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THE FLUID EQUILIBRIUM TIDE IN STARS AND GIANT PLANETS

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Abstract. Many extrasolar planets orbit very close to their parent star, so that they experience strong tidal interactions; by converting mechanical energy into heat, these tides contribute to the dynamical evolution of such systems. This motivates us to seek a deeper understanding of the processes that cause tidal dissipation, which depend both on the internal structure and the physical properties of the considered bodies. Here, we examine the equilibrium tide, i.e. the hydrostatic adjustment to the tidal potential, in a rotating fluid planet or star. We derive the adiabatic velocity field induced by the tidal perturbation and discuss in particular how the quality factor Q characterizing the dissipation is linked whith the turbulent viscosity of convection zones. Finally, we show how the results may be implemented to describe the dynamical evolution of the system.

Keywords: stars: binaries (including multiple): close, stars: planetary systems

1 Introduction

When an extrasolar planet orbits very close to its host star, both components experience strong tidal interactions, which govern their orbital evolution. The dynamical evolution of a binary system is driven by the conversion of its mechanical energy into heat. Provided the system loses no angular momentum, it tends to the state of minimum energy in which the orbits are circular, the rotation of the components is synchronized with the orbital motion, and the spins are aligned. However, in very close systems of star-planet kind, such final state cannot be achieved: instead, the planet spirals toward the star and may eventually be engulfed by it (Hut 1981; Levrard et al. 2009). To predict the fate of a binary system, one has to identify the dissipative processes that achieve the conversion of kinetic energy into thermal energy, from which one may then draw the characteristic times of circularization, synchronization and spin alignment. Before reviewing these processes, let us recall the two types of tides which operate in stars and in the fluid parts of giant planets. The *equilibrium tide* designates the large-scale flow induced by the hydrostatic adjustment of the star in response to the gravitational force exerted by the companion (Zahn 1966a). On the other hand, the *dynamical tide* corresponds to the eigenmodes (gravity, inertial, or gravito-inertial waves) that are excited by the tidal potential (Zahn 1975; Goldreich & Nicholson 1989; Ogilvie & Lin 2004; Goodman & Lackner 2009; Rieutord & Valdettaro 2010; Barker & Ogilvie 2010). These tides experience two main dissipative mechanisms: turbulent friction in convective regions and thermal dissipation acting on the gravito-inertial modes excited in radiative zones.
We shall focus here on the equilibrium tide acting in the convective envelopes of solar type stars or giant planets.

2 Description of the problem

2.1 The system

We consider a system consisting of two bodies A and B, of mass m_A and m_B, and we undertake to describe the tide exerted by B on A, which we assume to be in fluid state, i.e. a star or a giant planet. Due to their mutual attraction, they move in elliptic orbits around their common center of mass, but it is often convenient to choose

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an inertial reference frame $\mathcal{R}_A$ whose origin is placed at the center of A (designated by the same letter) and whose axes $\{\mathbf{X}, \mathbf{Y}, \mathbf{Z}\}$ are fixed in space. We assume that all spins are aligned along the $(A\mathbf{Z})$ axis; therefore the position of B is entirely determined by the three following keplerian elements: the semi-major axis a of the relative orbit of B around A, its eccentricity e, and its true anomaly ν. The mean motion of B is denoted ω.

2.2 The tidal potential

The tidal force exerted by B on A derives from a potential, which may be expanded in spherical harmonics using the Kaula transform (Kaula 1962). Considering binary systems that are separated enough to allow the companion to be treated as a point mass (this approximation has been discussed by Mathis & Le Poncin-Lafitte 2009), the tidal potential U takes then, in this $q$uadrupolar aproximation, the following form:

$$U(r,\theta,\varphi) = -\frac{1}{4}\frac{m_B}{m_A+m_B}\omega^2 r^2 \sum_l G_{2,0,l-2}(e) P_2^2(\cos\theta)\cos(2\varphi - l\omega t)\,, \tag{2.1}$$

where (r,θ,φ) are the spherical coordinates attached to the inertial reference frame $\mathcal{R}_A$, and the functions $G_{2,0,l-2}(e)$ are polynomials in e whose lowest power is $|l-2|$. Each term of this expansion acts with a proper tidal frequency $\sigma = l\omega - 2\Omega$. The tidal potential (2.1) induces in the star (or the giant planet) pressure and density perturbations and a modified velocity field which obeys the classical equations of Navier-Stokes, mass conservation, Poisson and entropy.

