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MARKOV CHAIN MONTE-CARLO ORBIT COMPUTATION FOR BINARY ASTEROIDS

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Abstract. We present a novel method of orbit computation for resolved binary asteroids. The method combines the Thiele, Innes, van den Bos method with a Markov chain Monte Carlo technique (MCMC). The classical Thiele-van den Bos method has been commonly used in multiple applications before, including orbits of binary stars and asteroids; conversely this novel method can be used for the analysis of binary stars, and of other gravitationally bound binaries. The method requires a minimum of three observations (observing times and relative positions - Cartesian or polar) made at the same tangent plane - or close enough for enabling a first approximation. Further, the use of the MCMC technique for statistical inversion yields the whole bundle of possible orbits, including the one that is most probable.

In this new method, we make use of the Metropolis-Hastings algorithm to sample the parameters of the Thiele-van den Bos method, that is the orbital period (or equivalently the double areal constant) together with three randomly selected observations from the same tangent plane. The observations are sampled within their observational errors (with an assumed distribution) and the orbital period is the only parameter that has to be tuned during the sampling procedure. We run multiple chains to ensure that the parameter phase space is well sampled and that the solutions have converged. After the sampling is completed we perform convergence diagnostics. The main advantage of the novel approach is that the orbital period does not need to be known in advance and the entire region of possible orbital solutions is sampled resulting in a maximum likelihood solution and the confidence regions. We have tested the new method on several known binary asteroids and conclude a good agreement with the results obtained with other methods. The new method has been implemented into the Gaia DPAC data reduction pipeline and can be used to confirm the binary nature of a suspected system, and for deriving the mass of the binary system.

Keywords: binary, asteroid, MCMC

1 Introduction

The study of the motion of the objects in our Solar system has had one of the most profond impacts on our understanding of the physical world. Indeed it was Kepler's discovery in the 17$^{\rm th}$ century that the planets moved on ellipses with the Sun at one of the focii that produced a major philosophical revolution. Newton's formulation of the "universal" laws of gravitation were very general, in particular all masses were subject to the same acceleration regardless of their composition. This implies that the motions of multiple stellar systems could be viewed in the same manner as multiple asteroids systems.

For the simplest sytem consisting of just two gravitatioanlly bound masses celestial mechanics gives an exact analytical solution, namely a conic section depending on the initial conditions of the involved bodies. For two gravitationaly bound astronomical objects, whatever their nature, there are 2 distinct parts to the recovery of their orbits. First the determination of the apparent orbit, second the determination of the projected true orbit onto the plane perpendicular to the line of sight.

Many methods have been developed for the task of binary orbit determination especially for visual binary stars. We will not dwell on any except for the application of the Thielie–Ines–van der Bos method (TIvB) which has been applied very succesfully in the determination of the orbits of binary stars. This method requires a

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three observations only. When applying the method to binary asteroids, care must be taken in choosing the three points because the proximity of the object to earth introduces a parallax that can not be neglected.

T. N. Thiele observed that the usual orbital elements used to describe the objects in the solar system were not very appropriate for binary stars. In fact, the true orbit can be described with the help of the following four elements, the period, P, the time of passage through the periastron, T, the eccentricity, e, and the semimajor axis, a. The three custumary supplementary elements needed to define the projected orbit; inclination, i, longitude of the ascending node, Ω, and the argument of the perihelion, ω have no intrinsic meaning. T. N. Thiele (1883) proposed a more appropriate set of apparent elements. He considered the points P and R for which the eccentric anomaly is $0°$ and $90°$ and their projections on the apparent orbit P' and R'. The set of elements retained by Thiele is T, the mean motion, e, a the length to the point P', A, the position angle of P', b the length to the point R' and the position angle of R'. Then Starting from Kepler's equation, he proceeded to describe a method for obtaining the true orbit of the companion star in a binary system relying on three observational points and the constant of areal velocity (Thiele 1883). In 1926 I.T. A. Innes reformulated Thiele's method in rectangular coordinates instead of polar coordinates, or $A_1 = a\cos A$, $A_2 = a\sin A$, $B_1 = b\cos B$, $B_2 = b\sin B$, which simplified the equations. W.H. van den Bos modified the method for use with the Innes constants. The resulting equations can be found in their 1926 paper (van de Bos 1926). Orbits are usually described by the Innes constants A, B, F, and G. The TIvB method uses only three point out of a possibly larger set. Once a first orbit is determined, it can be differentially corrected to fit the remaining points. Or, another three points can be drawn from the set of observations and a new orbit solution obtained. Again corrections to the orbit can be sought and the initial orbit improved. This procedutre can be continued until a satisfactory orbit is found.

