SF2A 2013
L. Cambrésy, F. Martins, E. Nuss and A. Palacios (eds)

ORBIT DETERMINATION METHODS IN VIEW OF THE PODET PROJECT

F. Deleflie[1], D. Coulot[2], R. Decosta[1], A. Vienne[1] and P. Richard[3]

Abstract. We present an orbit determination method based on genetic algorithms. Contrary to usual estimation methods mainly based on least-squares methods, these algorithms do not require any *a priori* knowledge of the initial state vector to be estimated. These algorithms can be applied when a new satellite is launched or for uncatalogued objects

We show in this paper preliminary results obtained from an SLR satellite, for which tracking data acquired by the ILRS network enable to build accurate orbital arcs at a few centimeter level, which can be used as a reference orbit. The method is carried out in several steps: *(i)* an analytical propagation of the equations of motion, *(ii)* an estimation kernel based on genetic algorithms, which follows the usual steps of such approaches: initialization and evolution of a selected population, so as to determine the best parameters. Each parameter to be estimated, namely each initial keplerian element, has to be searched among an interval that is preliminary chosen.

Keywords: orbit determination, analytical propagation, genetic algorithms, space debris

1 Introduction

The main goal of this study consists in finding a way, to compute an orbit from tracking data, when no *a priori* information on the trajectory is available at all. In that case, classical methods such as least-squares can not be used any more (since in that case the function to be minimized can not be linearized in the neighborhood of the *a priori* values of the parameters). Moreover, the usual methods may suffer from many drawbacks which can frequently make them be unappropriated; they are often merely based on keplerian modelings, and can hence not be applied over time scales longer than a couple of hours.

On the contrary, even if other kinds of difficulties have to be managed, methods based on genetic algorithms are supposed to be valid for all dynamical configurations, since the algorithm itself is independent from the orbit propagator used to compute the cost function. With an efficient dynamical modeling, they can be used over different periods of time, from a couple of minutes (for Too-Short Arcs, TSA) up to a couple of days or weeks.

We provide here the preliminary results we obtain with measurement on the Lageos-1 satellite, tracked by the International Laser Ranging Service, the ILRS (Pearlman, 2002) network.

2 Orbital modeling

To keep a reasonable computation time for the propagation step, we use an analytical approach to get orbital element time series. The modeling is supposed to be valid whatever the values of the eccentricity or the inclination, small (even equal to zero), or large: the model is written in a set of equinoctial elements (Deleflie, 2013), namely: a, $\xi = \Omega + \omega + M$, $e\cos(\Omega + \omega)$, $e\sin(\Omega + \omega)$, $i_x = \sin\frac{i}{2}\cos\Omega$, $i_y = \sin\frac{i}{2}\sin\Omega$, where a, e, i, Ω, ω, M stand for the classical keplerian elements.

The whole analytical modeling is governed by the set of mean initial conditions, whereas it is the corresponding osculating initial conditions $E(t_0)$ that are adjusted by the genetic algorithm, and that can be directly compared to the reference orbits, if they are expressed in the same reference frame.

[1] IMCCE, Observatoire de Paris, UPMC, univ. Lille1, CNRS. F-75014 Paris, France
[2] IGN/LAREG, Université Paris Diderot, Sorbonne Paris Cité, Paris, France
[3] Centre National d'Etudes Spatiales, Toulouse, France

3 Multi-Objective Genetic Algorithm (MOGA) used

3.1 Description

The Multi-Objective Genetic Algorithm (MOGA) used here is the ϵ-MOEA (Deb et al., 2003). Between two successive iterations, some vectors of initial conditions are replaced by other ones and the best ones are archived. The evolution through the iterations of the set of initial conditions is governed by mutations (random small changes in vectors of possible initial conditions) and by crossover (mix two vectors of possible initial conditions) ; the probabilities of changes are usual parameters of the approach. At the end of the iteration procedure, a set of solutions is supplied (Coello Coello et al., 2007).

As in many orbit determination algorithms, an evaluation is made up of several steps:

- ϵ-MOEA provides a vector of initial conditions, randomly chosen among a large set of possible (osculating) initial values ;
- These initial conditions are used to propagate an analytical orbit over the period when tracking or astrometric data are available ;
- The analytical orbit, as time series of orbital elements, is used to compute predicted measurements, that can be compared to the available data sets, at the same epochs of the observations ;
- These predicted measurements are compared to the true data ;
- The values of the cost functions are computed.

