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PLANCK RESULTS - NON GAUSSIANITY CONSTRAINED WITH MINKOWSKI FUNCTIONALS

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Abstract. We use a morphological tool, the Minkowski Functionals, to constrain non Gaussianity in the Cosmic Microwave Background (CMB) temperature anisotropies measured by the *Planck* satellite. Two types of non Gaussianities are studied, primordial non Gaussianity with the estimate of the 'local' bispectrum parameter $f_{\rm NL}^{\rm local}$, and non Gaussianity induced by cosmic strings, related to the string tension $G\mu/c^2$. We use a Bayesian method to constrain these specific models, on Wiener filtered maps and we account for secondaries and foregrounds using the linear properties of the functionals. We find $f_{\rm NL}^{\rm local} = 4.2 \pm 20.5$ (1σ) consistent with primordial Gaussianity and with bi-spectrum based estimator results. We show no evidence for cosmic strings, with $G\mu/c^2 < 6.0 \times 10^{-7}$ (95% C.L.). Minkowski Functionals prove to be a nice complementary tool to bispectrum estimators in the case of primordial non Gaussianity and an efficient estimator for cosmic strings induced non Gaussianity.

Keywords: Cosmology: Observations, Cosmic Microwave Background, Gaussianity, Methods: Statistical, Numerical

1 Introduction

Non Gaussianity (NG), i.e. any deviation of the CMB anisotropies from the purely Gaussian distribution, is a much powerful probe to constrain fundamental theories and the origin of structure in the universe as it gives access to extreme high energy physics. In particular it could bring new information on the inflation paradigm, and discriminate among simple or more complex mechanisms for the generation of the cosmological perturbations in the early universe (see review in Bartolo et al. 2004). NG could also probe topological defects like cosmic strings, the results of symmetry-breaking phase transitions in the early universe, forming at the end of inflation. They are predicted in supersymmetric and grand unified theories and in higher-dimensional theories for the origin of our universe, such as brane inflation.

Primordial non Gaussianity is easily parametrised when considering the non-linear 'local' coupling parameters $f_{\rm NL}^{\rm local}$, $g_{\rm NL}$, ... which appear in the perturbative development of the *primordial* curvature perturbation around the Gaussian prediction (Okamoto & Hu 2002),

$$\Phi(x) = \phi_{\rm L}(x) + f_{\rm NL}^{\rm local}(\phi_{\rm L}^2(x) - \langle\phi_{\rm L}^2(x)\rangle) + g_{\rm NL}\,\phi_{\rm L}^3(x) + ..., \tag{1.1}$$

where $\phi_{\rm L}(x)$ is the linear Gaussian part of the Bardeen curvature. We will focus in this paper on constraining the first parameter, $f_{\rm NL}^{\rm local}$. We do not study here the other numerous models to characterize primordial NG, which can be based on theoretical predictions, or which define different triangle shapes for the harmonic transform of the 3-point correlation function – the bi-spectrum.

Estimators of $f_{\rm NL}^{\rm local}$ have been developed using the measure of the bi-spectrum (in particular Komatsu et al. 2005), they have been shown to be optimal estimators as they saturate the Cramér-Rao inequality for a weak non Gaussianity (Babich 2005). On *Planck* data, bi-spectrum based estimators obtained the constraint $f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$ (68 % C.L.) (Planck Collaboration XXIV 2013).

However, alternative statistics to the bi-spectrum exist, that can serve as checks and diagnoses of the results obtained from the bi-spectrum. Indeed, various and numerous systematics and contaminants can bias the results such as inhomogeneous noise, beam asymmetries and foreground residuals, as well as secondary

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anisotropies. Most important foreground residuals are induced by our galaxy, but point sources (mostly unresolved radio galaxies), or Cosmic Infrared Background (CIB – mostly unresolved starburst galaxies forming a diffuse background) could as well bias results. Eventually, secondary anisotropies such as integrated Sachs-Wolfe effect (ISW), Sunyaev-Zeldovich effect (SZ) and weak gravitational lensing are all contributing to non Gaussianity. Hence, using different probes is a necessary consistency test, as they will be differently affected by those systematics.

