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THE BAADE-WESSELINK PROJECTION FACTOR OF THE δ-SCUTI STARS AI VEL AND β CAS

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Abstract. The Baade-Wesselink method of distance determination is based on the oscillations of pulsating stars. After determining the angular diameter and the linear radius variations, the distance is derived by a simple ratio. The linear radius variation is measured by integrating the pulsation velocity (hereafter $V_{\rm puls}$) over one pulsating cycle. However, from observations we have only access to the radial velocity ($V_{\rm rad}$) because of the projection along the line-of-sight. The projection factor, used to convert the radial velocity into the pulsation velocity, is defined by: $p = V_{\rm puls}/V_{\rm rad}$. We aim to derive the projection factor for two δ-Scuti stars, the high amplitude pulsator AI Vel and the fast rotator β Cas. The geometric component of the projection factor is derived using a limb-darkening model of the intensity distribution of AI Vel, and a fast rotator model for β Cas. Then, by comparing the radial velocity curves of several spectral lines forming at different levels in the atmosphere, we derive directly the velocity gradient (in a part of the atmosphere of the star) using SOPHIE/OHP data for β Cas and HARPS/ESO data for AI Vel, which is used to derive a dynamical projection factor for both stars. We find $p = 1.44 \pm 0.05$ for AI Vel and $p = 1.41 \pm 0.25$ for β Cas. By comparing Cepheids and δ-Scuti stars, these results bring valuable insights into the dynamical structure of pulsating star atmospheres.

Keywords: δ-Scuti, projection factor, spectroscopy, modelling

1 Introduction

Determining distances in the Universe is not a trivial task. From our Galaxy to the Virgo Cluster, distances can be derived using the Period-Luminosity relation (PL) relation of Cepheids (Riess et al. 2009a,b). However, this relation has to be calibrated using for instance the Baade-Wesselink method of distance determination (Storm et al. 2011a,b). The principle of this method is simple: after determining the angular diameter and the linear radius variations of the star, the distance is derived by a simple ratio. Angular diameter variations can be measured using interferometry (Kervella et al. 2004) or using the infrared surface brightness relation (Gieren et al. 1998, 2005). When determining the linear radius variation of the Cepheid by spectroscopy, one has to use the so-called projection factor in order to convert the radial velocity into pulsation velocity. There are in principle three sub-concepts involved in the Baade-Wesselink projection factor: (1) the geometric projection factor p_0 directly related to the limb-darkening of the star (see Section 3), (2) the correction $f_{\rm grad}$ due to the velocity gradient between the spectral line forming region and the photosphere -this quantity can be derived directly from observations by comparing different lines forming at different level in the atmosphere- (see Sections 2 and 4) and (3) the correction $f_{\rm og}$ due to the relative motion between the *optical* and *gas* layers associated to the photosphere (see Section 5). For a detailed analysis of the p-factor decomposition refer to Nardetto et al. (2007). The projection factor is then defined by $p = p_0 f_{\rm grad} f_{\rm o-g}$. In the following, we apply this decomposition of the projection factor (originally developed for Cepheids) to the δ Scuti stars AI Vel and β Cas (Guiglion et al., 2012, A&A, to be submitted). The determination of the projection factor is very pertinent to the study of this class of variable stars since it follows a PL relation (McNamara et al. 2007; Poretti et al. 2008).

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2 Spectroscopic observations of δ Scuti stars

AI Vel (HD 69213, A9IV/V) is a multi-periodic high-amplitude δ-Scuti star with $T_{\rm eff} = 7\,430$ K, $\log g = 3.72$ and is located at $d = 98$ pc according to the Hipparcos catalogue (van Leeuwen 2007). Its fundamental pulsation period is $P = 0.111574$ days (Walraven et al. 1992). According to the same study, AI Vel oscillations are characterized by 2 radial modes and 3 non-radial modes. We observed AI Vel during 4 nights in January 2011 (9th, 10th, 11th, 12th) using the HARPS[i] spectrograph of the La Silla 3.6 meters telescope (ESO). Our set is composed of 104 high resolution spectra, with a mean S/N of 140, a resolution R = 80 000 and a wavelength domain from 3 780 to 6 910 Å. We identified 53 metallic unblended spectral lines relevant for the determination of radial velocities.