Now, the method will provide one orbit only for any three points chosen. The determined orbit may or possibly may not pass through the other observed data points. Besides, preliminary orbit determination is highly nonlinear and regardless of the success of the inversion method an estimation of the errors on the determined elements is not obtained. Furthermore, for asteroids, as pointed out above, the inherent parallax in the data does not allow the choice of the three points required by the method to be arbitrary. The three points must lie close to or in the same tangent plane, ie the plane perpendicular to the line of sight. So from the determined orbit it still must be established how to pass from one tanget plane to another. The simple procedure applicable to distant binary stars needs a little modification, this is done in e.g. Hestroffer (2005). We propose here to use a Markov Chain Monte Carlo approach which has the power of determing a bundle of orbits, not just one, which will contain the most likely orbit of the binaries and has the added benifit of providing the errors on the determined elements. To do so the parameter phase space is sampled according to the measurement errors as described in (Virtanen 2001).

Knowledge of the asteroid pair's orbit has fundamental implications for our knowledge of these bodies and their evolution. Indeed it is the best method to derive the mass of asteroids with good precision (excluding results from space probes that are limited to only a handfull of bodies), and moreover knowledge of the true orbit in addition to the apparent one can bring insights on formation scenarii.

2 Gaia's contribution

A number of binary asteroids will be observed in the course of the Gaia mission. Particularly the resolved binaries (when the two components are well separated and can be detected as two different bodies) or astrometric binaries (that is when the system appears as a single object but the binarity can be detected by a wobble of the photocenter around the center of mass) are of high interest (Hestroffer 2010, 2002; Tanga 2012). For those objects improved orbits, masses and possibly densities can be computed. It has been shown by Tanga (2012) that asteroid binaries both with large (≈ 100 km) and small (≤ 10 km) primary bodies will be detectable by Gaia, given favourable geometric conditions. For example, binary asteroids with 0.3 arc second seperation or larger and small magnitude difference (< 2 mag) between the two components will be detected by Gaia's sky mapper (see Fig. 1); in other cases both components will still be resolved but on a single window transmitted to ground.

3 Thiele-Innes-van den Bos

The Thiele-Innes-van den Bos method directly provides, if it exists, a Keperian orbit expressed for example in terms of the 7 elements $\mathbf{q} = (a, e, i, \Omega, \omega, P, M = (m_1 + m_2))$, repectively the semimajor axis, the eccentricity, the inclination, the arguement of the ascending node, the argument of perihelion, the orbital period, and the

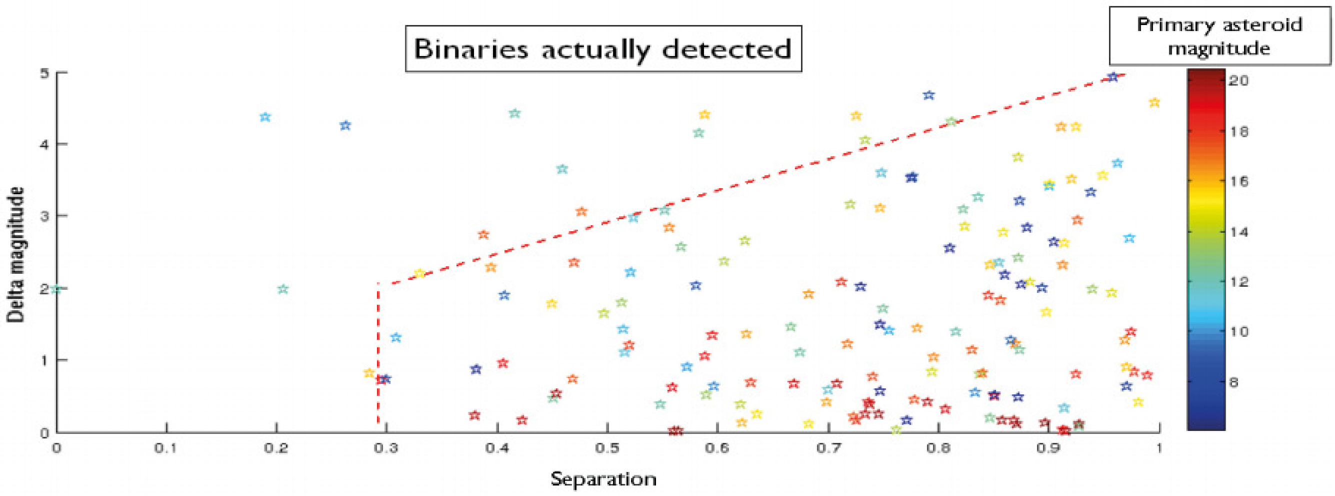

Fig. 1. Binary asteroids detectable by Gaia directly through the sky mapper with a 2 by 2 pixel binning. Angular separation vs. magnitude difference (Tanga 2012).