Here we focus on a complete set of morphological tools, the Minkowski Functionals (hereafter MFs) introduced in cosmology by Mecke et al. (1994) and widely used since then as probes of primordial non-Gaussianities (Schmalzing & Buchert 1997; Schmalzing & Gorski 1998; Hikage et al. 2006). For CMB studies, MFs provide a nice complement to the bi-spectrum: they are defined in real space, which makes a robust implementation for MFs in practice much easier than for the bi-spectrum, defined in harmonic space, especially for the mask treatment; MFs are sensitive to the full hierarchy of higher-order correlations, instead of third order only, and can provide additional information on all the non-linear coupling parameters beyond $f_{\rm NL}^{\rm local}$, and on other NG models. However, MFs only probe angular averages of higher order statistics, leaving out the angular dependences, at variance with the bi-spectrum, and can only poorly disentangle between various models of NG.

In this paper, we also studied another source of NG, the cosmic strings. They are defined by an enormous energy per unit length μ and generate a number of observable effects, including gravitational lensing and a background of gravitational waves. Here we shall focus on their non-Gaussian signature, dominantly a second order NG (at the level of the 4-point correlation function, or its equivalent in the harmonic domain, the tri-spectrum). We usually constrain the existence of cosmic strings with the magnitude of the gravitational perturbations they produce on sub-horizon scales: $\dfrac{G\mu}{c^2} = \left(\dfrac{\eta}{m_{\rm Pl}}\right)^2$, where η is the energy scale of the string-forming phase transition and $m_{\rm Pl} \equiv \sqrt{\hbar c/G}$ is the Planck mass. Previous NG constraints on this parameter were obtained on *WMAP* data, for example with an analytic trispectrum estimator $G\mu/c^2 < 1.1 \times 10^{-6}$ (95% C.L.) (Fergusson et al. 2010).

This paper is organised as follows. In section 2, we define MFs and the Bayesian method we use to constrain NG models. In section 3, we describe the *Planck* data and its characteristics, while our results are detailed in section 4. Section 5 summarises and discusses the results and future improvements.

2 Minkowski functionals: definitions and method

2.1 Minkowski Functionals on the sphere

Minkowski Functionals of a random field are defined using specific partitions, the excursion sets. For a field $f(x)$ of zero average and variance σ_0^2 on the two-dimensional sphere $\mathbb{S}^2$, an overdense excursion set is defined as

$$\Sigma \equiv \{x \in \mathbb{S}^2 |\, f(x) > \nu\sigma_0\}. \tag{2.1}$$

Its boundary is

$$\partial\Sigma \equiv \{x \in \mathbb{S}^2 |\, f(x) = \nu\sigma_0\}. \tag{2.2}$$

The three MFs on the sphere are then

$$\text{Area}: V_0(\nu) = \frac{1}{4\pi}\int_\Sigma \mathrm{d}\Omega, \qquad \text{Perimeter}: V_1(\nu) = \frac{1}{4\pi}\frac{1}{4}\int_{\partial\Sigma} \mathrm{d}l, \qquad \text{Genus}: V_2(\nu) = \frac{1}{4\pi}\frac{1}{2\pi}\int_{\partial\Sigma} \kappa\, \mathrm{d}l, \tag{2.3}$$

where dΩ and dl are respectively elements of solid angles (surface) and of angle (distance), κ is the geodesic curvature. Note that the Genus can also be expressed as the number of components* in the excursion minus the number of holes in the excursion. We construct a fourth functional – $N_{\rm cluster}$, $V_3(\nu)$, being for $\nu > 0$, the number of components in the excursion. Symmetrically, for $\nu < 0$, it is the number of underdense components (or the number of components in the excursion $\{x \in \mathbb{S}^2 |\, f(x) < \nu\sigma_0\}$).