3.2 Parameterization

The chromosomes represent the initial state of the solution: each one is then made up of six initial orbital keplerian elements, and each chromosome determines a unique orbit. Let us note that the MOGA runs on a population of constant size, but also with an archive of variable size, to keep knowledge of the best solutions found so far. At each iteration, two children may be generated (depending upon the probability of crossover) from two parents randomly taken from both the population and the archive, and mutations may also occur (depending on the probability of mutation).

For this first attempt, we chose a population of 400 chromosomes, with fixed intervals for each keplerian initial element:

- semi–major axis $a \in [12200; 15600]$ km
- eccentricity $e \in [0; 0.1]$
- inclination $i \in [0; 180°[$
- angles Ω, ω $M \in [0; 360°[$

Let us note that to reduce computation time, the search for the initial eccentricity has been reduced to an interval with a wideness of 0.1, and the search of the initial semi major axis to intervals large of a few thousands of kilometers.

The crossover probability has been set up to $p_C = 0.9$, and the mutation probability to $p_m = 1/6 \simeq 0.16667$. The stop condition is the total number of iterations which is here set up to 500 000 (this corresponds to a total CPU time of the order of 30 hours).

3.3 Genetic algorithm handled with range data: the SLR satellite Lageos-1

We use a set of Satellite Laser Ranging (SLR) data acquired on Lageos-1 by 29 stations of the tracking network of the ILRS, and including 2034 measurements, over eight days in April 2012 (from MJD 56 024 to 56 031 included). We consider here two objectives to be optimized: we search not only for the best vector of initial conditions (the RMS of differences between predicted measurements and the real data has to be minimized); but also, we search for an optimal sub-network of SLR stations so that the number of SLR stations involved in the computation has to be maximized.

