DOES INERTIA DETERMINE THE MAGNETIC GEOMETRY OF LOW-MASS STARS?

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Abstract. M dwarfs are of prime interest for stellar dynamo theories. They indeed span a wide range of parameters, in terms of relative depth of the convection zone (the lowest-mass M dwarfs being fully convective) and rotation. The number of magnetic field measurements on M dwarfs has been rapidly growing in the past few years, trends are emerging and now need to be understood in the framework of dynamo theory. We detail the analogy between latest anelastic dynamo simulations by Gastine et al. and observations of M-dwarf magnetism, focusing on field geometries derived from spectropolarimetric observations. In geodynamo models, the relative importance of inertia in the force balance is known to have a strong impact on the magnetic field geometry. This can be quantified by the so-called "local Rossby number", which has been found to be a rather universal quantity that allows to separate dipolar and multipolar dynamo models. We discuss its relevance in setting the field geometry of M dwarfs and the transition towards a bistable regime.

Keywords: Dynamo, Stars: magnetic field, Stars: rotation, Stars: low-mass

1 Introduction

M dwarfs – the lowest-mass stars of the main sequence – are of prime interest to study stellar dynamos operating in physical conditions quite remote from the solar case. During the past few years, their surface magnetic fields have been investigated using two complementary approaches: spectroscopy in unpolarized light from which the average magnetic field strength can be derived, and spectropolarimetry which provides a constraint on the geometry of the field at large and intermediate scales (for recent reviews see Donati & Landstreet 2009; Reiners 2012). Using the latter approach combined with Zeeman-Doppler imaging (ZDI, Semel 1989), the study of a sample of about 20 M0-M8 dwarfs points towards a broad variety of magnetic field geometries: partly-convective stars as well as a few fully-convective ones feature complex magnetic structures (Donati et al. 2008; Morin et al. 2010), while most fully-convective ones host a strong axial dipole component (Morin et al. 2008a,b). Explaining such a diversity in the magnetic field geometry is one of the main goals of stellar dynamo theory.

In geodynamo models, the "local Rossby number" – defined by $\text{Ro}_l = u_{\text{rms}}/\Omega l$, l being the typical flow lengthscale – which measures the relative contribution of inertia and Coriolis force in the global balance, has been found to be a rather universal quantity that allows to separate dipolar and multipolar dynamo models. A sharp transition between these two types of dynamo indeed occurs around $\text{Ro}_l \simeq 0.1$ (Christensen & Aubert 2006). However, recent studies employing stress-free mechanical boundary conditions (more appropriate when modelling stellar dynamos) question this view as they found that a dipolar and a multipolar magnetic field can coexist at the same parameter regime depending on the initial condition (e.g. Busse & Simitev 2006), leading to multipolar solutions even for $\text{Ro}_l < 0.1$ (Schrinner et al. 2012). Although most of these studies have been conducted under the Boussinesq approximation (i.e. assuming constant reference state), the parametric study of Gastine et al. (2012) shows that these results remain valid when the effect of moderate density stratification are taken into account.

Several recent studies have shown that dynamo action in planets and low-mass stars share a number of similarities (e.g. Goudard & Dormy 2008; Christensen et al. 2009). Here we discuss the analogy between the anelastic dynamo models of Gastine et al. (2012) and spectropolarimetric observations of M dwarfs (see Morin et al. 2010, and references therein), thereby extending the discussion on possible bistability among very-low-mass stars of Morin et al. (2011a).

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2 The dynamo model

We consider MHD simulations of a conducting anelastic fluid in spherical shells rotating at a constant rotation rate Ω about the z-axis. Following Gilman & Glatzmaier (1981), the governing MHD equations are nondimensionalised using the shell thickness $d = r_o - r_i$ as the reference lengthscale and Ω^{-1} as the time unit. Our dynamo model results are then characterised by several dimensionless diagnostic parameters. The rms flow velocity for instance is given by the Rossby number $\text{Ro} = u_{\text{rms}}/\Omega d$. Following Christensen & Aubert (2006), we also employ the aforementioned local Rossby number $\text{Ro}_l = \text{Ro} \, \bar{\ell}_u/\pi$, that is known to be a more appropriate measure to quantify the impact of inertia on the magnetic field geometry. The mean spherical harmonic degree $\bar{\ell}_u$ is obtained from the kinetic energy spectrum and relates to the typical flow lengthscale l through:

$$l = \pi d/\bar{\ell}_u \quad \text{with} \quad \bar{\ell}_u = \sum_{\ell} \ell \frac{\langle \vec{u}_\ell \cdot \vec{u}_\ell \rangle}{\langle \vec{u} \cdot \vec{u} \rangle},\tag{2.1}$$

where \vec{u}_{ℓ} is the flow velocity at a given spherical harmonic degree ℓ and the brackets correspond to an average over time and radius. The magnetic field strength is measured by the Elsasser number $\Lambda = B_{\rm rms}^2/\rho\mu\lambda\Omega$, where ρ is the density, and μ and λ are the magnetic permeability and diffusivity. The geometry of the surface field is quantified by its dipolarity $f_{\rm dip}$ that measures the ratio of the magnetic energy of the dipole to the magnetic energy contained in spherical harmonic degrees up to $\ell_{\rm max} = 11$. The dimensionless MHD equations are advanced in time with the spectral code MagIC (Wicht 2002; Gastine & Wicht 2012) that uses the anelastic formulation of Lantz & Fan (1999) and has been validated against several dynamo benchmarks (Jones et al. 2011). We rely in the following on the results of the parameter study of Gastine et al. (2012).

3 Spectropolarimetric observations

Spectropolarimetric observations of 23 active M0-M8 dwarfs with rotation periods ranging from 0.4 to 19 days have been carried out. For each star at least one time-series of unpolarized and circularly polarized spectra sampling a few rotation periods has been obtained. The data reduction and analysis is detailed by Donati et al. (2006, 2008) and Morin et al. (2008a,b, 2010).

The relative importance of inertia with respect to the Coriolis force in the convection zone of these stars is assessed through an empirical Rossby number given by

$$\mathrm{Ro}_{\mathrm{emp}} = \frac{P_{\mathrm{rot}}}{\tau_{\mathrm{conv}}},\tag{3.1}$$

where τ_{conv} is the empirical turnover timescale of convection based on the rotation-activity relation (Kiraga & Stepien 2007). This Rossby number misses explicitly the flow lengthscale l involved in Ro_l. However, as τ_{conv} is based on the average convective turnover time it encompasses this scale information to some extent. We thus use Ro_{emp} as our best available proxy for Ro_l.

For each obtained spectrum, an average line profile with increased signal-to-noise ratio is computed using the least-squares deconvolution technique (LSD, Donati et al. 1997). Each time-series of LSD profiles is modelled with ZDI, resulting in a map of the large-scale component of the surface magnetic field vector that satisfies a maximum-entropy criterion. The large-scale magnetic fields of most of these stars fall into two distinct groups: one is dominated by a strong axial dipole and the other by a much weaker and non-axisymmetric field.

Similarly to Morin et al. (2011b), we define an Elsasser number based on the averaged unsigned large-scale magnetic field $\langle B_V \rangle$ which roughly characterises the ratio of the Lorentz and Coriolis forces. We also consider the fraction of the magnetic energy that is recovered in the axial dipole mode in ZDI maps. The spatial resolution of such maps mostly depends on $v \sin i$ and the actual degree ℓ_{max} up to which the reconstruction can be performed ranges from 4 to 10, although very little energy is recovered in modes with $4 < \ell \leq 10$. We therefore directly compare this quantity to the dipolarity employed in numerical models and term them both f_{dip} . We however note that in simulations, f_{dip} does not strongly depend on the chosen ℓ_{max} , whereas for the observation-based dipolarity, considering the ratio of magnetic energy in the axial dipole relative to the total magnetic energy derived from unpolarized spectroscopy (instead of the large-scale magnetic energy derived from spectropolarimetric data with ZDI) would lead to much lower values of f_{dip} (cf. Reiners & Basri 2009). We attribute this difference to the low magnetic Reynolds number (Rm ~ 100 - 500) accessible by numerical simulations which does not allow for a significant small-scale field to be generated – hence the weak dependence of f_{dip} on ℓ_{max} – while in stellar interiors large-scale and small-scale dynamo action likely coexist (e.g. Cattaneo & Hughes 2009).