β Cas (HD 432) is a low-amplitude δ-Scuti star with a spectral type of F2III/IV (Rhee et al. 2007). The star is located at $d = 16.8$ pc (van Leeuwen 2007). In the literature, the rotational velocity $v \sin i$ is found to be from 71 to 85 km s^{-1} (Bernacca & Perinotto 1970; Uesugi & Fukuda 1970; Schröder et al. 2009). The effective temperature ranges from 6 877 K to 7 178 K (Gray et al. 2001; Rachford & Foight 2009). Riboni et al. (1994) measured the period using photometry: $P = 0.101036676 \pm 0.000000053$ days. We observed β Cas with the OHP 1.93 meter telescope (SOPHIE[ii]) during one night in september 2011 (30th). We collected 241 high resolution spectra with a mean S/N of 100, a resolution R = 75 000 and a wavelength domain from 3 872 to 6 943 Å. We identified only 8 unblended spectral lines relevant for the spectral analysis because of the strong broadening of the lines.

For both stars, the centroid radial velocity $RV_{\rm c}$ (or the first moment radial velocity) and the line depth D are derived as a function of the pulsation phase for each selected spectral line. These observations are used in Section 4 to apply a dynamical correction to the geometric projection factor. In the following we assume that the non-radial modes for both stars have an negligible impact on the projection factor. A detailed analysis of this assumption will be done in a following paper.

3 The geometric projection factor p_0

In the case of a limb-darkened pulsating star in rotation with a one-layer atmosphere, the projection factor is purely geometric and we have $p = \frac{V_{\rm puls}}{V_{\rm rad}} = p_0$. The radial velocity is then defined by:

$$V_{\rm rad} = \frac{1}{\pi {\rm R}^2} \int_{x,y\,\in {\rm D_R}} I(x,y,\lambda) \cdot V_{\rm puls} \cdot \sqrt{1 - \frac{(x^2+y^2)}{{\rm R}^2}}\, dxdy, \tag{3.1}$$

where $\rm D_R$ is the surface of the stellar disc of linear radius R and $I(x,y,\lambda)$ the limb-darkened intensity distribution considered at the wavelength of observation λ defined by $I(x,y,\lambda) = I_0(1 - u_\lambda + u_\lambda \sqrt{1-(x^2+y^2)})$ where u_λ is the linear limb-darkening coefficient from Claret & Bloemen (2011). We assume that the projection factor is constant with the pulsating phase. In other words, the geometric projection factor can be calculated at any pulsation phase, whatever the pulsation velocity considered (one can consider for instance $V_{\rm puls} = 1$). Considering $T_{\rm eff} = 7\,400$ K and $\log g = 3.5$, we find $u_{\rm R} = 0.474 \pm 0.025$ in the R-band. We deduce the value of the geometric projection factor for AI Vel: $p_0 = 1.43 \pm 0.01$. This value is larger compared to Cepheids (typically 1.36 to 1.41, see Fig.1 (left)).

Concerning β Cas, its high rotation rate leads to a distortion of its intensity distribution and of its geometrical shape. The geometric projection effect depends on the inclination of the rotation axis of the star compared to the line-of-sight. If the rotation axis of the star is along the line-of-sight ($i = 0°$), the star is pole-on and is seen as a circle. For $i > 0°$ the star has an ellipsoidal shape. Using the fundamental parameters from Che et al. (2011) and the rotating stars model by Domiciano de Souza et al. (2002, 2012), we derive the intensity distribution in the continuum for different inclinations of the star (from $i = 0°$ to $i = 90°$ with a step of 5°) and for three wavelengths: $\lambda = 6\,000, 6\,500$ and $7\,000$ Å. Using these intensity maps, we can easily calculate the

[i] High Accuracy Radial velocity Planetary Search project developed by the European Southern Observatory (ESO).

