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COLLECTIVE EXCITATIONS IN THE NEUTRON STAR INNER CRUST

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Abstract. We study the spectrum of collective excitations in the inhomogeneous phases in the neutron star inner crust within a superfluid hydrodynamics approach. Our aim is to describe the whole range of wavelengths, from the long-wavelength limit which can be described by macroscopic approaches and which is crucial for the low-energy part of the spectrum, to wavelengths of the order of the dimensions of the Wigner-Seitz cells, corresponding to the modes usually described in microscopic calculations. As an application, we will discuss the contribution of these collective modes to the specific heat in comparison with other known contributions.

Keywords: collective modes, neutron star crust, pasta phases, specific heat

1 Neutron star cooling and the inner crust

It has been known for a long time that within the different structures inside a neutron star we can find superfluid and superconducting ones. The first observational indications were the glitches, and more recently observations of the surface thermal emission have been discussed in this context. The latter is an observable, which depends on heat transport properties and is thus very sensitive to the superfluid and superconducting character of the different structures inside the star. The properties of the crust thereby influence the cooling behavior mainly during the first 50-100 years, the crust thermalisation epoch (Gnedin et al. 2001). During this stage the core evacuates heat by strong neutrino cooling and the crust is not yet thermalized with the cold core. The outer crust and the envelope have high thermal conductivity such that the inner crust matter plays an essential role.

The inner crust is composed of nuclear clusters, unbound neutrons and ultrarelativistic electrons. Close to the core, probably nuclear clusters start to deform, from an almost spherical shape, they could form tubes or slabs immersed in homogeneous neutron rich matter at the different densities (Ravenhall et al. 1983). These phases are commonly called the nuclear pasta. In this work we focus on these inhomogeneous phases in the inner crust.

2 Superfluid hydrodynamics approach

The spectrum of collective excitations shall be discussed within a superfluid hydrodynamics approach. Here we mention only the main ideas, more details can be found in Di Gallo et al. (2011). The advantage of the approach is that the wavelengths are not limited to the size of the Wigner-Seitz cell as in standard microscopic (quasiparticle random-phase approximation, QRPA) calculations (Khan et al. 2005). Our aim is to describe also the low-energy part of the spectrum, which is determined by modes whose wavelengths are longer than the periodicity of the inhomogeneous structure. These modes can give important contributions to thermal properties (Aguilera et al. 2009; Pethick et al. 2010; Cirigliano et al. 2011). However, in contrast to Pethick et al. (2010); Cirigliano et al. (2011), we are not only interested in the long wavelength limit, but we want to study more in detail the effect of the inhomogeneous structure on the excitations. This means in particular that the phase of the superfluid order parameter, the density variations and the velocities characterizing the collective modes are not plane waves, but have a more complicated spatial dependence.

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Let us summarize the assumptions on which our approach is based. First of all, we neglect temperature effects. This assumption is justified if we consider temperatures well below the superfluid energy gap, $T \ll 1$ MeV. Under this assumption, neutrons and protons form two superfluids, while the normal-fluid component made of thermally broken pairs is absent. Furthermore, we assume that the Cooper pairs are not broken by the excitation of the collective modes themselves, i.e., the excitation energies should not exceed the pairing gap of 1 MeV. Finally, the use of superfluid hydrodynamics is only valid if all spatial variations, those corresponding to the inhomogeneous structures and the wavelengths of the collective modes under consideration, are larger than the coherence length of the Cooper pairs.

At the present stage of our work, we shall completely neglect the Coulomb interaction between the protons. As long as we focus on the neutron modes, this is acceptable, but one should nevertheless keep in mind that the neutron modes are coupled to the lattice phonons which cannot be described without Coulomb interaction.

For completeness, we mention that we use a non-relativistic framework because all relevant velocities are much smaller than the speed of light.

The hydrodynamic equations can be derived from local conservation laws. Particle number conservation for neutrons and protons leads to two continuity equations, while energy and momentum conservation give the Euler equations. The acceleration of the fluids, described by the Euler equations, depends on the fluid densities through the equation of state (EOS). The latter depends on the nuclear interaction. Following Avancini et al. (2009), we use a relativistic mean-field (RMF) model, the so-called DDHδ model, to calculate the EOS. For the study of collective modes, it is sufficient to linearize the hydrodynamic equations around stationary equilibrium.

In the case of uniform matter, one easily obtains in this way an eigenvalue problem for two coupled sound modes. Due to the nuclear interaction (including entrainment), these modes do not describe pure proton or neutron waves, but combinations of both.

3 Collective modes in a periodic slab structure

In our model, the inhomogeneous phases we are interested in are described as mixed phases where a neutron gas (phase 1) coexists with a dense phase (phase 2) containing protons and neutrons. Inside each phase, the densities are supposed to be constant, with a sharp interface at the phase boundaries. Although this is a crude approximation, it contains the essential features of the structure of the inner crust.