3 The adiabatic and dissipative equlibrium tides

Treating the tides as a small amplitude perturbation of the hydrostatic structure of the object, we may separate the problem in two parts:

- first, an *adiabatic system* (I) in phase with the perturbing potential U (eq. 2.1) : it corresponds to the adiabatic tide, i.e. the star's response to the tidal excitation, ignoring all dissipative processes.
- second, a *dissipative system* (II) in quadrature with the perturbing potential : it corresponds to the star's response due to the dissipative processes (here, the turbulent friction due to the convective motions).

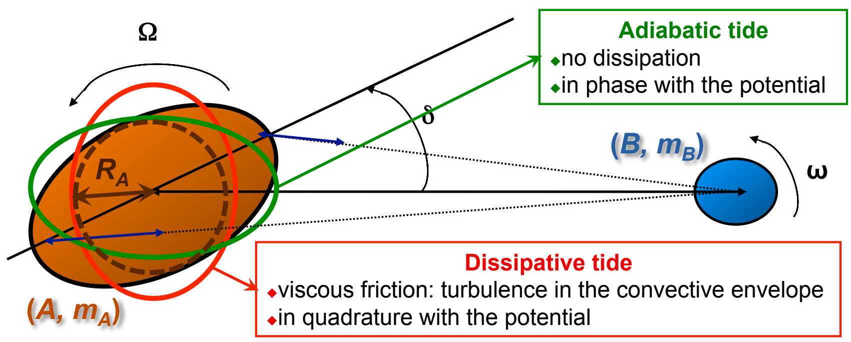

Fig. 1. Tidal interactions in a binary system. Body B exerts a tidal force on body A, which adjusts itself with a phase lag δ, because of internal friction. This adjustment may be split in an adiabatic component, which is in phase with the tide, and a dissipative one, which is in quadrature.

4 The adiabatic tide

First of all, each body suffers a deformation due to its hydrostatic adjustment to the tidal perturbation described by an adiabatic response potential ϕ_{I}. This adjustment is quantified by the second-order Love number $k_2 = \phi_{\mathrm{I}}(R_A)/U(R_A)$, where ϕ_{I} and U are taken at the surface of the planet. In the absence of dissipation, this deformation induces a velocity field (represented in Fig. 2), which is in phase with the tidal potential and thus does not lead to a net exchange of angular momentum.

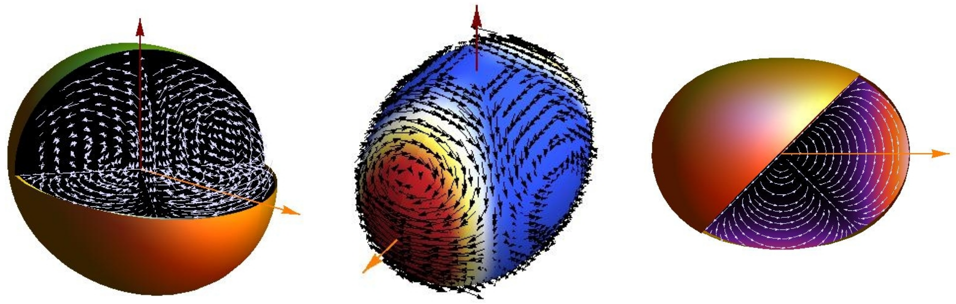

Fig. 2. Representations of the total (poloidal and toroidal) adiabatic equilibrium tide velocity field. The red and orange arrows repectively indicate the direction of the primary rotation axis and of the line of centers. **First:** 3–D view of the total (poloidal and toroidal) adiabatic equilibrium tide velocity field (white arrows). **Second:** Representation of this velocity field at the surface of the primary (black arrows); the color-scaled background represents the normalized tidal potential intensity (blue and red for the minimum and maximum values respectively). **Third:** View of the velocity field (white arrows) in its equatorial plane of symmetry; the color-scaled background represents the velocity value (black and orange for the minimum and maximum values respectively). **Fourth:** View of the velocity field (white arrows) in its meridional plane of symmetry; the color-scaled background represents the velocity value (black and purple for the minimum and maximum values respectively).

5 The dissipative tide

We assume that the convective motions and the tidal flow are separated enough in temporal and spatial scales that their interaction can be described by an eddy viscosity η acting on the tidal velocity field. We use here the prescription of Zahn (1989) for the turbulent viscosity. In convective regions, viscous friction acts on the adiabatic tide velocity field and leads to a redistribution of mass, which is no longer in phase with the tidal potential (Zahn 1966b, 1989). This process is at the origin of the tidal dissipation in solar-type stars and giant planets; it is quantified by the ratio k_2/Q, where Q is the tidal dissipation factor. k_2/Q is a function of tidal frequency, as shown in Fig. 3.

