Dynamics of rapidly rotating stars in gravitational contraction

Delphine Hypolite, Michel Rieutord

Institut de Recherche en Astrophysique et Planétologie, Toulouse
SF2A 2015

4 juin 2015

Context

What is the dynamical state of a star reaching the Main Sequence ?

- Surface abundance anomalies
- Rotational mixing of elements

Young stars
Motivations

- Dynamical history of stars
- Initial conditions for MS $\rightarrow$ PMS
  - Temperature/entropie fiels
  - Internal rotation profile $\Omega_0(r, \theta)$
- 2D

The model

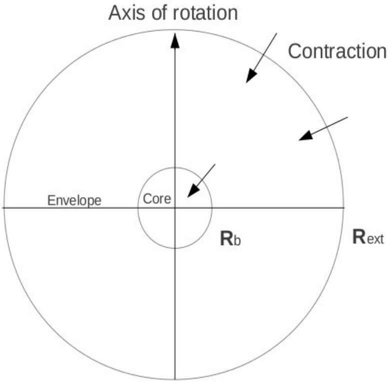


FIGURE: Incompressible and viscous fluid in rotation within two spherical shells.

Gravitational contraction induced flow

Equations

Equations of moment and mass conservation

In the co-rotating frame

$$\left\{ \begin{array}{c} \frac{\partial \vec{w}}{\partial t} + 2\vec{\Omega} \wedge \vec{w} + (\vec{w} \cdot \vec{\nabla})\vec{w} = -\vec{\nabla}\Pi + \nu\Delta\vec{w} - \dot{\vec{\Omega}} \wedge \vec{r} \\ \vec{\nabla} \cdot \vec{v} = 0 \end{array} \right. \quad (1)$$

in the incompressible case. The boundary conditions read

$$\left\{ \begin{array}{c} v_r(r = R_b) = -V_a \\ v_r(r = R_{ext}) = -\eta^2 V_a \end{array} \right. \qquad \left\{ \begin{array}{c} v_{\mathsf{tangent}}(r = R_b) = 0 \\ v_{\mathsf{tangent}}(r = R_{ext}) = \text{Solid} \\ \text{or Stress-Free} \end{array} \right.$$

Gravitational contraction induced flow

Equations

With the Kelvin-Helmoltz timescale $\frac{R_{ext}}{V_a}$, the lenght scale R_{ext} and the velocity scale V_a

Dimensionless equations

$$\begin{cases} Ro\frac{\partial \vec{u}}{\partial \tau} + \vec{e}_z \wedge \vec{u} + Ro(\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla}p + E\Delta\vec{u} - \dot{\omega}\vec{e}_z \wedge \vec{r} \\ \vec{\nabla}\cdot\vec{u} = 0 \end{cases} \quad (2)$$

The following dimensionless numbers appear

- The Rossby number : $Ro = \frac{V_a}{2\Omega_0 R_{\text{ext}}} \ll 1$
- The Ekman number : $E = \frac{\nu}{2\Omega_0 R_{\text{ext}}^2} \ll 1$

and the dimensionless rotation rate acceleration of the core :

$$\dot{\omega} = \frac{\dot{\Omega} R_{\text{ext}}}{2\Omega V_a}$$

Numerical method

Spectral

Spherical harmonics base for the horizontal direction :

$$\vec{u} = \sum_{l=0}^{+\infty} \sum_{m=-l}^{+l} u_m^l(r_j)\vec{R}_l^m + v_m^l(r_j)\vec{S}_l^m + w_m^l(r_j)\vec{T}_l^m$$

où $\begin{cases} \vec{R}_l^m & = & Y_l^m \vec{e_r} \\ \vec{S}_l^m & = & \vec{\nabla} Y_l^m \\ \vec{T}_l^m & = & \vec{\nabla} \wedge \vec{R}_l^m \end{cases}$

and radial discretisation onto the Gauss-Lobatto grid of N_r points associated with Chebyshev polynomials.

Gravitational contraction induced flow

Equations

We set $Ro = 0 \leftrightarrow$ steady and linear problem

We write $\vec{u} = -\frac{\eta^2}{r^2}\vec{e}_r + \vec{u'}$

Vorticity equation

$$\vec{\nabla} \wedge \left(\vec{e}_z \wedge \vec{u'} - E\Delta\vec{u'}\right) = 2\left(\frac{\eta^2}{r^3} - \dot{\omega}\right) \underbrace{\cos\theta\vec{e}_r}_{\sqrt{\frac{4\pi}{3}}\vec{R}_{l=1}^{m=0}} + \left(\frac{\eta^2}{r^3} + \dot{\omega}\right) \underbrace{\sin\theta\vec{e}_\theta}_{-\sqrt{\frac{4\pi}{3}}\vec{S}_{l=1}^{m=0}} \quad (3)$$

with

$$\dot{\omega} = \frac{9}{4\tilde{\rho}\eta}$$

from the momentum theorem and a boundary layer analysis.

