Gravito-inertial modes of oscillation in a differentially-rotating radiative zone

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Journées de la sf2a

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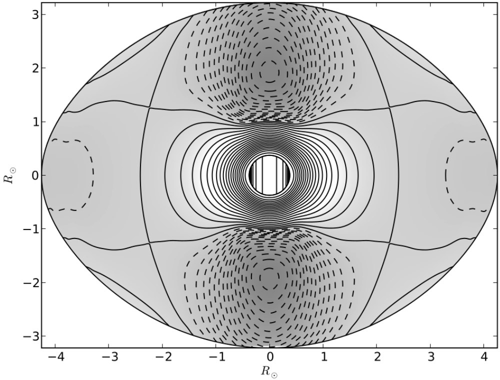
Massive or intermediate-mass stars are usually rotating fast and differentially.



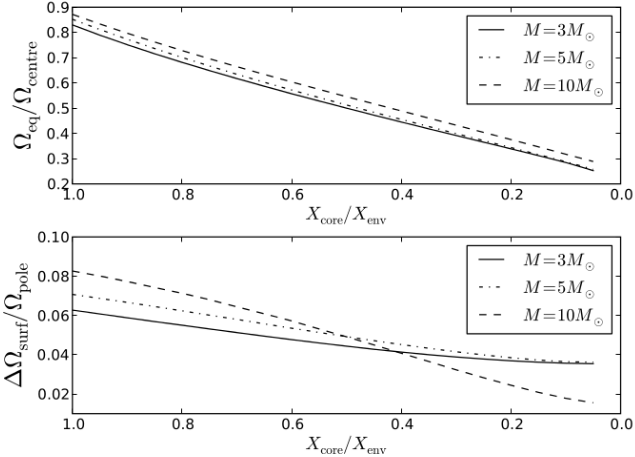
◁ Rotation profile of Regulus ($4.1M_\odot$) with the ESTER code.

- Fast rotation → flattening,
- Differential rotation
  → fast core, slow surface.

Espinosa Lara & Rieutord (2013)

Gravito-inertial modes → restored by buoyancy and Coriolis forces. They allow us to probe deep layers in the radiative zone of massive stars, and are excited by the internal κ-mechanism or tidal effects from close-in planets.

Differential rotation is expected in all kinds of stars, and increases through their evolution:



Espinosa Lara & Rieutord (2013)

→ impact on the gravito-inertial modes?

We solve the oscillations' eigenproblem using a simplified model:

- We use the Boussinesq approximation,
- We impose a linear temperature gradient: $\boldsymbol{\nabla} T = -\beta \boldsymbol{r}/R$

Equations

- We normalize equations using

$$L_{\mathrm{ref}} = R, \qquad t_{\mathrm{ref}} = \Omega(R)^{-1}, \qquad \Theta_{\mathrm{ref}} = \beta R$$

- Normalized equations read

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$$

$$\partial_t \Theta - r u_r = \frac{\mathrm{E}}{\mathrm{Pr}} \Delta\Theta$$

$$\partial_t \boldsymbol{u} + \Omega \partial_\phi \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} + s\left(\boldsymbol{u}\cdot\boldsymbol{\nabla}\Omega\right)\boldsymbol{e}_\phi = -\frac{1}{\rho_0}\boldsymbol{\nabla} P + \mathrm{E}\nabla^2\boldsymbol{u} - N^2\Theta\boldsymbol{r}$$

using the dimensionless parameters

$$N^2 = \frac{\alpha\beta g_0}{\Omega(R)^2}, \qquad \mathrm{Pr} = \frac{\nu}{\kappa}, \qquad \mathrm{E} = \frac{\nu}{\Omega(R)R^2}.$$

- stars → Pr $\ll 1$, E $\ll 1$, simulations → Pr$\sim 10^{-4} - 1$, E$\sim 10^{-9} - 10^{-6}$.

Radial rotation profile

We impose a stable radial Brunt-Väisälä frequency profile, *Chandrasekhar (1961)*

$$\nabla T = -\beta \boldsymbol{r}/R \quad \Rightarrow \quad n(r) = N \times r.$$

$\rightarrow$ first compilations of oscillations in a fluid where the differential rotation is obtained from the baroclinic flow.