[ii] Spectrographe pour l'Observation des Phénomènes des Intérieurs stellaires et des Exoplanètes developed by the Observatoire de Haute Provence (OHP).

geometric projection factor. Indeed, for an ellipsoid, $V_{\rm rad}$ is then defined by:

$$V_{\rm rad} = \frac{1}{\pi \mathrm{R}^2} \int_{x,y \in \mathrm{D_R}} I(x,y,\lambda) \cdot \sqrt{1 - \left(\frac{x^2}{\mathrm{a}^2} + \frac{y^2}{\mathrm{b}^2}\right)} dxdy, \tag{3.2}$$

where a and b are the semi-major and semi-minor axis of the ellipse, respectively. In Figure 1 (right), we show the geometric projection factor (p_0) as a function of i. This relation is extremely interesting because it shows that the inclination of a fast rotating star can have an impact of more than 10% on the projection factor. Of course, it depends also on the rotation velocity of the star: the larger the rotation velocity (for a given inclination), the lower the projection factor. Using the inclination found by Che et al. (2011), $i = 19.9 \pm 1.9°$, we finally find a geometric projection factor for β Cas of $p_0 = 1.41 \pm 0.02$ (averaged over the three wavelengths considered).

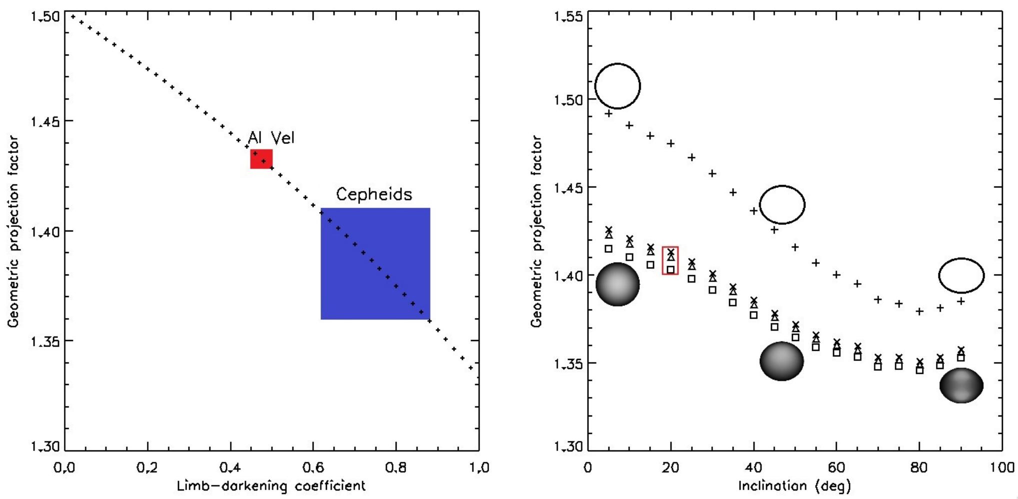

Fig. 1. Left: p_0 as a function of the limb-darkening parameter u_λ. The red box indicates the uncertainty on p_0 for the δ Scuti AI Vel. The blue box indicates the typical values of u_λ and p_0 for Cepheids. The dots corresponds to the relation provided by Nardetto et al. (2006). **Right:** p_0 as a function of the inclination of the fast rotating star β Cas for three different wavelength ($\lambda = 6\,000$ Å (□), $\lambda = 6\,500$ Å (△), and $\lambda = 7\,000$ Å (×)). The red box indicates the uncertainty on p_0 for β Cas. The case of an uniform elonged disc is over-plotted (+) and we find that $p_0 = 1.5$ for $i = 0°$ as expected for a circular uniform disc.

4 The dynamical structure of δ-Scuti stars

By comparing the 2K-amplitude (defined as the amplitude of the first moment radial velocity curve, hereafter $\Delta RV_{\rm c}$) with the depth of the 53 spectral lines selected in the case of AI Vel, one can measure directly the atmospheric velocity gradient in the part of the atmosphere where the spectral line are formed. Note indeed, that in order to quantify the impact of velocity gradient on the projection ($f_{\rm grad}$) we do not need to derive the velocity gradient over the whole atmophere, but only at the location of the forming regions of the spectral lines used to derive the distance of the star.