total mass of the system which is the sum of the mass of the primary m_1 and the mass of the companion m_2. As afore mentioned three observational points are necessary at a minimum in the same tangent plane, (x_i, y_i), where x and y are cartesian positions of the companion relative to the primary at times t_i for $i = 1, 2, 3$ and the orbital period or the areal constant. The compution procedes from the fundamaental equation relating the eccentric anomaly E anf the double areal constant c to the orbital period P see e.g Hestroffer (2010):

$$
\begin{aligned}
t_k - t_l - \Delta_{lk}/c &= [E_k - E_l - \sin(E_k - E_l)]/n \\
\Delta_{pk} &= x_l y_k - X_k y_l \\
n &= \pi/P
\end{aligned}
$$

This trascendental set of equations can be solved iteratively, given P, for the areal constant c from which the orbital elements can be computed.

4 MCMC with Thiele-Innes-van den Bos

The inverse problem of computing orbits of binary asteroids simialrly to the one of individual objects (Muinonen 1993; Virtanen 2001) can be taken to be probablistic in nature and therefore treated using the Bayesian approach and appropriate statistical methods. In particular we have developed a novel method combining the well known Thiele–Innes method (Aitken 1918; Argyle 2004) and numerical methods. Principally we use the Metropolis-Hastings (Chib 1995) algorithm to sample the parameters of the Thiele-Innes method. From the whole set of N observations $i = (x_i, y_i)$ made at observation times t_i (where $i = 1 \ldots N$) we randomly select three observations from the same tangent plane and a staring orbital period P. We refer to those seven parameters as the sampling parameters denote by $S = (x_1, x_2, x_3, y_1, y_2, y_3, P)$. From the three selected observations and the period we compute a starting orbital elements $\mathbf{q} = (a, e, i, \Omega, \omega, P, M = (m1+m2))$ using the Thiele- Innes method. Once a starting orbit has been computed we start Markov chain Monte–Carlo (MCMC) sampling of the parameters S by adding random deviates to the selected three observations and the orbital period. In practice at each iteration t in a chain a new candidate sampling parameters are proposed using so-called proposal densities. In particular we make use of Gaussian proposal densities for all the seven sampling parameters. For the cartesian coordinates we use proposal densities that are centered around the last accepted sampling parameters in the chain and the size of the proposal density is proportional to the observational noise: $x_i^{(c)} \propto N(x_i^{(t-1)}, \sigma x_i)$, $y_i^{(c)} \propto N(y_i^{(t-1)}, \sigma y_i)$ (where i = 1; 2; 3) for x and y coordinates respectively. For the orbital period we use a normal distribution that is centered around the last accepted period $P^{(c)} \propto N(P^{(t1)}, \sigma_P)$. The size of the proposal density for the orbital period σ_P is the only parameter to be tuned in the method (one could also consider other distributions), but in general in most of the cases, an educated guess of the size of that parameter can be made. Once a new candidate sampling parameters have been generated $S^{(c)} = (x_1^{(c)}, x_2^{(c)}, x_3^{(c)}, y_1^{(c)}, y_2^{(c)}, x_3^{(c)}, P^{(c)})$ the M-H acceptance coefficient a_r is used to accept or reject the sample parameters. The acceptance coefficient a_r is expressed as:

$$a_r = \frac{p_p(\mathbf{q}^c)}{p_p(\mathbf{q}^{t-1})} \frac{|J^{t-1}|}{|J^c|}$$

where J^c and J^{t-1} are the Jacobians from the sampling parameters for the candidate and the last accepted orbit respectively. $\mathbf{q}^c$ and $\mathbf{q}^{t-1}$ are the p.d.f.s. for the candidate and the last accepted orbit respectively. Next the candidate parameters are accepted or rejected based on the Metropolis-Hastings criteria:

$$\begin{aligned} &\text{if } a_r \geq 1, \quad \text{then} \quad \mathbf{q}_t = \mathbf{q}^c \\ &\text{if } a_r < 1, \quad \text{then} \quad \mathbf{q}_t = \begin{cases} \mathbf{q}^c \text{ with probability } a_r \\ \mathbf{q}_t = \mathbf{q}_{t-1} \text{ with probability } 1 - a_r \end{cases} \end{aligned}$$

In practice, if the new orbit produces a better fit to the full observational data set, it is always accepted. If it produces a worse fit, it is accepted with the probability equal to a_r. The sampling is repeated until a large enough number of orbits have been obtained. After the sampling is completed convergence diagnostics has to be performed to insure that the stationary distribution was reached and to test for the length of burn-in period (the time required for the chain to reach the stationary). The obtained distributions of the orbital parameters reflect the properties of orbital element uncertainties.