Since the densities are discontinuous at the phase boundaries, the hydrodynamic equations have to be supplemented by appropriate boundary conditions. The first condition is that the pressure must be continuous, i.e., at a given point of the interface, the pressures in phase 1 and phase 2 must be equal: $P_1 = P_2$. The second condition is that the interface remains well defined, i.e., the normal components of the velocities of neutron fluid in phase 1 and of the neutron and proton fluids in phase 2 must be equal: $v_{\perp n1} = v_{\perp n2} = v_{\perp p1}$.

We will restrict ourselves to the simplest geometry which is a structure of periodically alternating slabs with different proton and neutron densities ("lasagna" phase), see Fig. 1.

The equilibrium properties, i.e., the densities n_{n1}, n_{p1}, n_{n2} and n_{p2}, and the slab thicknesses L_1 and L_2, are input parameters taken from Avancini et al. (2009).

The excitations are then obtained by solving in each slab the linearized hydrodynamic equations together with the boundary conditions at the interfaces between neighboring slabs. In addition, the periodicity of the structure is taken into account by the Floquet-Bloch boundary condition $\vec{v}_A(\vec{r} + \vec{R}, t) = e^{i\vec{q}\cdot\vec{R}}\, \vec{v}_A(\vec{r}, t)$ where $\vec{q}$ is the Bloch momentum and R_z must be a multiple of L. Here we have written the Bloch condition for the neutron ($A = n$) and proton ($A = p$) velocities, but analogous equations exist for the other oscillating quantities (deviations of the chemical potentials, densities, and pressure from their equilibrium values).

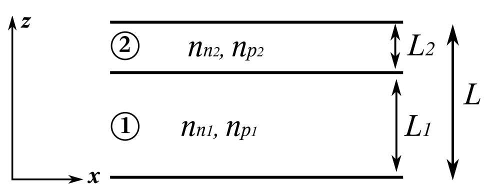

Fig. 1. Diagram representing the 1D (lasagna) structure.

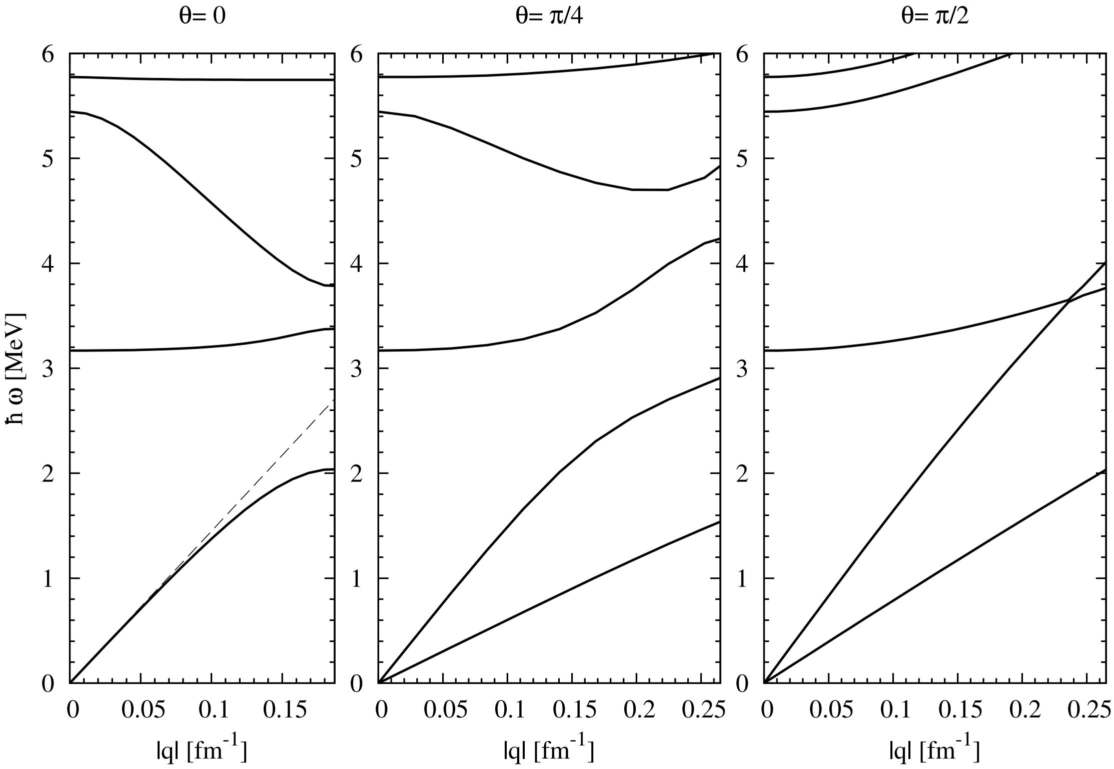

Fig. 2. Dispersion relations for waves propagating in different directions, from left to right: perpendicular to the interface, angle of $\pi/4$ and parallel to the interface.