The baroclinic flow yields a simple solution, provided we use no-slip inner and outer boundary conditions: *Rieutord (2006)*

$$\Omega = 1 + \int_r^1 \frac{n^2(r')}{r'} = 1 + \frac{N^2}{2}(1 - r^2).$$

We consider a spherical radiative zone for $1 > r > 0.35$. We use $N^2 < 10$ to limit the differential rotation to a factor 5.

Methods used

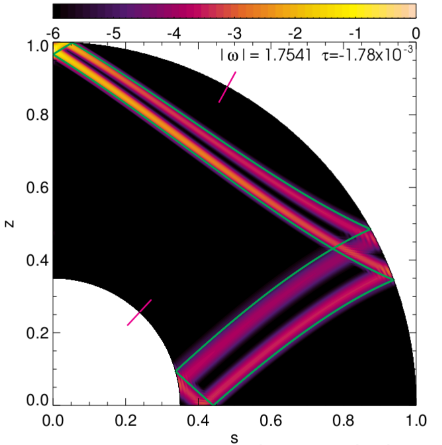
We combine two methods to solve this eigenproblem:
• we compute the eigenvalues and associated vector considering finite dissipations,

• setting the dissipations to zero, the eigensystem reduces to a second-order partial differential equation: *Colombo (1976)*

$$(N^2 z^2 - \Omega_p^2)\frac{\partial^2 p}{\partial s^2} - (2N^2 sz - 2\Omega N^2 sz)\frac{\partial^2 p}{\partial s \partial z} + (4\Omega^2 - 2\Omega N^2 s^2 + N^2 s^2 - \Omega_p^2)\frac{\partial^2 p}{\partial z^2} = 0.$$

This equation is of mixed type: gravito-inertial modes do not necessarily span the whole radiative zone → we may compute characteristics and turning surfaces in the star.