Stratification

Rieutord 2006

Vorticity, energy and mass conservation equations using the Boussinesq approximation

$$\left\{\begin{array}{c} \vec{\nabla} \times (\vec{e}_z \wedge \vec{u} - E\Delta\vec{u} - B\theta_T\vec{r}) \\ = \underbrace{2(\frac{\eta^2}{r^3} - \dot{\omega})\cos\theta\vec{e}_r + (\frac{\eta^2}{r^3} + \dot{\omega})\sin\theta\vec{e}_\theta}_{\text{contraction torque}} \underbrace{-Bn_T^2(r)\sin\theta\cos\theta\vec{e}_\phi}_{\text{baroclinic torque}} \\ (n_T^2(r)/r)u_r = B\tilde{E}_T\Delta\theta_T \\ \vec{\nabla}\cdot\vec{u} = 0 \end{array}\right. \quad (4)$$

with the dimensionless number $B = \frac{V}{V_a}$

and $V = \frac{\epsilon\mathcal{N}^2 R_{ext}}{2\Omega_0}$, $\epsilon = \frac{\Omega_0^2 R_{ext}}{g_S}$, $\mathcal{N}^2 = \frac{\alpha T_* g_S}{R_{ext}}$, $\tilde{E}_T = \frac{\kappa\epsilon}{V R_{ext}} \sim \frac{E}{\mathrm{Pr}}$

Stratification

Rieutord 2006

Vorticity, energy and mass conservation equations using the Boussinesq approximation

$$\left\{\begin{array}{c} \vec{\nabla}\times(\vec{e}_z\wedge\vec{u}-E\Delta\vec{u}-B\theta_T\vec{r}) \\ =\underbrace{2(\frac{\eta^2}{r^3}-\dot{\omega})\cos\theta\vec{e}_r+(\frac{\eta^2}{r^3}+\dot{\omega})\sin\theta\vec{e}_\theta}_{\text{contraction torque}}\underbrace{-Bn_T^2(r)\sin\theta\cos\theta\vec{e}_\phi}_{\text{baroclinic torque}} \\ (n_T^2(r)/r)u_r=B\tilde{E}_T\Delta\theta_T \\ \vec{\nabla}\cdot\vec{u}=0 \end{array}\right. \quad (4)$$

with the dimensionless number $B=\frac{V}{V_a}$

and $V=\frac{\epsilon\mathcal{N}^2R_{ext}}{2\Omega_0}$, $\epsilon=\frac{\Omega_0^2R_{ext}}{g_S}$, $\mathcal{N}^2=\frac{\alpha T_*g_S}{R_{ext}}$, $\tilde{E}_T=\frac{\kappa\epsilon}{VR_{ext}}\sim\frac{E}{\mathrm{Pr}}$

Baroclinic flow
Rieutord 2006

The Brunt-Väisälä frequency profile is set

$$n^2(r) = (\alpha(r-\eta)+\beta(r-\eta)^2+\gamma(r-\eta)^3)^2$$

if $r \in [\eta; 1]$.

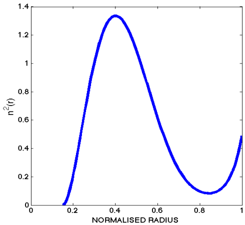


FIGURE: Brunt-Väisälä frequency profile for $\eta = 0.15$

according to 1D CESAM2k models profile.

BC=solid

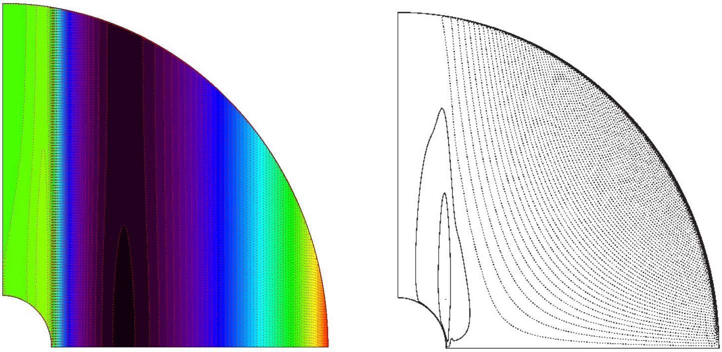


FIGURE: $\delta\Omega$ and ψ for $\mathrm{Pr} = 10^{-4}$, $E = 10^{-7}$, $\eta = 0.15$, $\tilde{\rho} = 10$ and $B = 0$

