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STOCHASTIC EXCITATION OF GRAVITO-INERTIAL WAVES IN ROTATING STARS

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Abstract. Stochastic gravity waves have been recently detected and characterised in stars thanks to asteroseismology and they may play an important role in the evolution of stellar angular momentum. Moreover, the observational study of the CoRoT hot Be star HD 51452 suggests a potential strong impact of rotation on stochastic excitation of gravito-inertial waves in rapidly rotating stars. In this work, we present our exploration of the action of rotation on stochastic wave excitation by turbulent convection in stellar interiors. Using a local model, we study stochastic excitation as a function of the waves' Rossby number and we demonstrate that the excitation presents two different regimes for super-inertial and sub-inertial frequencies. Consequences for rapidly rotating stars and the transport of angular momentum in their interiors are discussed.

Keywords: hydrodynamics – waves – turbulence – stars: rotation – stars: evolution

1 Introduction

Gravity waves propagate in the stably stratified radiative core of low-mass stars and the external radiative envelope of intermediate-mass and massive stars (e.g. Aerts et al. 2010). When they (or their signatures) are detected thanks to helioseismology and asteroseismology, they allow us to probe the stellar internal structure and differential rotation (e.g. Garcia et al. 2007; Bedding et al. 2011; Beck et al. 2012; Deheuvels et al. 2012; Mosser et al. 2012; Neiner et al. 2012b). Furthermore, these waves are able to transport and deposit a net amount of angular momentum because of their damping and corotation resonances (e.g. Talon & Charbonnel 2005; Alvan et al. 2013; Mathis et al. 2013a). If frequencies of excited gravity waves are around the inertial frequency 2Ω, where Ω is the angular velocity of the star, which is assumed to be uniform here, their propagation is strongly affected by the Coriolis acceleration and they become gravito-inertial waves (hereafter GIWs; e.g. Dintrans & Rieutord 2000; Mathis 2009; Ballot et al. 2010).

GIWs have been discovered thanks to the CoRoT satellite in the hot Be star HD 51452 (Neiner et al. 2012a). In this star, which rotates close to its critical angular velocity, GIWs are probably excited stochastically by turbulent convection in the core or/and in the subsurface convection zone, since the star is too hot to excite them with the κ-mechanism. Moreover, the detected GIWs with the largest amplitudes have frequencies in the sub-inertial domain (*i.e.* $\sigma < 2\Omega$), which are the most influenced by the Coriolis acceleration. Therefore, this discovery points out how necessary it is to understand the action of rotation on stochatic excitation of GIWs in stellar interiors, which has been poorly explored until now (Belkacem et al. 2009).

To reach this objective, in Mathis et al. (2013b) we chose to generalise the work by Goldreich & Kumar (1990) and Lecoanet & Quataert (2013) taking the action of the Coriolis acceleration into account. This work is summarized here. First, in Sect. 2, we present the local rotating set-up in which we describe the different regimes of the dynamics of GIWs. Next, in Sect. 3. their stochastic excitation by turbulent convective flows is studied and we identify and discuss the impact of rotation.

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2 Gravito-inertial waves in stellar interiors

2.1 The studied set-up and the Poincaré wave equation

In Mathis et al. (2013b), we chose to study a cartesian region, centered on a point M of a radiation-convection interface. We introduce Θ the angle between the local effective gravity $\vec{g}_{\rm eff}$, which is the sum of the self-gravity $\vec{g}$ and of the centrifugal acceleration $1/2\,\Omega^2\vec{\nabla}s^2$ (s being the distance from the rotation axis), and the rotation vector $\vec{\Omega}$ (see left panel of Fig. 1). Mx, My and Mz are the axes along the local azimuthal, latitudinal and vertical (*i.e.* along $\vec{g}_{\rm eff}$) directions respectively. This "f-plane" (Pedlosky 1982) is co-rotating with the stellar angular velocity $\vec{\Omega}$ and we introduce the two components of this vector along the vertical and latitudinal directions

$$f = 2\Omega\cos\Theta \quad \text{and} \quad \widetilde{f} = 2\Omega\sin\Theta. \tag{2.1}$$

Following Gerkema & Shrira (2005), we go beyond the so-called *traditional approximation* and both components are taken into account to ensure a complete and correct treatment of GIWs' dynamics, both in the radiative and in the convective regions, and of their coupling with inertial waves (see also Gerkema et al. 2008).