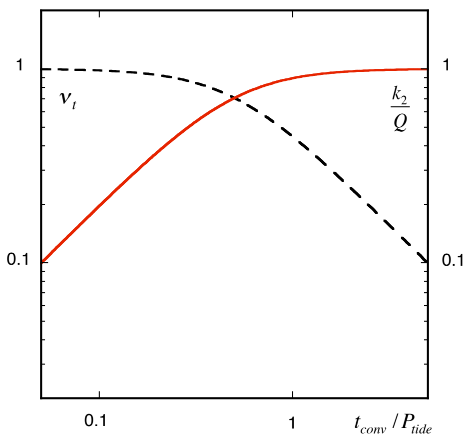

Fig. 3. The two regimes of turbulent dissipation. As long as the local convective turn-over time remains shorter than the tidal period ($t_{\rm conv} < P_{\rm tide}$), the turbulent viscosity ν_t (in black dashed line) is independent of the tidal frequency, and the inverse quality factor k_2/Q (in red continuous line) varies proportionally to the tidal frequency (σ_l) (so does also the tidal lag angle). When $t_{\rm conv} > P_{\rm tide}$, ν_t varies proportionally to the tidal period, whereas k_2/Q does no longer depend on the tidal frequency. ν_t and k_2/Q have been scaled by the value they take respectively for $t_{\rm conv}/P_{\rm tide} \to 0$ and $\to \infty$.

6 Dynamical evolution

Due to dissipation, the tidal torque has non-zero average over the orbit, and it induces an exchange of angular momentum between each component and the orbital motion. This exchange govern the evolution of the semi-major axis and the eccentricity of the orbit, and that of the angular velocity of each component (see for example Mathis & Le Poncin-Lafitte 2009). Depending on the initial conditions and on the planet/star mass ratio, the system evolves either to a stable state of minimum energy (where all spins are aligned, the orbits are circular and the rotation of each body is synchronized with the orbital motion) or the planet tends to spiral into the parent star. For simplicity, we give here the results for the coplanar case, where the orbital and rotational spins are aligned. The evolution of the semi-major axis a, of the eccentricity e and of the angular velocity Ω of component A (I_A denotes its moment of inertia), is governed by the following equations:

$$\begin{aligned}
\frac{1}{t_{\rm sync}} &\equiv -\frac{1}{(\Omega-\omega)\,I_A}\frac{{\rm d}\,(I_A\,\Omega)}{{\rm d}t} = \frac{1}{(\Omega-\omega)\;I_A}\,\frac{8\pi}{5}\,\frac{\mathcal{G}m_B^2R^5}{a^6}\sum_l\left\{\frac{k_2(\sigma_l)}{Q(\sigma_l)}\left[\mathcal{H}_l(e)\right]^2\right\},\\
\frac{1}{t_{\rm circ}} &\equiv -\frac{1}{e}\,\frac{{\rm d}e}{{\rm d}t} = \frac{1}{\omega}\,\frac{1-e^2}{e^2}\,\frac{4\pi}{5}\,\frac{\mathcal{G}m_BR^5}{a^8}\sum_l\left\{\left[2\left(1-\frac{1}{\sqrt{1-e^2}}\right)+(l-2)\right]\frac{k_2(\sigma_l)}{Q(\sigma_l)}\left[\mathcal{H}_l(e)\right]^2\right\},\\
\frac{1}{a}\,\frac{{\rm d}a}{{\rm d}t} &= -\frac{2}{\omega}\,\frac{4\pi}{5}\,\frac{\mathcal{G}m_BR^5}{a^8}\sum_l\left\{l\,\frac{k_2(\sigma_l)}{Q(\sigma_l)}\left[\mathcal{H}_l\,(e)\right]^2\right\},
\end{aligned}$$

where: $\mathcal{H}_l\,(e) = \left(\dfrac{15}{32\,\pi}\right)^{1/2} G_{2,0,l-2}\,(e).$

7 Conclusion

We have rigorously separated equilibrium and dynamical tides (pseudo-resonances, present in Zahn 1966a, have been removed). We also have confirmed the divergence-free property of the tidal velocity field (critized by Scharlemann 1981). The results obtained here are valid even if the components are far from synchronism and evolve on highly elliptical orbits. This work represents a consistent treatment of the equilibrium tide, taking into account the hydrodynamical properties of the considered body. That allowed us to get the equations of the dynamical evolution of the system in function of a dissipative quality factor Q depending on the tidal frequency. Those results have been presented in a submitted article (Remus et al. 2011). The next work will integrate differential rotation and obliquity of the axis of rotation.

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