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THE LARGE-SCALE MAGNETIC FIELD IN PROTOPLANETARY DISCS

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Abstract. We study the time evolution of the large scale poloidal magnetic field in a global model of thin accretion disc, with a particular attention to protoplanetary discs. With transport coefficients usually used (coming from a crude vertical averaging), the magnetic field strength does not increase radially inward, leading to weak magnetic fields in the inner part of the disc. We show that with more precise transport coefficients taking into account the vertical structure of the disc as obtained by Guilet & Ogilvie (2012, 2013), the magnetic field can significantly increase radially inwards. The magnetic field profile adjusts in such a way as to reach an equilibrium value of the plasma β parameter (the ratio of the midplane thermal pressure to the magnetic pressure) in the inner parts of the disc. This value of β depends strongly on the disc aspect ratio and the magnetic Prandtl number, and we find that it corresponds to a plasma β parameter in the range $10^4 - 10^7$ for protoplanetary discs. Such a magnetic field strength is expected to have a significant impact on the dynamics of the protoplanetary disc by increasing the strength of MHD turbulence and allowing the launch of an outflow.

Keywords: protoplanetary discs, accretion discs, magnetic fields, MHD, ISM: jets and outflows.

1 Introduction

The presence of a magnetic field in protoplanetary discs has important consequences for driving turbulence and for launching an outflow. Both processes lead to angular momentum transport and are thus crucial to explaining the accretion rate onto the central star. The presence of a strong magnetic field in the inner parts of protoplanetary discs is furthermore strongly suggested by observations of powerful collimated jets from T-Tauri star systems (Ferreira et al. 2006). Such a magnetic field can also affect the evolution of planets embedded in the disc by changing their migration rate and direction (Terquem 2003; Guilet et al. 2013). The strength of the magnetic field remains, however, very uncertain both from an observational point of view (since direct measurements of its strength are still lacking) and from a theoretical perspective.

The evolution of a large scale magnetic field in an accretion disc has been a long standing theoretical problem since Lubow et al. (1994) found that the diffusion of this field was much more efficient than its advection in a geometrically thin disc. As a consequence, they found an almost uniform magnetic field strength presumably leading to a negligibly weak magnetic field in the inner parts of the disc, which is problematic for magnetically-driven jet models. In Guilet & Ogilvie (2012, 2013) we showed that the vertical structure of the disc could lead to a faster advection and a slower diffusion of the magnetic field than previously thought. This suggests that the magnetic field could increase radially inwards and could therefore potentially solve this problem. Determining the radial structure and intensity of the magnetic field requires however the study of a global model of an accretion disc. Such a global model is the subject of this article and will be described in more detail in a forthcoming paper (Guilet & Ogilvie in preparation).

The physical and numerical setup used are described in Section 2. In Section 3, we present the magnetic field configuration obtained when using the transport coefficients of Guilet & Ogilvie (2012). We discuss the consequences for protoplanetary discs and conclude in Section 4.

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2 Formalism

We study the evolution of the large-scale poloidal magnetic field around a thin accretion disc (with a small aspect ratio $h \ll 1$). We assume that no outflow is launched from the disc such that outside the disc the density vanishes and the magnetic field configuration is current free. We also neglect the currents present in the star such that the magnetic field originates from currents inside the disc and from currents at infinity that create a uniform background magnetic field. We compute the time evolution of the magnetic flux function ψ as defined by Ogilvie (1997), proportional to the magnetic flux threading the disc, and related to the magnetic field by:

$$B_r = -\frac{1}{r}\frac{\partial\psi}{\partial z}, \tag{2.1}$$

$$B_z = \frac{1}{r}\frac{\partial\psi}{\partial r}, \tag{2.2}$$

The time evolution of the magnetic flux function in the equatorial plane of the disc is determined by transport processes in the accretion disc through:

$$\frac{\partial\psi}{\partial t} + v_\psi \frac{\partial\psi}{\partial r} = 0, \tag{2.3}$$

where v_ψ is the transport velocity of the magnetic flux. This transport rate of magnetic flux has a component due to the advection by the accreting matter, and a second component due to the diffusion of the magnetic field by an effective turbulent resistivity. In the simple vertical average derived by Lubow et al. (1994) the transport rates are independent of the magnetic field strength. The advection velocity is assumed to be the same as that of mass (density averaged), which for a steady state disc far from the boundary is

$$v_{\rm adv} = -\frac{3}{2}\frac{\nu}{r}, \tag{2.4}$$

where ν is the effective turbulent viscosity. The diffusion velocity of the magnetic field is

$$v_{\rm diff} = \frac{\eta}{H}\frac{B_{rs}}{B_z}, \tag{2.5}$$

where η is the effective turbulent resistivity, H is the vertical scale height of the disc, and B_{rs} is the radial magnetic field at the surface of the disc. B_{rs} is proportional to the height integrated current inside the disc and can be computed from the magnetic flux distribution by inverting an integral equation (see Lubow et al. 1994 and Ogilvie 1997).