In order to find a relation between $\Delta RV_{\rm c}$ and D, we consider the second night of observation for which we have almost a complete cycle (Figure 2, top left). We perform a linear regression according to the relation $\Delta RV_{\rm c} = a_0\, D + b_0$. We obtain $\Delta RV_{\rm c} = [-0.40 \pm 0.53] D + [32.87 \pm 0.23]\,\mathrm{km\,s^{-1}}$ (Figure 2, top right). In principle, $f_{\rm grad}$ depends on the spectral line considered (Nardetto et al. 2007): $f_{\rm grad} = b_0/(a_0 D + b_0)$. Here, we find that $f_{\rm grad}$ is typically the same for all spectral lines ($f_{\rm grad} = 1.01 \pm 0.01$) which is consistent with no correction of the projection factor due to the velocity gradient.

Figure 2 (bottom left) presents the interpoled $RV_{\rm c}$ curve of β Cas in the case of the FeI spectral line ($\lambda = 4\,508.288$ Å). We clearly see an increase of the amplitude of the radial velocity curve ($\sim 4.6 \pm 0.9\%$ per cycle). Moreover the radial velocity curves have several minima and maxima and we can easily deduce a period of pulsation. We find $P = 0.10046 \pm 0.00054$ days. Our value is in a good aggrement with Riboni et al. (1994).

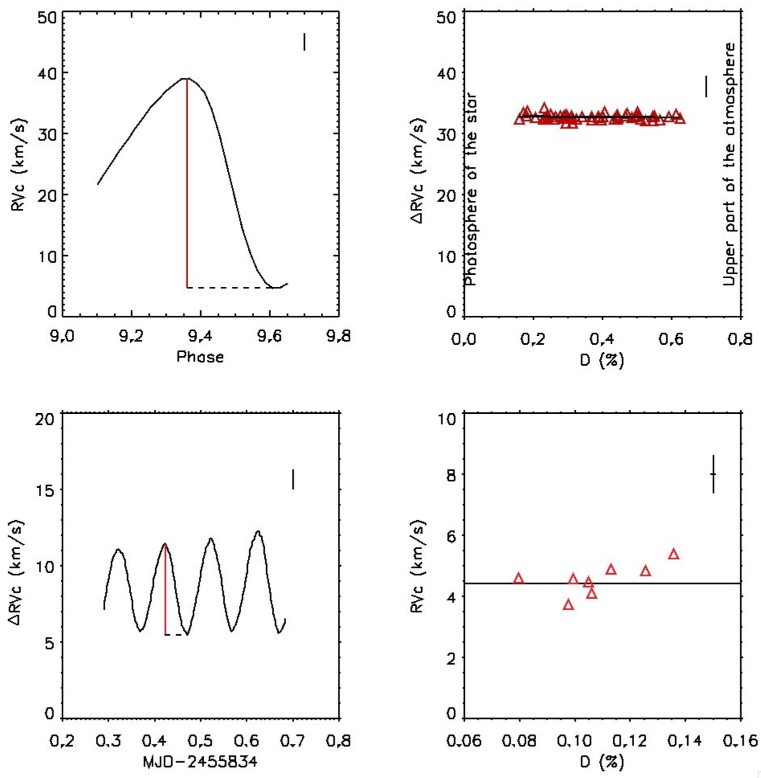

Fig. 2. Top: RV_c as a function of the pulsation phase for the night 2 (FeII spectral line, 5 234.625 Å) and amplitude of RV_c curves as a function of the spectral line depth. **Bottom:** Interpoled RV_c *vs.* MJD for the FeI spectral line $\lambda = 4\,508.288$ Å and ΔRV_c *vs.* D with corresponding statistical uncertainties. The solid line corresponds to the average of the values.