Analytical expressions for a Gaussian or weakly non Gaussian random field can be derived in terms of ν (see e.g. Vanmarcke 1983; Matsubara 2010). These analytical expressions represent useful descriptions of the MFs as they can be factorized as $V_k(\nu) = A_k v_k(\nu)$, $v_k(\nu)$ being a function of the threshold and A_k a function of only the shape and amplitude of the C_ℓ (see Ducout et al. (2012) for precise definitions). We will use here the normalized – with respect to the C_ℓ – MFs (v_k) to focus on the non Gaussian deviations.

*A component is a connected subset of the excursion.

2.2 Bayesian estimation of specific non Gaussian models

While MFs are sensitive to any departure from the Gaussian statistics, and are used as a generic test for non Gaussianity in *Planck* (Planck Collaboration XXIII 2013), they can also be used to constrain specific models of NG, using a Bayesian approach. The method is described in Ducout et al. (2012) for the estimate of $f_{\rm NL}^{\rm local}$, but it can quite similarly be applied for the string tension measure $G\mu/c^2$. The aim is to compare the normalised functionals v_k measurements on the data to "theoretical" predictions of these functionals for different models and amplitudes of non Gaussianity in order to obtain their posterior distribution.

We measure v_k over an ensemble of $n_{\rm th} = 26$ threshold values ν_i between $\pm\nu_{\rm max} = \pm 3.5$, defining a vector $\hat{v}_k \equiv \{\hat{v}_k(\nu_1), \cdots, \hat{v}_k(\nu_{n_{\rm th}})\}$ for each functional. We combine the 4 functionals into one vector of $4\times n_{\rm th}$ elements, $\hat{y} = \{\hat{v}_i, \hat{v}_j, \cdots\}$.

For a given NG model parametrised by an amplitude f (hence here, $f_{\rm NL}^{\rm local}$ or $G\mu/c^2$) the Bayes formula is

$$P(f|\,\hat{y}) = \frac{P(\hat{y}\,|f)P(f)}{\int P(\hat{y}\,|f)P(f){\rm d}f}. \tag{2.4}$$

We shall take a flat prior for the parameter f, with $P(f)$ being a constant over a reasonable range of values for f, mostly determined from previous experiments ($f_{\rm NL}^{\rm local}$ between -100 and +100 and $G\mu/c^2$ between 0 and 1e-6). The evidence $\int P(\hat{y}\,|f)P(f){\rm d}f$ is being considered as a normalisation factor.

The likelihood $P(\hat{y}\,|f)$ is a multivariate Gaussian, this allows us to define a simple χ^2 test for f and the posterior becomes

$$P(f|\,\hat{y}) \propto \exp\left[-\frac{\chi^2(\hat{y},f)}{2}\right] \qquad \text{with} \qquad \chi^2(\hat{y},f) \equiv \left[\hat{y} - \bar{y}(f)\right]^T C^{-1}\left[\hat{y} - \bar{y}(f)\right] \tag{2.5}$$

where $\bar{y}(f)$ is the model under test.

This model is constructed with direct measurements on realistic simulations of the data, integrating beam effects, inhomogeneous noise, component separation weighting and masks. For $f_{\rm NL}^{\rm local}$, the NG simulations are described in Elsner & Wandelt (2009), 1000 simulations were available for the Gaussian and non Gaussian parts of the CMB, based on *WMAP7*[†] best-fit power spectrum (Komatsu et al. 2011). For the string tension, only 2 string maps were available (Ringeval & Bouchet 2012), the simulations hence consisted mostly in the variation of the *Gaussian*[‡] part of the CMB. So $\bar{y}(f) \equiv \langle\hat{y}(f)\rangle$ is the mean of y measured over m_f simulated maps with a given level of non-Gaussianity f.

Since the considered level of non-Gaussianity is weak, one can compute the covariance matrix C in a Monte-Carlo fashion relying on 10^4 *Gaussian* simulated maps of the CMB (to which we add, when relevant, beam effects, noise, masks).

