, but a priori this does not preclude anything on the relevance for the core-collapse events. We expect that, since in classical supernova events, temperatures and density stay relatively low compared with those discussed above, that our extended EOS will be more relevant in events forming a black hole. This would be in agreement with the first studies by Sumiyoshi et al. (2009). It has to be seen whether the EOS dependence shows up in the neutrino and gravitational wave signal of those events. Of course, for a full simulation, the deleptonization and neutrino-matter interaction rates have to be evaluated coherently. This will be kept for future work.

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