SATELLITE FORMATION :
SPREADING OF RINGS BEYOND THE ROCHE RADIUS

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Abstract. When a disk of solid particles (like Saturns rings) spreads beyond the Roche radius (inside which planetary tides prevent aggregation), satellites form and migrate away. After a quick derivation of the Roche radius, here we describe analytically this process. When the spreading is fast, only one large satellite forms, as was the case for Pluto and Earth. When the spreading is slow, a retinue of satellites appear with masses increasing with distance to the Roche radius, in excellent agreement with Saturns, Uranus, and Neptunes satellite systems. This suggests that Uranus and Neptune used to have massive rings that disappeared to give birth to their regular satellites. Thus, we argue in Crida & Charnoz (2012) that most regular satellites in the Solar System probably formed in this way.

Keywords: Solar System: satellites, Saturn, accretion disks

1 Introduction

Planets form in proto-planetary discs of gas and dust around young stars. Giant planets open gaps in this disc, and are then surrounded by a circum-planetary disc. It is generally thought that their satellites form inside this disc, like a miniature Solar System, although the delivery and growth of solids are different (e.g. Canup & Ward 2002, 2006; Sasaki et al. 2010; Mosqueira & Estrada 2003a,b). However, these models can not explain the particular mass-distance distribution of the regular satellites of the giant planets, in particular the three following features : (i) no satellite is found at an orbital radius smaller than ~ 2.5 planetary radii ; (ii) the masses of the satellites increase with orbital distance, starting from 0 at this particular radius ; (iii) a pile-up of small bodies is observed just outside this radius, while the number density of satellites decreases outwards.

Recently, Charnoz et al. (2010) have shown that 5 small moons of Saturn, orbiting just outside the rings, were actually born from ring material spreading beyond the outer edge of the rings. This model has been extended using numerical simulations to all the medium size moons of Saturn, explaining their stochastic composition (Charnoz et al. 2011). Here, we study analytically the spreading of a dynamically cold, self-gravitating ring of solid particles. The SF2A conference being held this year in Montpellier, the city of Édouard Roche, we first show in section 2 why the particles constituting the rings aggregate only beyond a given distance to the central planet, called the Roche radius. In section 3, the equations describing the spreading of the rings beyond the Roche radius and the evolution of the subsequent forming moons are derived. This model is compared to the observed satellite systems in section 4, where the agreement is found to be very good for most planets of the solar system.

2 Tides and the Roche radius

Consider two spheres of radius a, mass m and density ρ, orbiting around a planet of mass M at a distance r, the two spheres being in contact, aligned along the radial direction (see Fig. 1). Their orbital angular velocity

is given by Kepler's law: $\Omega = \sqrt{GM/r^3}$. In the frame corotating with the spheres, they feel the centrifugal acceleration and the gravity from the central planet. These two forces balance exactly at r, but the spheres are actually at distances $r \pm a$ from the central object. Thus, they feel a gravitational force $F_g = GMm/(r \pm a)^2$ towards the planet, and a centrifugal force $F_c = m\Omega^2(r \pm a)$ in the opposite direction. To first order in a/r, the total force felt by the sphere located at $r \pm a$ is $F_t = \pm 3\Omega^2 am$. This is the tidal force, that tends to separate the spheres.

On the other hand, the spheres gravitationally attract each other, with the force $F_{sg} = \mp Gm^2/(2a)^2$. Consequently, self-gravity wins over tides if and only if $|F_{sg}| > |F_t|$ which, using $m = \frac{4}{3}\pi a^3 \rho$, is equivalent to:

$$r > \left(\frac{9M}{\pi\rho}\right)^{1/3} \equiv r_{\mathrm{R}} \qquad (2.1)$$

Self-gravitating aggregates are only possible beyond a particular radius, called the Roche radius.

Fig. 1. Sketch of the centrifugal force (F_c, blue) and gravitational force (F_g, red) acting on two spheres in orbit around a central mass.
The resultant total force is the tidal force (yellow).

It should be noted that this simple model is not the original derivation by (Roche 1849). He considered equipotentials of a liquid sphere, and found $r_{\mathrm{R}} = 2.456\,R(\rho_p/\rho)^{1/3}$, where R and ρ_p are the radius and density of the central planet, respectively. The two expressions are almost identical. In the case of the rings of Saturn ($M = 5.86 \times 10^{24}$ kg, $\rho = 600$ kg.m^{-3}), Eq. (2.1) gives $r_{\mathrm{R}} \approx 140\,000$ km, which is the outer edge of the main rings. This is why the rings do not coalesce into one large moon. They are prevented to do so by the tides. We call such a system a *tidal disc*.