With those properties, the posterior probability distribution function $P(f_{\rm NL}^{\rm local}|\,\hat{y})$ is expected to be very close to a Gaussian. Indeed, in the weakly non Gaussian regime, $\bar{y}_i(f_{\rm NL}^{\rm local})$ is, at first order, a linear function of $f_{\rm NL}^{\rm local}$. For the string tension posterior however, this is not the case, the posterior has a far more complex shape, as $\bar{y}_i(G\mu/c^2)$ is a more complicated function of $G\mu/c^2$.

3 Planck data, systematics, foregrounds, secondaries and simulations

The *Planck*[§] mission (Planck Collaboration I 2013), has been designed to measure the Cosmic Microwave Background temperature anisotropies with unprecedented sensitivity and resolution over the whole sky. Sensitivity is not limited by noise but by the capacity to remove foregrounds, in particular the Galactic signal from the CMB one. In this paper, we report results obtained on *Planck* nominal data, acquired between 12 August 2009 and 27 November 2010 (Planck Collaboration XXIV 2013; Planck Collaboration XXV 2013).

The pixelisation scheme adopted for *Planck* processed maps is HEALPix[¶] (Górski et al. 2005). Maps were used at $N_{\rm side} = 1024$ for primordial NG constraints and at $N_{\rm side} = 2048$ for cosmic strings, restraining $\ell_{\rm max}$ to 2000 in both cases. We applied a Wiener filter to the map ($W_{\rm M}$).

[†] The bias due to the difference between *Planck* and *WMAP* power spectra is accounted for, at a level of $\Delta f_{\rm NL}^{\rm local} \sim 3$.
[‡] The lensed – hence non Gaussian – CMB was varied in fact.
[§] `http://www.esa.int/Planck`
[¶] Available at http://healpix.jpl.nasa.gov

Table 1. $f_{\rm NL}^{\rm local}$ estimates obtained with MFs on *Planck* `SMICA` map. Foreground and secondaries effects are evaluated in terms of $f_{\rm NL}^{\rm local}$.

	$f_{\rm NL}^{\rm local}$	Source	Corresponding $\Delta f_{\rm NL}^{\rm local}$
Lensed raw map	19.1 ± 19.3		–
Lensing subtracted	8.5 ± 20.5	Lensing	+10.6
Lensing+**PS subtracted**	7.7 ± 20.3	Point sources	+0.8
Lensing+**CIB subtracted**	7.5 ± 20.5	CIB	+1.0
Lensing+**SZ subtracted**	6.0 ± 20.4	SZ	+2.5
All subtracted	4.2 ± 20.5	All	+14.9

Several component separation methods (Leach et al. 2008) have been used to remove the Galaxy from cosmological data, we report here the results obtained with `SMICA`, the results obtained on different methods being globally consistent. Galactic residuals and radio point sources were taken into account by the use of the U73 mask ($f_{\rm sky} = 73\%$ of the sky is unmasked). The smallest point sources holes were inpainted.

Additionally, we estimated the average impact of some residual foregrounds and secondaries (FG) using the linear properties of MFs (Ducout et al. 2012) and foregrounds models processed through the *Planck* simulation pipeline (FFP6 simulations, see Planck Collaboration ES (2013)). Point sources (PS), CIB and SZ clusters[‖] signals can be introduced as a simple additive bias $\Delta\bar{y}^{\rm PS,\cdots}$ on MF curves, measured on simulations, following:

$$\hat{y} = \hat{y}^{\rm FGsubtracted} + \Delta\bar{y}^{\rm PS} + \Delta\bar{y}^{\rm CIB} + \Delta\bar{y}^{\rm SZ}. \tag{3.1}$$

Finally, the lensing by the large scale structures of the Universe was accounted for, as it has an important NG contribution. This lensing contribution was based on *Planck* measurement of the lensing, using simulations (FFP6 simulations). In the case of $f_{\rm NL}^{\rm local}$, we just evaluated it as another additive bias in our data $\hat{y}$, verifying that MFs are linear with the lensing at first order and that primordial non-Gaussianity and lensing contributions are additive. In the case of the string constraints, it was easier to directly include the lensing contribution in the model $\bar{y}(G\mu/c^2)$.