5 Preliminary results

To validate the method proposed here, we have computed orbital distributions for some well-constrained systems. In particular we selected the binary asteroids 1998 WW_{31} and 2000 QL_{251} as test cases since for these, the orbital elements have been determined by other methods already reported in the litterature.

In November 1998 WW_{31} was discovered by R. Millis and collaborators at Kitt Peak National Observatory but was reported as a single object in the Kuiper belt. The observations provided a short arc only. The asteroid was subsequently included in follow up studies with the Canada France Hawaii Telescope. Further observations along with reprocessed previous observations finally showed 1998 WW_{31} to be a binary system. Based on observations from the Hubble Space Telescope and the available ground based data, the secondary's orbital parameters relative to the primary were found to be the following, $P = 574 \pm 10$ days, $a = 22300 \pm 800$ km, $e = 0.817 \pm 0.05$ (Veillet 2002).

The primary of 2000 QL_{251} was discovered by M. W. Buie from Cerro Tololo Observatory, La Serena. Hubble Space Telescope observations allowed Noll, Stephens, Grundy, and Levison to detect its companion (Noll 2006). Grundy (2009) give the following elements $a = 5002 \pm 27$ km, $P = 56.451 \pm 0.025$ days, $e = 0.4871 \pm 0.0065$.

The two figures below present preliminary results for those two objects from the MCMC runs. These orbital elements agree with results obtained by others. We expect to improve on these when the implementation of the method is complete and more robust for various cases.

6 Conclusions

We have proposed a novel method for obtaining the orbits of resolved binary asteroids. This method is based on the work of Thiele–Innes–van Bos and Monte Carlo Markov Chain statistics. The method is still being implemented but has already been succesfully applied to a few selected binaries. The preliminary results are very promising and show strong convergence.

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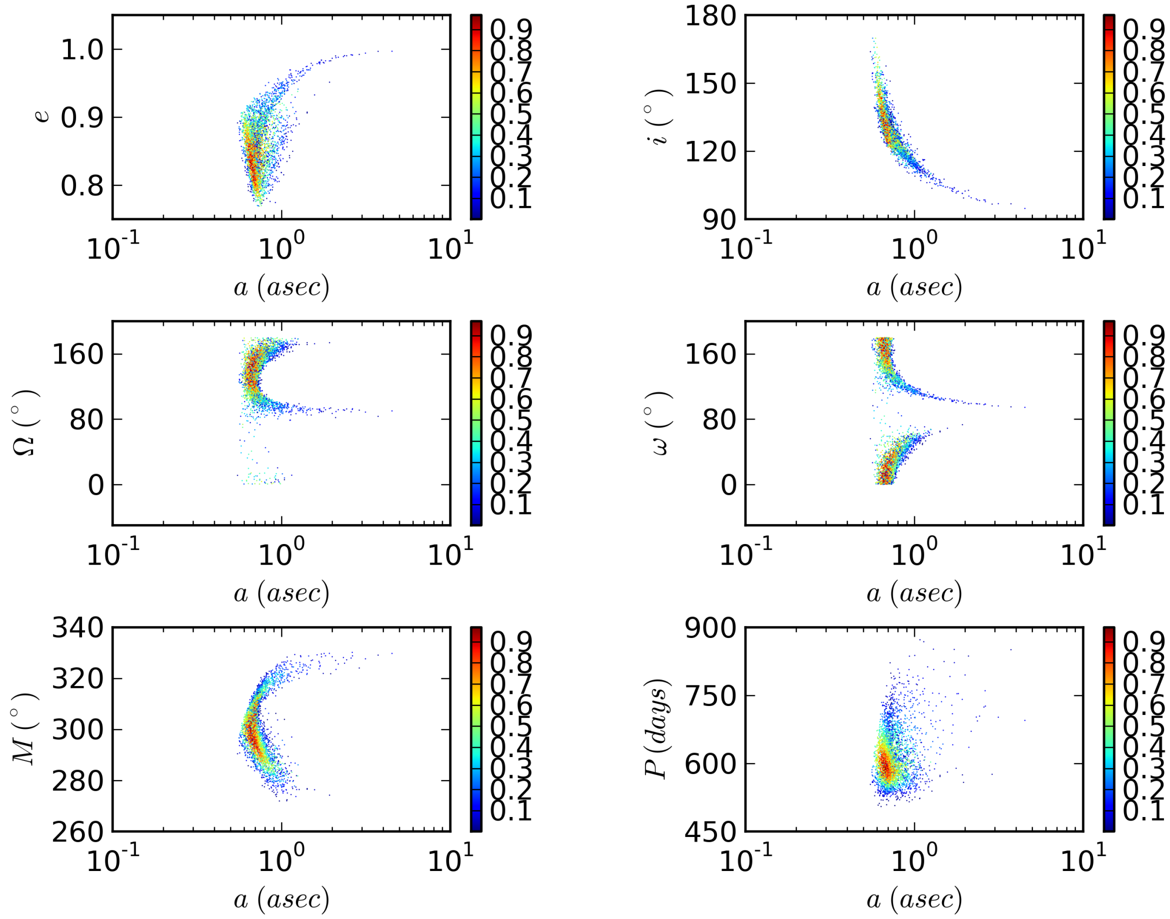