3 Analytical model

Be T_{R} the orbital period at the Roche radius r_{R}, F the flow of mass from the rings through r_{R}, and $\tau_{\mathrm{disc}} = M_{\mathrm{disc}}/FT_{\mathrm{R}}$ the normalised life time of the tidal disc (M_{disc} being its mass). We also note $q = M_{\mathrm{sat}}/M$ and $D = M_{\mathrm{disc}}/M$ the normalised masses of the satellite and the disc respectively (M being still the mass of the central object).

3.1 Continuous regime

Assume that 1 satellite forms by self-gravitating aggregation of the material beyond r_{R}. Then, its mass is obviously $M_{\mathrm{sat}} = Ft$, where t is the time. It is well-known (Lin & Papaloizou 1979) that the satellite will then feel a torque from the tidal disc, given by $\Gamma = \dfrac{8}{27}q^2\Sigma r^4\Omega^2\Delta^{-3}$, where Σ is the surface density of the disc, and $\Delta = (r - r_{\mathrm{R}})/r_{\mathrm{R}}$. Thus, the migration rate of the satellite is given by:

$$\frac{d\Delta}{dt} = \frac{2^5}{3^3}\,q\,D\,T_{\mathrm{R}}^{-1}\Delta^{-3} \qquad (3.1)$$

Using $q(t) = Ft/M$, this leads to :
$$q(t) = \left(\frac{\sqrt{3}}{2}\right)^3 \tau_{\mathrm{disc}}^{-1/2}\Delta(t)^2 \ . \qquad (3.2)$$

We call this the *continuous regime*, in which the mass of the satellite grows linearly with time, while its distance to rings is proportional to the square root of its mass.

This model is valid as long as the Roche radius is within the feeding zone of the satellite. The feeding zone is generally considered to have a width of 2 Hill radii, where the Hill radius is $r_{\mathrm{H}} = r(q/3)^{1/3}$. The condition of validity of the continuous regime is thus $\Delta < 2(q/3)^{1/3}$. Using Eq. (3.2), this is equivalent to:

$$\Delta < \sqrt{\frac{3}{\tau_{\mathrm{disc}}}} \equiv \Delta_{\mathrm{c}} \qquad\qquad q < \frac{3^{5/2}}{2^3}\tau_{\mathrm{disc}}^{-3/2} \equiv q_{\mathrm{c}} \qquad (3.3)$$

As the rings start spreading, $\Delta = q = 0$ and this condition is fulfilled. The continuous regime happens at the beginning of the process. When $\Delta > \Delta_{\mathrm{c}}$, the satellite is too far to accrete the material that flows from the rings. It keeps then migrating outwards at constant mass, while a new satellite forms at the Roche radius.
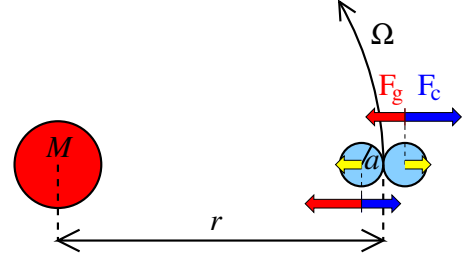
3.2 Pyramidal regime

Say satellites of the critical mass are produced regularly at the critical distance (this is the natural outcome of the continuous regime). They migrate outwards slower and slower as they go away. Thus, they approach each other. If their mutual distance decreases below 2 Hill radii, they merge. This gives birth to satellites twice more massive, who migrate outwards twice faster, but eventually approach each other, and merge further. This gives birth to satellites four times more massive, and so on... We call this the *pyramidal regime*, as a hierarchical history of mergers governs the evolution of the system, like a genealogical tree.

Using Eq. (3.1), we show that in the pyramidal regime, the mass is doubled (a merging event takes place) every time the distance Δ is multiplied by $2^{5/9}$. This means that $q \propto \Delta^{9/5}$. In addition, the number density of satellites is proportional to $1/\Delta$. This explains the pile-up of small satellites just beyond the Roche radius, starting with $q = 0$ at $r = r_{\mathrm{R}}$.

Beyond the 2:1 mean motion resonance with the rings (located at $\Delta_{2:1} = 0.58$), Eq. (3.1) doesn't apply anymore, and the migration of the satellites is dominated by the tides from the central planet. Still, the equations can be solved, and lead to $q \propto r^{3.9}$. In the end, satellites formed in the pyramidal regime have their masses such that:

$$q \propto \mathcal{Q}(\Delta) = \begin{cases} (\Delta/\Delta_{2:1})^{9/5} & \Delta < \Delta_{2:1} \\ \left(\frac{\Delta+1}{\Delta_{2:1}+1}\right)^{3.9} & \Delta > \Delta_{2:1} \end{cases} \qquad (3.4)$$

4 Application

Amazingly, the critical mass or distance (Eq. (3.3)) corresponding to the end of the continuous regime depends on only one parameter: τ_{disc}. The spreading of the disc can be modelled as a viscous process, with a time scale $t_\nu = r_{\mathrm{R}}^2/\nu$. Daisaka et al. (2001) provide a prescription for ν in the case of a tidal disc, that lead to $\tau_{\mathrm{disc}} = t_\nu/T_{\mathrm{R}} = 0.0425/D^2$.