To study GIWs' dynamics, we write the linearised equations of motion of the stellar stratified fluid on the non-traditional f-plane assuming the Boussinesq and the Cowling approximations (Gerkema & Shrira 2005; Cowling 1941). We introduce the GIWs' velocity field $\vec{u} = (u, v, w)$, where u, v and w are the components in the local azimuthal, latitudinal and vertical directions respectively. Eliminating the horizontal components of the velocity, the pressure and the buoyancy, the equation for the vertical velocity is obtained

$$\partial_{t,t}\left[\nabla^2 w\right] + 4\left(\vec{\Omega}\cdot\vec{\nabla}\right)^2 w + N^2\nabla_H^2 w = 0, \tag{2.2}$$

where ∇_H^2 is the horizontal Laplacian and $N^2(z) = -\dfrac{\overline{g}_{\rm eff}}{\overline{\rho}}\dfrac{{\rm d}\overline{\rho}}{{\rm d}z}$ is the Brunt-Väisälä frequency, $\overline{\rho}$ being the hydrostatic density profile. We then consider a monochromatic GIW with a frequency σ that propagates in the direction $(\cos\alpha, \sin\alpha)$ in the (Mxy) plane (see left panel of Fig. 1). Introducing the reduced horizontal coordinate $\chi = x\cos\alpha + y\sin\alpha$ (with $\partial_x = \cos\alpha\,\partial_\chi$ and $\partial_y = \sin\alpha\,\partial_\chi$) and substituting $w(\vec{r}, t) = W(\vec{r})\exp[i\sigma\,t]$, we get the Poincaré equation for GIWs:

$$\left[N^2(z) - \sigma^2 + f_s^2\right]\partial_{\chi,\chi}W + 2ff_s\,\partial_{\chi,z}W + \left[f^2 - \sigma^2\right]\partial_{z,z}W = 0, \tag{2.3}$$

where $f_s = \widetilde{f}\sin\alpha$. We note that GIWs' dynamics is a non-separable bidimensionnal problem because of the mixed derivative $\partial_{\chi,z}$ (see also Dintrans & Rieutord 2000). However, it is possible in our set-up to introduce the transformation

$$W = \Psi(z)\exp\left[ik_\perp\left(\chi + \tilde{\delta}z\right)\right] \quad \text{where} \quad \tilde{\delta} = -\frac{ff_s}{f^2 - \sigma^2}, \tag{2.4}$$

that leads to

$$\frac{{\rm d}^2}{{\rm d}z^2}\Psi + k_V^2(z)\,\Psi = 0, \quad \text{where} \quad k_V^2(z) = k_\perp^2\left[\frac{N^2(z) - \sigma^2}{\sigma^2 - f^2} + \left(\frac{\sigma\,f_s}{\sigma^2 - f^2}\right)^2\right], \tag{2.5}$$

$k_\perp$ being the wave number in the χ direction. This enables us to use the method of vertical modes as in the non-rotating case and the modal functions W_j that verify boundary conditions constitute an orthogonal and complete basis (Gerkema & Shrira 2005). This leads to the allowed frequency spectrum $\sigma_- < \sigma < \sigma_+$ (*i.e.* $k_V^2 > 0$), where

$$\sigma_\pm = \frac{1}{\sqrt{2}}\sqrt{\left[N^2 + f^2 + f_s^2\right] \pm \sqrt{\left[N^2 + f^2 + f_s^2\right]^2 - (2fN)^2}}. \tag{2.6}$$