4 Results

4.1 Primordial NG - $f_{\rm NL}^{\rm local}$

Results are summarized in Table 4.1 and Minkowski functionals curves are showed on Fig. 1, without including the lensing subtraction ("Lensed curves"). The constraints obtained are consistent with bispectrum-based estimators results, even if we do not include FG contributions, taking into account a larger error bar. Moreover, results are quite robust against Galactic residuals: constraints obtained on other component separation methods (`NILC` and `SEVEM`), with different sky coverages, fairly agreed within a maximum $\Delta f_{\rm NL}^{\rm local} = 1$.

This consistency between different estimators, different component separation methods, masks, assess for the robustness of these results.

4.2 Cosmic strings tension - $G\mu/c^2$

Due to the non linearity of MFs with $G\mu/c^2$ and the small number of string simulations, the posterior distribution is quite complex and noisy (see Fig. 2), we then evaluated the posterior at $n_{\rm NL} = 51$ values of $G\mu/c^2$, between 0 and 10×10^{-7}, to obtain the estimate $\widehat{G\mu/c^2}$. This estimation is stable and have been validated in realistic conditions with the *Planck* String Challenges described in Planck Collaboration XXV (2013). We eventually integrated the posterior to report credible intervals. Results are summarized in Table 2, for raw data (lensing subtracted) and foreground subtracted data (PS, CIB and SZ subtracted).

[‖]The SZ signal does not include the SZ×lensing NG contribution.