Then, assuming the mass of the putative tidal disc that gave birth to a satellite system was 150% of the total mass of the present satellites, we can apply our model to all the planets of the Solar System.

Saturn: Today's Saturn's rings would produce tiny moonlets of $q_c = 10^{-18}$. But even if one takes $D \approx 4 \times 10^{-4}$ (more than the mass of all the regular satellites, including Titan), $\tau_{\mathrm{disc}} \approx 2 \times 10^8$, and $\Delta_c < 0.01$; so, this system should be dominated by the pyramidal regime. In fact, the satellite system of Saturn represented in the $q - \Delta$ plane is shown in Fig. 2, and is very well fitted by Eq. (3.4) (dashed line): considering the pair Janus-Epimetheus as 1 object, no satellite is further than 60% from the expected mass. This could be considered as the smoking gun, the observational proof that Saturn's regular satellite have been formed by the spreading of the rings in the pyramidal regime.
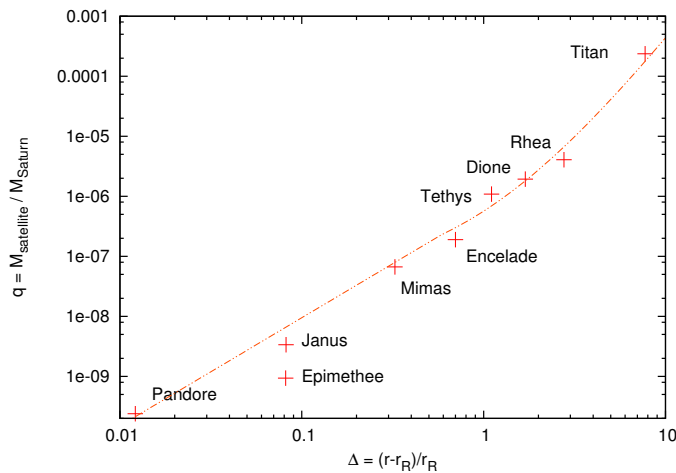

Fig. 2. Mass - Distance distribution of the regular satellites of Saturn. The dashed curve is given by Eq. (3.4)

Actually, it has been shown by Salmon et al. (2010) that Saturn's rings could have been much more massive in the past. As a matter of fact, Canup (2010, see also Crida & Charnoz (2010)) propose a scenario for their origin in which they must have been at least as massive as half Titan, so that she suggested that their spreading may have given birth to the satellite inside Tethys. Last, Lainey et al. (2012) have shown that the tidal dissipation inside Saturn is higher than previously thought, which allows the outward migration of all the satellites from the Roche radius to their present orbit within the age of the Solar System (Charnoz et al. 2011). Put together, these works make a consistent scenario for the formation of the rings and satellite system of Saturn, which is well supported by observations and theory.

Uranus and Neptune The satellite systems of the two ice giants follow the same mass-distance distribution. In the case of Neptune, the fit is remarkable, except for Despina, whose mass seems to be three times too high. In the case of Uranus, the distribution is more scattered around the theoretical curve ; however, this system is chaotic, and still evolving : collisions are supposed to occur quite frequently, and ringlets may accrete into new satellites. This can be seen as the pyramidal regime still at play.

Therefore, we claim that Uranus and Neptune used to have massive rings, that have spread, given birth to the regular satellites observed today, and vanished.

Earth-Moon : In the case of the Earth's Moon forming disc, we would have $D \approx 0.02$, $\tau_{\mathrm{disc}} \approx 125$, and then $q_c \approx D$. This means that only one satellite forms in the continuous regime, that absorbs all the mass in the tidal disc. Obviously, the Earth has only one large satellite...

The structure of the Moon forming disc is much more complex than a peaceful, cold tidal disc. However, once it has condensed, our calculations show that this disc preferentially forms one single Moon than many.

The case of a possible Charon forming disc around Pluto is similar.

5   Conclusion

The formation of satellites through the spreading of a tidal disc beyond the Roche radius appears to be a generic process in the Solar System, from the terrestrial to the giant planets. Whether only one large satellite forms or a series of small satellites appears, is only controlled by the rapidity of the spreading (τ_{disc}), itself determined by the mass of the disc. But the physical process at play is intrinsically the same.

This work has been published recently, and all the details of the calculations can be found in the supplementary material of Crida & Charnoz (2012, `http://arxiv.org/abs/1301.3808`).

A video presenting this mechanism in a pedagogical way for the general audience is available in English at `http://vimeo.com/55613414` or in French (`http://vimeo.com/64293772`).

Realisation : Frédéric Durillon — animea , under creative commons licence.

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