Moreover, in convective regions, which excite GIWs, the local vertical number (Eq. (2.5)) becomes

$$k_{\rm CZ}^2 \equiv k_\perp^2\frac{\sigma^2}{\left(\sigma^2 - f^2\right)^2}\left[\left(f^2 + f_s^2\right) - \sigma^2\right] = k_\perp^2\frac{\left[\left[\widetilde{R}_{\rm o}(\Theta, \alpha)\right]^{-2} - 1\right]}{\left(1 - R_{\rm o}^{-2}\cos^2\Theta\right)^2}, \tag{2.7}$$

where we defined a local wave Rossby number $\widetilde{R}_{\rm o} = R_{\rm o}\left[\cos^2\Theta + \sin^2\alpha\sin^2\Theta\right]^{-1/2}$ expressed as a function of the wave's Rossby number $R_{\rm o} = \sigma/2\Omega$. We identify for $\alpha = \pi/2$ the allowed frequency domain for inertial waves

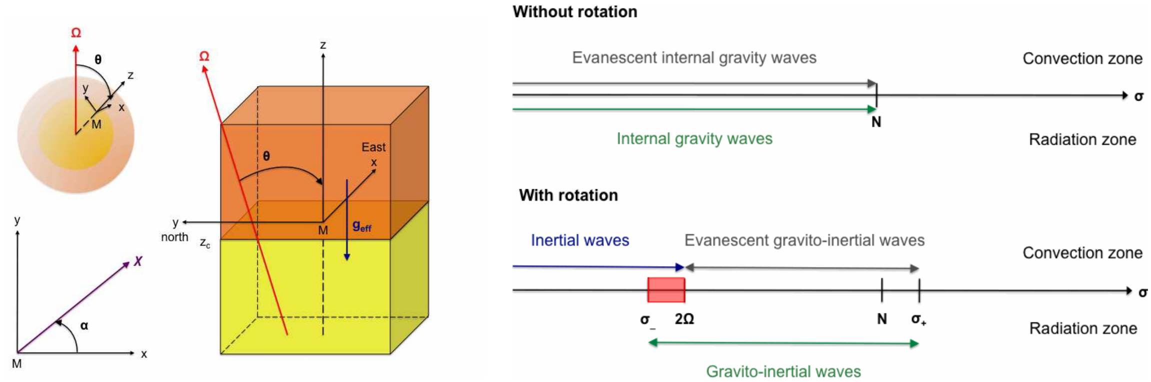

Fig. 1. Left: local studied "f-plane" reference frame. The radiative and convective regions are respectively in yellow and orange. This case corresponds to a low-mass star. The vertical structure of this set-up can be inverted to treat the case of intermediate-mass and massive stars. Right: low frequency spectrum of waves in a non-rotating ($\Omega = 0$, top) and in a rotating star (bottom). For each case, waves in the convective and radiative zones are indicated at the top and bottom, respectively. (Taken from Mathis et al. (2013b).)

$0 < \sigma < 2\Omega$ (i.e. $R_{\rm o} < 1$). On one hand, in the local super-inertial regime ($\widetilde{R}_{\rm o} > 1$), there are turning points $(z_{t;i})$ in the radiative zones for which $k_V^2\left(z_{t;i}\right) = 0$, *i.e.:*

$$N^2 = \sigma^2 \frac{\left[\sigma^2 - \left(f^2 + f_s^2\right)\right]}{\left(\sigma^2 - f^2\right)} \tag{2.8}$$

and GIWs are evanescent in convective zones. On the other hand, in the local sub-inertial domain ($\widetilde{R}_{\rm o} < 1$), GIWs propagate in the whole radiative zone and become propagative inertial waves in convective zones. These two behaviours (see right panel of Fig. 1) have been put forward by Dintrans & Rieutord (2000).