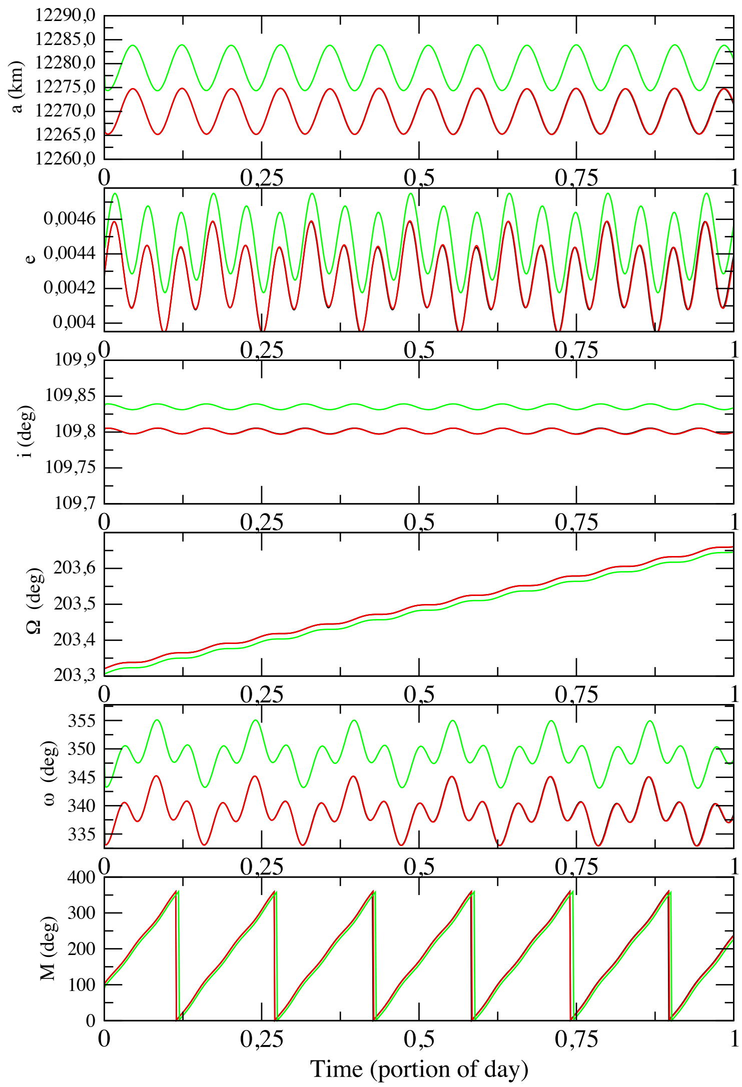

Fig. 1. Time series of each keplerian orbital elements $(a, e, i, \Omega, \omega, M)$, computed for the Lageos-1 satellite, in the inertial frame, where the equations of motion are propagated. X-axis: time (portion of day of 7th April 2012). Black curves correspond to the reference orbit (computed with the Gins CNES s/w) ; the red ones correspond to the trajectories obtained with our analytical modeling and with the reference initial conditions provided by Gins ; the green ones correspond to the trajectories obtained with our approach (with initial conditions estimated with the genetic algorithm).

The RMS of differences between the tracking data and their theoretical equivalent computed with the reference orbit (obtained from post-fit adjustment of a numerical integration with the CNES Gins software) is at the level of 2.15 cm. The adjusted initial conditions, seen as reference ones, are the following:

- $a_0^{\mathrm{ref}} = 12270.009$ km

- $e_0^{\mathrm{ref}} = 0.004261$
- $I_0^{\mathrm{ref}} = 109.801°$
- $\Omega_0^{\mathrm{ref}} = 203.323°$
- $(\omega_0 + M_0)^{\mathrm{ref}} = 76.616°$

The best results found by the MOGA are the following:

- $a_0 = 12274.840$ km: $\Delta a = 4.831$km
- $e_0 = 0.004408$: $\Delta e = 0.000147$
- $I_0 = 109.839°$: $\Delta I = 0.038°$
- $\Omega_0 = 203.306°$:$\Delta\Omega = 0.017°$
- $\omega_0 + M_0 = 76.538°$: $\Delta(\omega + M) = 0.078°$

Figure 1 shows the time series of each orbital element, over the given period of time: semi–major axis, eccentricity, inclination, longitude of the node, perigee, mean anomaly. The black curves correspond to the reference orbit obtained with the Gins s/w. The red curves correspond to the analytical propagation of the model presented above, but with the reference initial conditions. It appears that at this scale, the analytical simplified model that we use is suitable to handle the dynamics of the trajectory (at the level of the results). Finally, the green curves show the best trajectory that is found by the MOGA. The differences between the two sets of initial conditions (Δa, Δe, Δi, $\Delta\Omega$, $\Delta\omega$, ΔM above) are quite small with respect to each value to be determined ($3\ 10^{-4}$ on a, 3% on e, $3\ 10^{-4}$ on i, relatively), but induce as well a difference that is not compensated during the propagation. We should keep in mind that *(i)* this level of precision is good enough to use these initial values as *a priori* values in a least-squares adjustment ; *(ii)* genetic algorithms have anyway a good capability over the global scale, but locally they can be less accurate than other approaches ; *(iii)* better results are likely to be obtained when changing the parameterization of the MOGA (in a forthcoming paper); *(iv)* we compare sets of elements expressed in two different reference frames (inertial frame and true of date).

4 Conclusions

In this paper, we combined a MOGA and an analytical satellite motion theory to roughly adjust an orbit on tracking data, without any *a priori* knowledge of the values of the initial conditions to be retrieved. We tested the method on two kinds of data. Some further developments will be enhanced in the future *(i)* the analytical modeling will be improved by adding some significant terms in the model *(ii)* the parameterization of the MOGA will be refined, with a reduced set of chromosomes, and by empirically decreasing the mutation probability throughout the iterations; we will implement as well a better stopping condition to reduce the CPU required time. We will then test the capabilities of the algorithm in downgraded conditions (data sparse in time, very few number of data).

References

Coello Coello, C.A., G.B. Lomont, D.A. Von Veldhuizen, Evolutionary Algorithms for Solving Multi-Objective Problems, Second Edition, Springer, 2007.

Deb, K., M. Mohan, S. Mishra, A Fast Multi-objective Evolutionary Algorithm for Finding Well-Spread Pareto-Optimal Solutions, KanGAL Report Number 2003002, 2003.

Pearlman, M.R., Degnan, J.J.,, and Bosworth, J.M., The International Laser Ranging Service, Adv. In Sp. R., Vol 30, No 2, pp 135-143, 2002.

Deleflie, F., Finalized Analytical Satellite Theory, CNES Internal Report, 2013.