Fig. 1. Left: Relative dipole strength plotted against Ro_l in anelastic dynamo models. Red (grey) symbols correspond to simulations in thick (thin) shells ($r_i/r_o = 0.2$ and $r_i/r_o = 0.6$, respectively) and their size is scaled according to the value of the surface field, expressed in units of the square root of the Elsasser number. Each type of symbols corresponds to a given density contrast. The vertical lines mark the tentative limits for dipolar dynamos. **Right:** Relative dipole strength plotted against Ro_{emp} . Symbol sizes scale with the Elsasser number based on the average large-scale magnetic field derived from spectropolarimetric observations. The vertical dashed line marks the tentative limit for the dipolar regime. For the two stars exhibiting the largest temporal variations, the individual epochs are shown and connected by a vertical red line. Dotted red circles with black arrows correspond to stars from Morin et al. (2010) for which ZDI reconstruction could not be achieved, only upper limit for the rotation period and an estimate of dipolarity were derived.



Fig. 2. Left: Radial component of the surface magnetic field $B_r(r = r_0)$ and axisymmetric zonal flows $\overline{u_{\phi}}$ for a dipolar dynamo model (a) and a multipolar one (b) with similar density contrast $N_{\rho} \simeq 2$. The maps of B_r have been low-pass filtered up to $\ell_{\text{max}} = 10$. Magnetic fields are expressed in units of the square root of the Elsasser number and velocities in units of the Rossby number. Right: Radial component of the surface magnetic field of V374 Peg (c) and GJ 1245 B (d) recovered with ZDI from spectropolarimetric observations of Donati et al. (2006) and Morin et al. (2010), respectively. The field has been reconstructed up to $\ell_{\text{max}} = 10$ (4) for V374 Peg (GJ 1245 B). Surface differential rotation of V374 Peg has been derived by Morin et al. (2008b) from spectropolarimetric observations, while this was not possible for GJ 1245 B. Magnetic fields are expressed in units of the Elsasser number.

4 Results and conclusions

Spectropolarimetric observations of active M dwarfs and dynamo models show a broad variety of magnetic geometries (see Gastine et al. 2012; Morin et al. 2010, and references therein). In both cases, dipolar and multipolar large-scale magnetic fields are found to coexist at low Rossby numbers. Here we briefly discuss the analogy between these two results, the reader is referred to Gastine et al. (submitted) for more details.

We derive observation-based quantities aimed to reflect the diagnostic parameters employed in the numerical

models (Ro_l, Λ and f_{dip}), although these crude proxies are not expected to provide a direct quantitative match. Within these limits, we draw an interesting analogy between the observational parameters and their numerical counterparts: for large values of the Rossby numbers multipolar fields are found, while below a critical value around Ro_l ~ Ro_{emp} ~ 0.1, a bistable region exists where both dipolar and multipolar fields can be generated (see Fig. 1, 2). Several limitations must be noted though. (i) The spectropolarimetric sample is biased as all stars at high (low) Ro_{emp} are partly (fully) convective. Thus it is not yet clear if the change in f_{dip} observed around Ro_{emp} ~ 0.1 can be attributed to a threshold in Ro_l or rather to the drastic changes in stellar structure occurring at the fully-convective transition. (ii) As the numerical models of Gastine et al. (2012) do not attempt to model a tachocline, they might miss some important features of early M dwarfs magnetism. However this issue does not question the validity of the agreement between observations and simulations regarding the existence of a bistable dynamo regime at low Ro_l for fully-convective stars. (iii) In numerical models, the dipolar branch only exists for moderate density contrasts ($N_{\rho} \leq 2$), much below the stratification of stellar interiors. Different assumptions from those considered by Gastine et al. (2012) could possibly extend the dipolar regime towards higher stratifications, for instance by using different values of Prandtl numbers (Simitev & Busse 2009) or radius-dependent properties (e.g. thermal and ohmic diffusivities).

The analogy between numerical models and magnetic properties of M dwarfs can be further assessed with additional observations, as it implies that: (i) stars with multipolar fields can be found over the whole parameter range where also dipole-dominated large-scale fields are observed; (ii) in the bistable domain, stars on the multipolar branch have a much stronger surface differential rotation than those on the dipolar branch.

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