2.2 *Propagation in rotating stellar convective zones*

2.2.1 Super-inertial regime $\left(\widetilde{R}_{\rm o} > 1\right)$

In the case of the super-inertial regime, there are two turning points ($z_{t;1}$, $z_{t;2}$ with $z_{t;1} < z_{t;2}$) in the radiative zone for which $k_V^2 = 0$ (see Eq. 2.8). In the convective region, GIWs are evanescent and we obtain the complete bidimensionnal expression for the vertical velocity using asymptotic methods, Eq.(2.7) and Eq. (2.4):

$$W\left(z > z_{\rm c}, \chi\right) = \frac{A}{\sqrt{k_\perp p}} \exp\left[-k_\perp p\left(R_{\rm o}\right)\left(z - z_{\rm c}\right) - \Delta\left(z_{t;2}, z_{\rm c}\right)\right] \cos\left[k_\perp\left(\chi + \tilde{\delta}\left(R_{\rm o}\right) z\right)\right], \tag{2.9}$$

where we define

$$p = \frac{\sqrt{|1 - \widetilde{R}_{\rm o}^{-2}|}}{|1 - R_{\rm o}^{-2}\cos^2\Theta|},\ \Delta\left(z_1, z_2\right) = \int_{z_1}^{z_2} |k_V| {\rm d}z' \ \text{ and } \ \tilde{\delta} = -\frac{R_{\rm o}^{-2}\cos\Theta\sin\Theta\sin\alpha}{R_{\rm o}^{-2}\cos^2\Theta - 1} \tag{2.10}$$

and we introduce A the amplitude of the wave. In this regime, p is the decay rate of the velocity in the convective zone. The evolution of p and $\tilde{\delta}$ as a function of $R_{\rm o}$ and Θ is plotted in Fig. 2. In the super-inertial regime, p increases with $R_{\rm o}$ (with $p \to 1$ for $R_{\rm o} \to \infty$, *i.e.* the non-rotating case). This means that the decay rate of the wave function in the convective zone decreases when the rotation rate increases until $R_{\rm o} = 1$. Moreover, $\tilde{\delta}$ tends to vanish at large $R_{\rm o}$, while it increases with the rotation rate until $R_{\rm o} = 1$. Indeed, as soon as rotation becomes important, the problem is non-separable in z and χ (see Eq. 2.3), while it is completely separable in the non-rotating case (i.e. $R_{\rm o} \to \infty$). Accordingly, modifications of GIW velocity fields occur. This shows the important impact of the Coriolis acceleration, which will modify the couplings of GIWs with convective flows.

2.2.2 Sub-inertial regime $\left(\widetilde{R}_{\rm o} < 1\right)$

In the case of the sub-inertial regime, waves are propagative in the whole domain without any turning point (*i.e.* $k_V^2 > 0$). In the convective region, we get the complete bidimensionnal expression for the vertical velocity