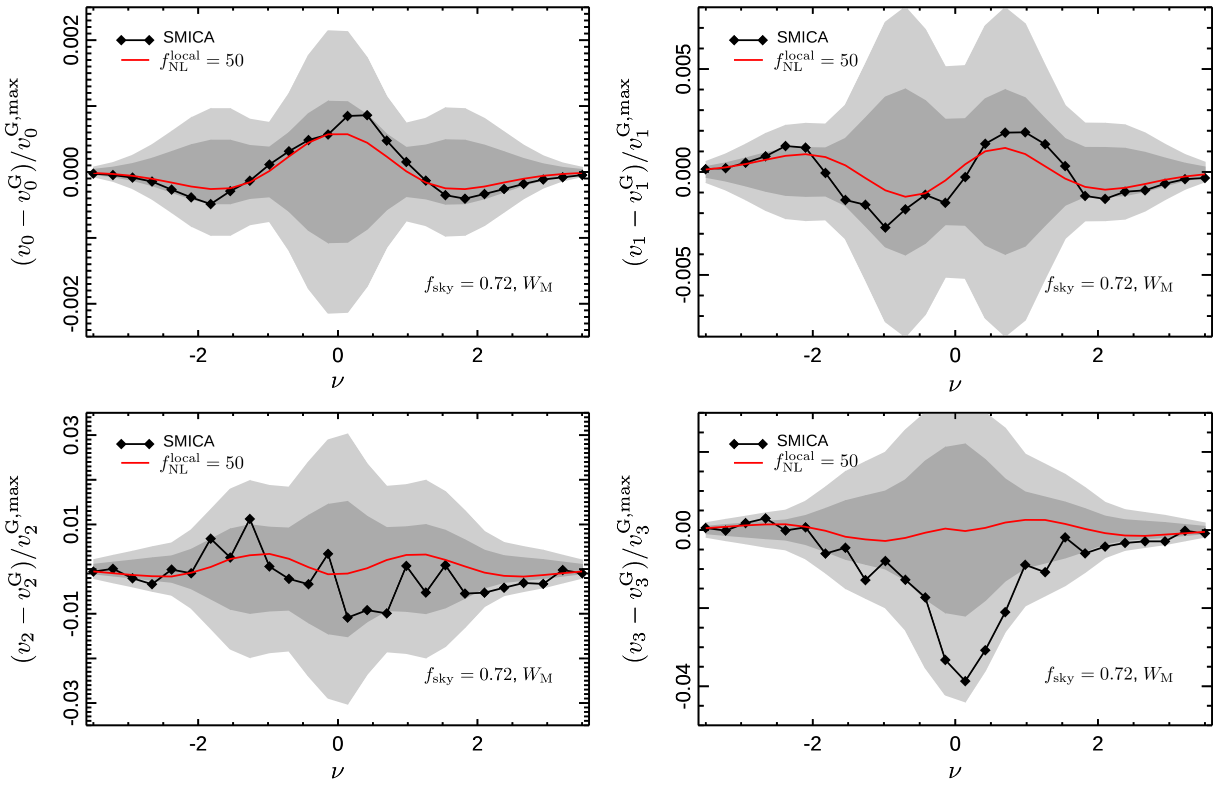

Fig. 1. The 4 Minkowski functionals curves measured on `SMICA` map. The curves are the difference of each normalized MF measured on the data to the averaged measure made on Gaussian *Planck* realistic simulations (not lensed). The difference curves are normalized by the maximum of the Gaussian curve. To compare the curves to the presence of primordial NG, the average difference curves for non Gaussian simulations with $f_{\rm NL}^{\rm local} = 50$ is also represented.

Table 2. MFs constraints obtained on $G\mu/c^2$, at the 95% C.I. The "Raw map" result includes only the lensing contribution to the data, while the "Foreground subtracted map" includes the lensing, Poissonian point sources, CIB and SZ clusters contributions.

	Raw map (lensing accounted for)	With FG correction
$G\mu/c^2$	$< 6.8 \times 10^{-7}$	$< 6.0 \times 10^{-7}$

5 Conclusions and perspectives

In this paper we have reported constraints on primordial NG $f_{\rm NL}^{\rm local}$ parameter and cosmic strings tension $G\mu/c^2$, obtained with Minkowski Functionals on *Planck* nominal data. Many biases have been considered, in particular foreground residuals and secondaries, as they can be easily handled with MFs.

For local primordial NG, MFs find $f_{\rm NL}^{\rm local} = 4.2 \pm 20.5$ (1σ), this confirms the results obtained with the optimal estimators (bispectrum) and assess for its robustness. Moreover using optimal filtering (enhancing information from the gradient of the map) and full mission data, the error bar should decrease by a factor of two.

For the string tension, MFs find $G\mu/c^2 < 6.0 \times 10^{-7}$, the most stringent constraint from NG estimators applied on *Planck* data at this moment. However, one important caveat is the small number of string simulations used to calibrate the estimator. The estimator being mostly sensitive to low-redshift strings (infinite strings, with redshifts between 0 and 30), it is here limited by cosmic variance. As low-redshift strings simulations are faster to produce than complete simulations, the robustness of this result should be assessed very soon.

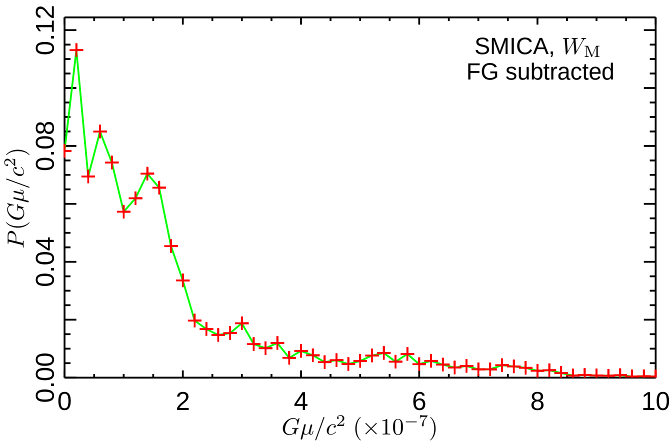

Fig. 2. Posterior distribution of the parameter $G\mu/c^2$ obtained with Minkowski functionals, on `SMICA` map, with a correction for foregrounds.

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