As for AI Vel, we need to consider several spectral lines forming at different levels in the atmosphere. In Figure 2 (bottom right), ΔRV_c is plotted as a function of D (see the vertical red line on the figure). Contrarily to AI Vel for which we had many spectral lines, the case of β Cas is more difficult: most of the spectral lines, broadened by the large rotation velocity of the star, are indeed blended. As a consequence, we cannot estimate properly the actual f_{grad} correction for β Cas: If we measure the velocity gradient as we did for AI Vel, we find $f_{\mathrm{grad}} = 0.64 \pm 0.82$. This value is simply to small to be relevant compared to what we know from Cepheids ($f_{\mathrm{grad}} \sim 0.95$) and what we obtain for AI Vel ($f_{\mathrm{grad}} \sim 1.00$). It would imply indeed, a huge velocity gradient in the atmosphere of the star which is not reasonnable. However, our data are consistent with a constant (no velocity gradient). If we make the assumption that $f_{\mathrm{grad}} = 1$ (or $a_0 = 0$) as for AI Vel, we obtain $b_0 = 4.07 \pm 0.25$ with a reduced χ^2 of 1.0, which provides us an uncertainty on f_{grad} of 0.14.

5 The $f_{\mathrm{o-g}}$ correction

The $f_{\text{o-g}}$ correction, which is the last component of the projection factor decomposition, cannot be measured from observations. To estimate the differencial velocity between the *optical* and *gas* layers at the photosphere of the star, we need an hydrodynamic model. However, modelling the pulsating atmosphere of δ Scuti stars is not simple because of: (1) cycle-to-cycle variations, (2) non-radial modes and (3) fast rotation in some cases (like for β Cas). However, $f_{\text{o-g}}$ have been studied intensively in the case of Cepheids (Nardetto et al. 2004, 2007, 2011), and it seems that there is a linear relation between $f_{\text{o-g}}$ and $\log P$: $f_{\text{o-g}} = [-0.023 \pm 0.005] \log P + [0.979 \pm 0.005]$. Moreover, we have a theoretical value of $f_{\text{o-g}}$ for the short-period β-Cepheid α Lup ($P = 0.2598$), $f_{\text{o-g}} = 0.99 \pm 0.01$, which seems to be consistent with the $\log P$-$f_{\text{o-g}}$ relation of Cepheids (Nardetto et al. 2012, in preparation). For our study, we propose to extend this law for the δ-Scuti β Cas and AI Vel. Considering $P = 0.11157$ days for AI Vel and $P = 0.10046 \pm 0.00054$ days for β Cas (from this paper), we find $f_{\text{o-g}} = 1.00 \pm 0.02$ for both stars.

6 Conclusion

We can now calculate the final projection factor p, using the relation $p = p_0 f_{\mathrm{grad}} f_{\mathrm{o\text{-}g}}$. We find $p = 1.44 \pm 0.05$ for AI Vel and $p = 1.41 \pm 0.25$ for β Cas. These results are consistent with the Pp relation $p = [-0.071 \pm 0.020] \log P + [1.311 \pm 0.019]$ from Laney & Joner (2009) applyed for classical and dwarf Cepheids (It corresponds to $p = 1.38 \pm 0.02$ for AI Vel and β Cas). To derive these values, they simply compared the distance of the stars derived from the PL relation with the ones obtained from the photometric version of the Baade-Wesselink method. The fact that our values of the projection factor are consistent at $1\,\sigma$ with the independant determinations by Laney & Joner (2009) strongly suggests that our assumptions are acceptable, namely (1) the projection factor decomposition can be applied to δ Scuti stars (at least AI Vel and β Cas), (2) the non-radial modes have a negligible impact on the projection factor and (3) the $f_{\mathrm{o\text{-}g}}$ we have considered is probably acceptable in the case of short period pulsating stars.

Interestingly, if we use the Pp obtained for classical Cepheids by Nardetto et al. (2007): $p = [-0.064 \pm 0.020] \log P + [1.376 \pm 0.023]$ in order to derive the projection factors of the two δ Scuti stars, we find $p = 1.44 \pm 0.01$, which is consistent with our values. This seems to show that the period-projection factor relation provided by Nardetto et al. (2007) is also applicable to short period pulsating stars and in particular to δ-Scuti.

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