More precise transport rates have been computed by Guilet & Ogilvie (2012) in an analysis that takes into account the vertical structure of the accretion disc. They found that the vertical structure has a large effect on the transport rate as compared to aforementioned vertical averages used in the past by Lubow et al. (1994), Heyvaerts et al. (1996) and Reynolds et al. (2006): for magnetic fields such that the magnetic pressure is smaller than the thermal pressure at the midplane of the disc, the advection is faster and the diffusion slower than these rough estimates. Indeed, the diffusion rate decreases with decreasing magnetic field strength because the magnetic field lines can bend over a larger height, while the advection velocity increases because of the faster radial velocity in the low density region away from the midplane. In the following, we use the transport rates obtained by Guilet & Ogilvie (2012).

In order to solve the time evolution of the magnetic flux distribution, we use a second order finite difference numerical algorithm (see Guilet & Ogilvie in preparation for more details). The radii are normalised by the radius of the inner edge of the disc, time by the Keplerian angular frequency at this inner edge and the surface density such that the mass accretion rate in the α disc model is equal to 1.

3 Equilibrium magnetic field profile

We compute the time evolution of the magnetic field profile from an initial condition where the magnetic field is uniform and equal to the background magnetic field. After a time shorter than the viscous time (and therefore shorter than the lifetime of protoplanetary discs), a stationary configuration is obtained, which is shown in

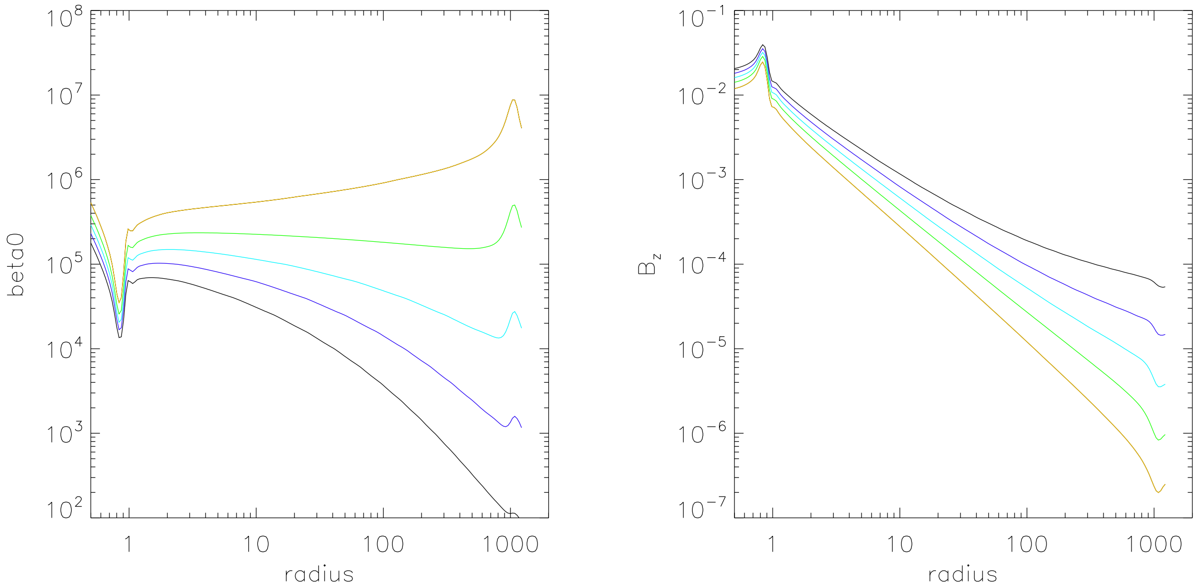

Fig. 1. Radial profiles of the plasma β parameter (ratio of midplane thermal pressure to the magnetic pressure, left panel) and the magnetic field strength (right panel) after a stationary state is reached. The disc is an α disc model with a uniform aspect ratio $h = 0.05$ and magnetic Prandtl number $\mathcal{P} \equiv \nu/\eta = 1$. The different colours show the results of simulations with different values of the background magnetic field strength, with β at the outer edge of the disc ranging from $\beta = 10^2$ (orange) to $\beta = 10^6$ (black).