BC=solid, we increase B

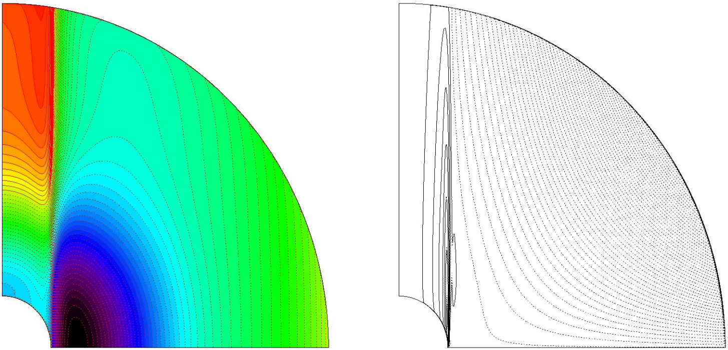


FIGURE: $\delta\Omega$ and ψ for $\mathrm{Pr} = 10^{-4}$, $E = 10^{-7}$, $\eta = 0.15$, $\tilde{\rho} = 10$ and $B = 10^4$

BC=solid, baroclinic differential rotation

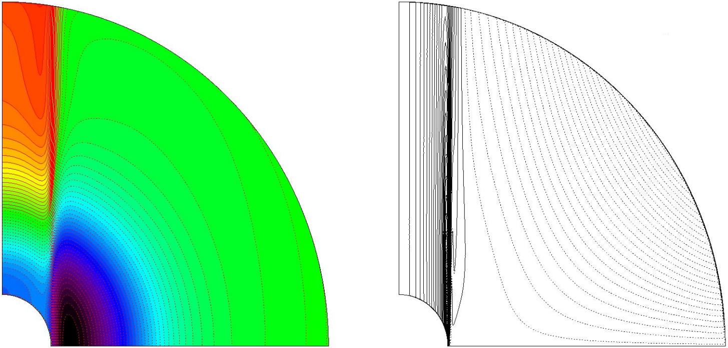


FIGURE: $\delta\Omega$ and ψ for $\mathrm{Pr} = 10^{-4}$, $E = 10^{-7}$, $\eta = 0.15$, $\tilde{\rho} = 10$ and $B = 10^5$

BC=solid, baroclinic meridional circulation

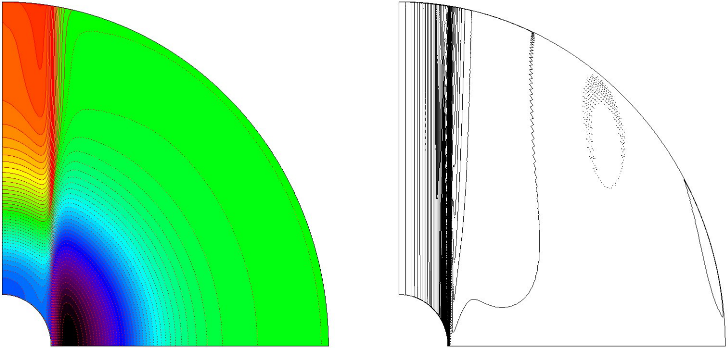


FIGURE: $\delta\Omega$ and ψ for $\mathrm{Pr} = 10^{-4}$, $E = 10^{-7}$, $\eta = 0.15$, $\tilde{\rho} = 10$ and $B_a = 10^7$

Critical B

BC=solid

On the differential rotation

$$B^{\delta\Omega}_{\text{crit}} \sim 3E^{-1/2}$$

On the meridional circulation

$$B^{\psi}_{\text{crit}} \sim 0.2E^{-1}$$

- If $B_\star < B_{\text{crit}}$,
  the spin-up flow dominates the dynamics,
  otherwise the baroclinic flow dominates.
- There is an intermediated regim where $\delta\Omega$ is generated by the stratification and ψ by the spin-up.