Fig. 2. Distribution of orbital elements for asteroid 1998 WW_{31}. Color corresponds to normalized probability density value.

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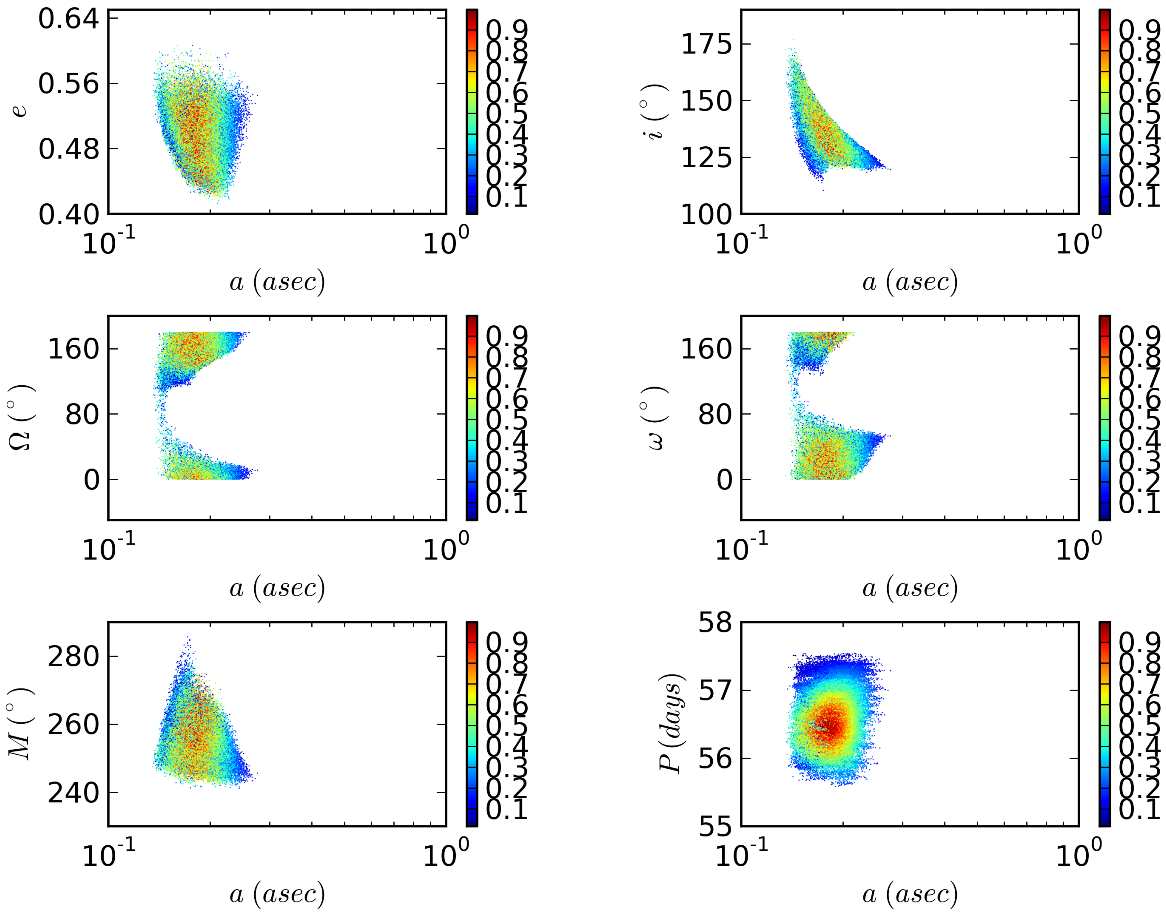

Fig. 3. Distribution of orbital elements for asteroid 2000 QL_{251}. Color corresponds to normalized probability density value.

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