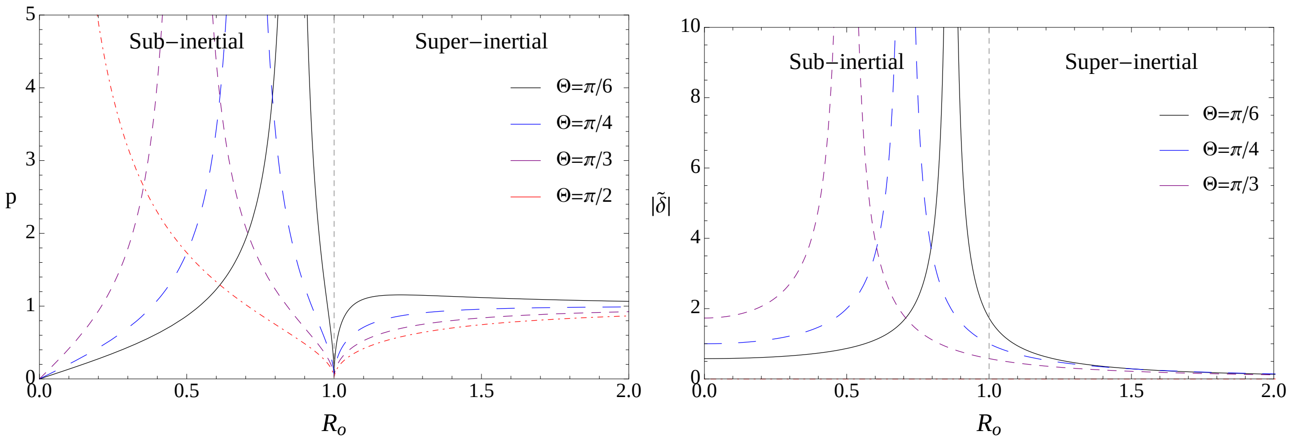

Fig. 2. Evolution of p (left) and $\tilde{\delta}$ (right) as a function of $R_{\rm o}$ for $\alpha = \pi/2$ and different inclination angle: $\Theta = \pi/6$ (black solid line), $\pi/4$ (blue long-dashed line), $\pi/3$ (purple dashed line) and $\pi/2$ (red dot-dashed line). $\tilde{\delta}$ vanishes for $\Theta = \pi/2$ (see Eq. (2.10)). The border between the sub-inertial and the super-inertial regimes is given by the grey dashed line. (Taken from Mathis et al. (2013b).)

for inertial waves using Eq. (2.7) and Eq. (2.4):

$$W\left(z > z_{\rm c}, \chi\right) = \frac{A}{\sqrt{k_\perp p}} \sin\left[k_\perp p\left(R_{\rm o}\right)\left(z - z_{\rm c}\right) + \Delta\left(z_b, z_c\right)\right] \cos\left[k_\perp\left(\chi + \tilde{\delta}\left(R_{\rm o}\right) z\right)\right], \tag{2.11}$$

where p, Δ and δ have been defined in Eq. (2.10). In this regime, p is the vertical wave number of the propagative inertial wave in the convective zone. The variations of p and $\tilde{\delta}$ as a function of $R_{\rm o}$ in the sub-inertial regime, shown in Fig. 2, imply that the velocity field becomes more and more oscillatory as the rotation rate increases until $R_{\rm o} = \cos(\Theta)$ where the denominators of p and $\tilde{\delta}$ vanish. $\Theta_{\rm c} = \cos^{-1}\left(R_{\rm o}\right)$ corresponds to the critical colatitude above which GIWs are trapped in the radiation zone in the sub-inertial regime (e.g. Mathis & de Brye 2012). This behaviour is very different from the one obtained in the super-inertial regime and will modify GIWs stochastic excitation by convective flows.

3 Stochastic excitation of gravito-inertial wave by turbulent convection

To obtain the amplitude (A) of GIWs excited by turbulent convective zones, we follow Goldreich & Kumar (1990) and Lecoanet & Quataert (2013). The equation for the vertical component of the wave velocity (w) is given for the radiative zone in Eq. (2.2) and for the convective region by

$$\partial_{t,t}\left[\nabla^2 w\right] + 4\left(\vec{\Omega}\cdot\vec{\nabla}\right)^2 w = \partial_t\left[\partial_z \vec{\nabla}\cdot\vec{F} - \nabla^2 F_z\right] = \partial_t \mathcal{S}, \tag{3.1}$$

where we have introduced the convective Reynolds stresses $\vec{F} = \vec{\nabla}\cdot\left(\vec{u}_{\rm c}\vec{u}_{\rm c}\right)$ and defined the corresponding source function $\mathcal{S}$ (Belkacem et al. 2009, see also Samadi & Goupil 2001), which vary as a function of the rotation rate. Using the orthogonality of GIW functions in our cartesian set-up, which has been demonstrated by Gerkema & Shrira (2005), we can derive a formal expression for the amplitude of GIWs depending on their super- or sub-inertial behaviour. Following Goldreich & Kumar (1990) and Lecoanet & Quataert (2013), we obtain