Stress-free boundary conditions

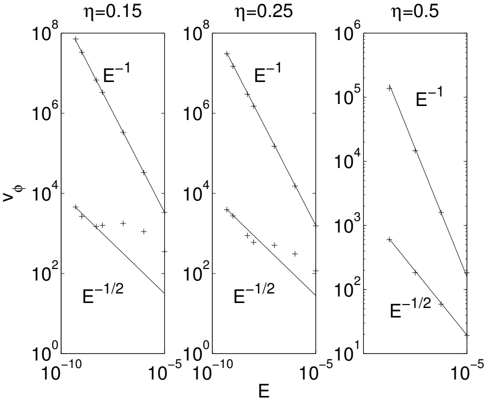


FIGURE: Amplitude of the azimuthal velocity in and out of the tangent cylinder of the steady state spin-up flow for different Ekman values (Hypolite & Rieutord 2014).

Spin-up transient flow

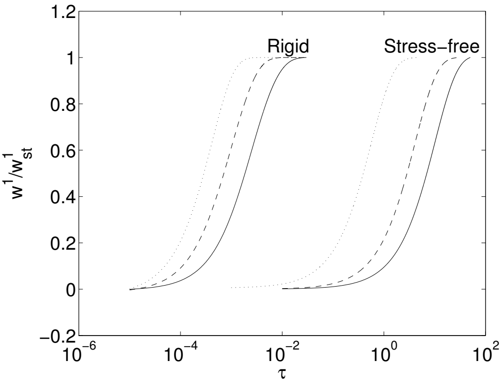
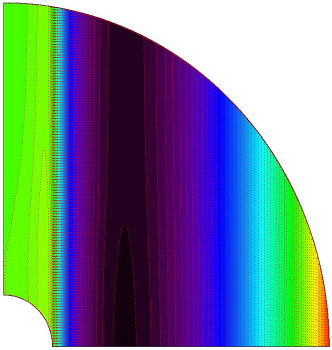


FIGURE: Time evolution of the main component of the azimuthal velocity as a function of time for different E (Hypolite & Rieutord 2014).

Spin-up transient flow



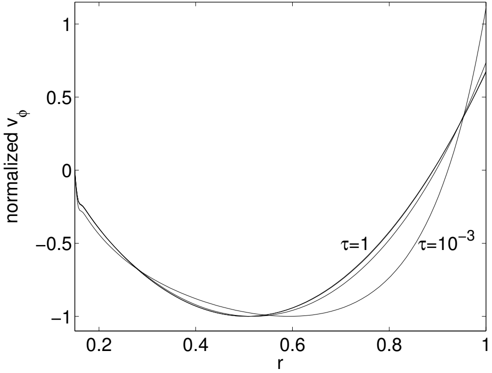


FIGURE: Differential rotation and azimuthal velocity of the transient spin-up flow at the equator (Hypolite & Rieutord 2014).

Time establishment of the baroclinic steady state

Baroclinic modes are dumped on an Eddington-Sweet timescale :

$$t_{ED} = \frac{t_{KH}}{\eta_{ED}}, \quad \eta_{ED} = \frac{4\Omega^2}{N^2}$$

If the rotation is slow, the steady state is reached in Gy.
For rapid rotation, $\eta_{ED} \sim 1$ the baroclinic steady state is established on a Kelvin-Helmholtz timescale.

Superposition on a pre existant baroclinic steady state

The time evolution pf the spin-up flow follows

$$\frac{A}{E}\left(1 - e^{-\tau \ln 10/\tau_{\mathrm{st}}}\right)$$

where $A = \mathcal{O}(1)$.
We deduce

$$\tau_c \sim \frac{k}{2A} E^{0.14}$$

The spin-up flow overwhelms the baroclinic one.

Superposition on a pre existant baroclinic steady state

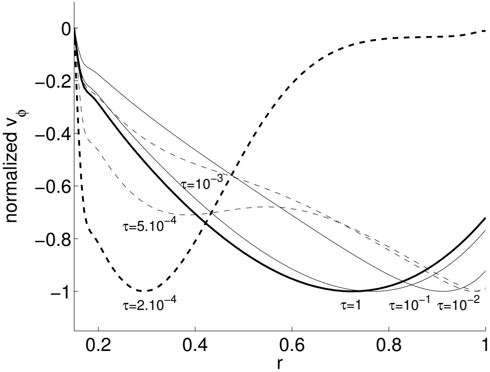


FIGURE: Radial profiles of azimuthal velocity at the equator at different times for $E = 10^{-5}$, $B = 10^2$, $Pr = 10^{-4}$, and $Ro = 10^{-5}$ (Hypolite & Rieutord 2014).

Conclusion

- initial conditions are erased → universal flow
- cylindrical differential rotation
- a Stewartson layer controling the flow amplitude within the envelope
- call for a compressible model → ESTER