4 Excitation spectrum

After working out the explicit equations, one obtains a 6×6 linear system of equations for the amplitudes of forward and backward running waves in the two slabs 1 and 2. The excitation spectrum of the eigenmodes is obtained by looking for those values of the frequency ω for which this system has a non-trivial solution (i.e., for which the determinant of the matrix vanishes). The frequencies depend, of course, on the Bloch momentum $\vec{q}$.

As an example, the spectrum as a function of $q = |\vec{q}|$ is displayed in Fig. 2 for different angles θ between $\vec{q}$ and the z axis. The total baryon number density is chosen to be $n_B = 0.48 n_0$, corresponding to a lasagna-type structure in the model of Avancini et al. (2009).

In the case $\theta = 0$, one observes one acoustic mode (having $\omega = 0$ for $q = 0$) and several optical ones (having $\omega > 0$ for $q = 0$). The slope of the acoustic one in the long wavelength limit ($q \to 0$) corresponds to an average sound velocity. The discrete spectrum of the optical modes at the point $q = 0$ is what one can obtain within the Wigner-Seitz approximation with periodic boundary conditions. However, the coupling between different Wigner-Seitz cells makes the energies q dependent, resulting in a continuous spectrum. In the case $\theta \neq 0$, a second acoustic mode appears. This mode describes an excitation of the denser phase with neutrons and protons moving out of phase. Since this mode penetrates only weakly into the neutron gas, its energy is almost independent of q_z, i.e., it is approximately proportional to $q \sin\theta$.

5 Application to specific heat

In the case of superfluidity, due to the pairing energy, the contribution of indiviual fermions (protons, neutrons) to the specific heat c_V is strongly suppressed ($\propto e^{-\Delta/T}$ since it costs energy of the order of Δ to break a pair). However, superfluidity induces collective low-energy excitations, for homogeneous matter these are called Bogoliubov-Anderson modes. In our case they correspond to the acoustic modes. From Fig. 3 it can be seen that indeed, for a typical temperature of 10^9 K, the contribution of the collective modes to c_V is much more

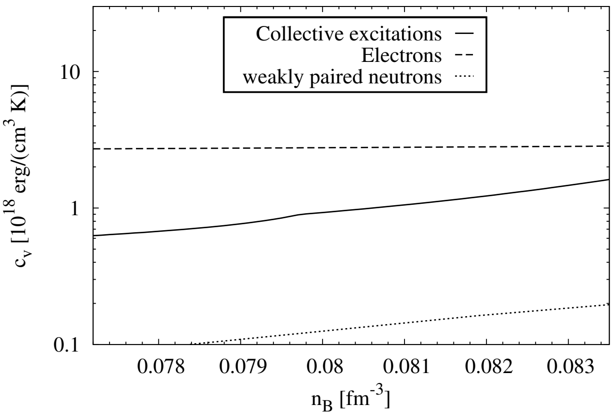

Fig. 3. Different contributions to the specific heat as a function of density for the lasagna-type structure at a temperature of $T = 10^9$K.

important than that of the individual neutrons (even if we assume that the latter are only weakly paired, dotted line) and comparable to that of the electrons (dashed line). At low temperatures, only the electrons and the acoustic modes contribute. While the electron contribution to the specific heat is linear in T, the contribution of the collective modes in homogeneous matter follows a T^3 law. In the slab structure, however, the second acoustic branch discussed in the preceding section gives a contribution proportional to T^2. At higher temperatures, the deviations from a linear dispersion relation become important, and also the optical modes start to contribute. However, one should keep in mind that, as mentioned in the beginning, our approach is in principle only valid at energies (and temperatures) below ~ 1 MeV ($\sim 10^{10}$ K).

6 Summary and outlook

In order to determine the thermal properties of the neutron star inner crust, the entire excitation spectrum has to be known. We have considered collective excitations taking into account the effects of superfluidity. With our approach, we try to build a bridge between the long wavelength limit ($|\vec{q}| \ll \pi/L$) of Pethick et al. (2010); Cirigliano et al. (2011) and the microscopic QRPA approaches (Khan et al. 2005) applying the Wigner-Seitz approximation ($|\vec{q}| > L$).

Our results for the slab structure (lasagna phase) show that the collective modes can give important contributions to the specific heat. For typical temperatures during the crust thermalization epoch, in particular the lowest lying acoustic mode(s) are important for the thermal properties.

Because of a couple of very restrictive simplifications, the present work has to be seen as an exploratory study which we plan to improve in the next future. In particular, it seems very important to include the Coulomb interaction between the protons. In addition, it will be necessary to consider more complicated geometries (2D tubes/rods, 3D droplets/bubbles) in order to be able to describe the entire inner crust.

The collective excitations are in fact not only interesting in the context of the specific heat. It could also be interesting to study their effect on neutrino-matter interactions.

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