$$\begin{aligned} A_{\rm sup} = & \frac{1}{\mathcal{N}}\frac{1}{2i\sigma\sqrt{\mathcal{A}}}\frac{1}{\sqrt{k_\perp p}}\int_{-\infty}^{t}{\rm d}\tau\int_{\mathcal{V}}{\rm d}x\,{\rm d}y\,{\rm d}\zeta\left\{\mathcal{S}\left(x,y,\zeta,\tau;\Omega\right)\exp\left[-i\,k_\perp\left[\left(x\cos\alpha + y\sin\alpha\right) + \tilde{\delta}\left(R_{\rm o}\right)\zeta\right] - i\sigma\tau\right]\right. \\ & \left.\times\exp\left[-k_\perp p\left(R_{\rm o}\right)\left(\zeta - z_{\rm c}\right) - \Delta\left(z_{t;2}, z_c\right)\right]\right\} \end{aligned} \tag{3.2}$$

and

$$\begin{aligned} A_{\rm sub} = & \frac{1}{\mathcal{N}}\frac{1}{2i\sigma\sqrt{\mathcal{A}}}\frac{1}{\sqrt{k_\perp p}}\int_{-\infty}^{t}{\rm d}\tau\int_{\mathcal{V}}{\rm d}x\,{\rm d}y\,{\rm d}\zeta\left\{\mathcal{S}\left(x,y,\zeta,\tau;\Omega\right)\exp\left[-i\,k_\perp\left[\left(x\cos\alpha + y\sin\alpha\right) + \tilde{\delta}\left(R_{\rm o}\right)\zeta\right] - i\sigma\tau\right]\right. \\ & \left.\times\sin\left[k_\perp p\left(R_{\rm o}\right)\left(\zeta - z_{\rm c}\right) + \Delta\left(z_b, z_c\right)\right]\right\}, \end{aligned} \tag{3.3}$$

where $\mathcal{V}$ and $\mathcal{A}$ are the volume and the horizontal cross-section of the box respectively and $\mathcal{N}$ is the orthogonality constant. This formal expressions demonstrate that rotation, through the Coriolis acceleration, modifies the stochastic excitation of GIWs, the control parameter being the wave's Rossby number $R_{\rm o} = \sigma/2\Omega$ (via p and δ, see Eq. (2.10)). On one hand, in the super-inertial regime ($\sigma > 2\Omega$, *i.e.* $R_{\rm o} > 1$), the evanescent behaviour of GIWs above (below) the external (internal) turning point becomes increasingly weaker as the rotation speed grows until $R_{\rm o} = 1$. The coupling between super-inertial GIWs and given turbulent convective flows is then amplified as described in Eq. (3.2). On the other hand, in the sub-inertial regime ($\sigma < 2\Omega$, *i.e.* $R_{\rm o} < 1$), GIWs become propagative inertial waves in the convection zone and the coupling between GIWs and the given turbulent source continue to be sustained (Eq. 3.3).

This result obtained in Mathis et al. (2013b) is of great interest for asteroseismic studies of rotating stars since GIW amplitudes are thus expected to be stronger in rapidly rotating stars. Moreover, the related transport of angular momentum, which until now was believed to become less efficient because of GIWs equatorial trapping in the sub-inertial regime (Mathis et al. 2008; Mathis 2009; Mathis & de Brye 2012), may be sustained thanks to the stronger stochastic excitation by turbulent convective flows. Finally, our prediction is coherent with results obtained in recent realistic numerical simulations of stochastic excitation of GIWs in stellar interiors (Rogers et al. 2012, and L. Alvan, private communication).

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