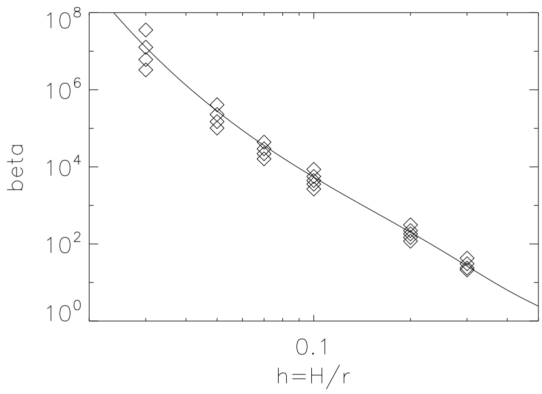

Fig. 2. Ratio of thermal pressure to magnetic pressure (β_0) near the inner edge of the disc as a function of the aspect ratio of the disc. The results of numerical simulations with several background magnetic field strengths are shown with diamond symbols. A semi-analytical prediction is shown with the full black line.

Figure 1. The different colours show results from simulations with different values of the background magnetic field strength corresponding to values of β in the range $10^2 - 10^6$ at the outer edge of the disc. The advection of magnetic flux by the accretion flow leads to a significant amplification of the magnetic field in the inner part of the disc, and the stationary magnetic field profiles increase radially inward. Interestingly, the evolution of the magnetic field favours a particular value of the ratio of thermal to magnetic pressure $\beta \sim 10^5$: if the background magnetic field is close to this strength at the outer edge of the disc, then the stationary magnetic field profile corresponds to a uniform thermal to magnetic pressure ratio (green line). If the background magnetic field is weaker (respectively stronger) than this at the outer edge, then the magnetic field increases more (respectively less) steeply inwards in order to reach this equilibrium value of β in the inner parts of the disc. This comes

from the fact that weak magnetic fields are advected faster and diffused at a slower rate than strong magnetic fields, and can therefore sustain a steeper radial profile of magnetic field strength.

This magnetic field strength is expected to depend mainly on the aspect ratio of the disc h and the magnetic Prandtl number $\mathcal{P} \equiv \nu/\eta$ through the parameter $\mathcal{P}h$ (Lubow et al. 1994; Guilet & Ogilvie 2012). In Figure 2, we show the dependence of the equilibrium value of β as a function of the aspect ratio of the disc with the expected value of the magnetic Prandtl for MHD turbulence of $\mathcal{P} = 1$. The equilibrium β is estimated by measuring β near the inner edge of the disc for different values of the background magnetic field (diamond symbols) and compared successfully to an analytical estimate using a self similar solution of the exterior magnetic field (see Guilet & Ogilvie in preparation for more details). Note that the equilibrium value of β depends steeply on the aspect ratio of the disc, and for parameters relevant to protoplanetary discs it lies in the range $10^4 - 10^7$.

4 Conclusion

We studied the global structure of the poloidal magnetic field in an accretion disc, with a particular attention to protoplanetary discs. In contrast to previous studies, we find that the transport coefficients taking into account the vertical structure obtained by Guilet & Ogilvie (2012) allow the magnetic field to significantly increase radially inwards. The magnetic field profile tends towards a stationary solution which favours a uniform equilibrium thermal to magnetic pressure ratio. This ratio depends on the aspect ratio of the disc and the magnetic Prandtl number of MHD turbulence and is typically in the range $\beta = 10^4 - 10^7$ for a protoplanetary disc, the large range coming from the steep dependence on the aspect ratio and the (somewhat uncertain) turbulent magnetic Prandtl number. Such a magnetic field strength is rather weak in the sense that the magnetic pressure remains significantly smaller than the thermal pressure at the disc midplane. It can however have profound consequences on the dynamics of protoplanetary discs. It could for example quench MRI turbulence and enable the launch of an outflow powerful enough to drive accretion at a rate compatible with observation (Bai & Stone 2013; Bai 2013). At larger radii where ambipolar diffusion is significant, it could by contrast foster the development of MRI turbulence (Simon et al. 2013). It is remarkable that the range of magnetic field strength found in our analysis coincides with the values needed in these studies in order to explain the observed mass accretion rates.

Note that we have not taken into account ambipolar diffusion and the Hall effect, which are expected to have an important effect in the outer parts of protoplanetary discs. In this calculation, we also did not describe the effect of an outflow, which could play an important role in driving accretion once such magnetic fields are present (Bai & Stone 2013; Bai 2013). Future work should take these processes into account in order to have a more precise description of the magnetic field evolution.

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