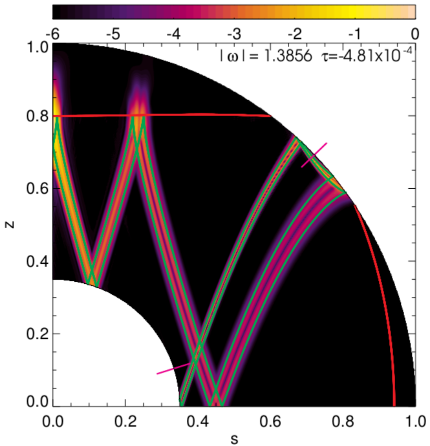
Mode classification



Nr=498 L=996 M=0 $E=1.0\times10^{-9}$ $P=1.0\times10^{-2}$ $N^2=0.45$

modes spanning the whole shell

→ **D modes**

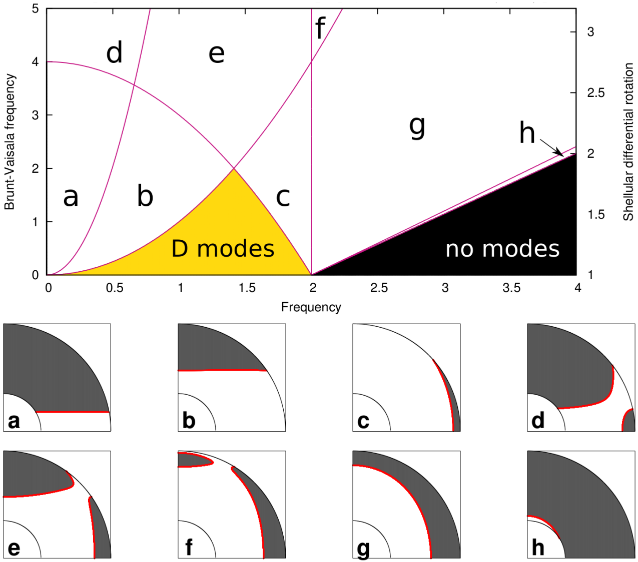


Nr=498 L=996 M=0 $E=1.0\times10^{-9}$ P=1.00

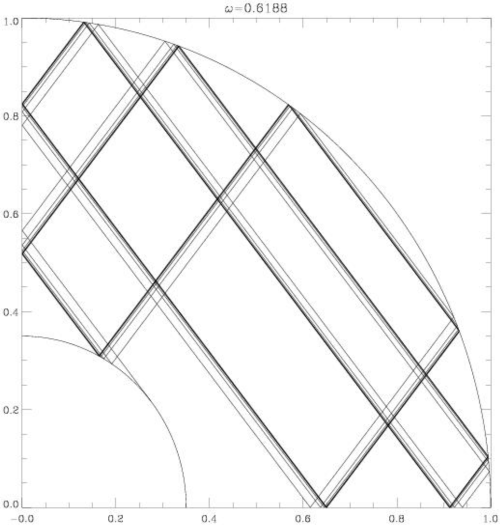
modes with turning surfaces

→ **DT modes**

Mode classification



Lyapunov exponents

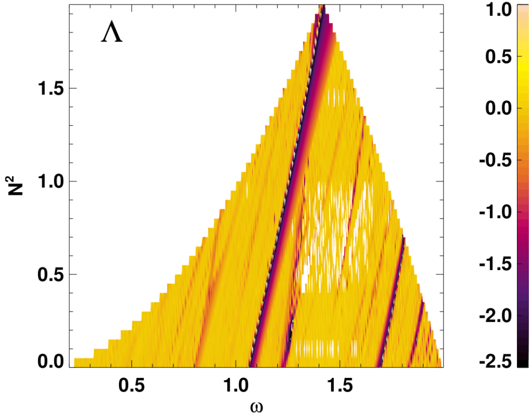


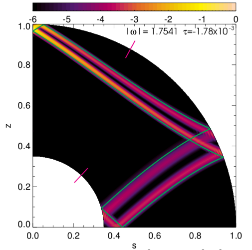
Characteristics tend toward an attractor
→ strength of the focusing quantified by the Lyapunov exponent Λ

$$dx_{n+1} = dx_n e^{\Lambda}.$$

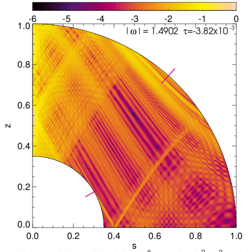
$\Lambda < 0 \rightarrow$ convergence toward an attractor,
Λ close to zero $\rightarrow$ regular mode.

Lyapunov exponents





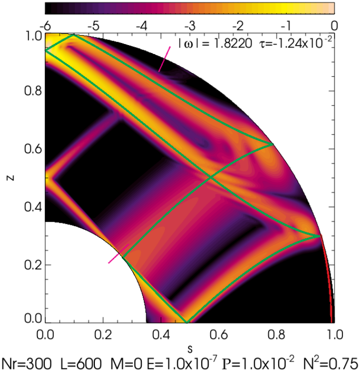
Nr=498 L=996 M=0 E=1.0x10^{-9} P=1.0x10^{-2} N^2=0.45



Nr=300 L=600 M=0 E=1.0x10^{-8} P=1.0x10^{-2} N^2=1.10

Example of axisymmetric modes

We follow a (gravito-)inertial mode from $N^2 = 0$ to $N^2 = 1$.



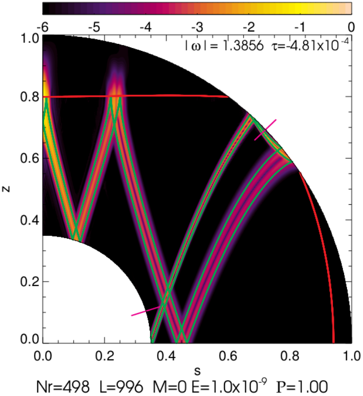
→ characteristics are straight lines at $N^2 = 0, \delta\Omega = 0$,
→ characteristics are curves when differential rotation is present.

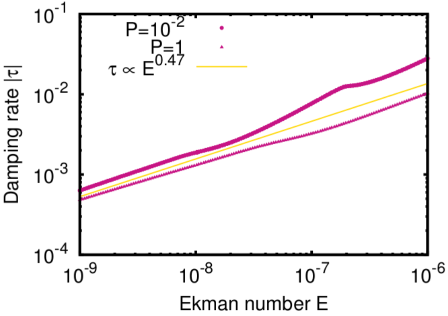
→ the characteristics cross the critical latitude for $N^2 \gtrsim 0.7$.

Video of the evolution of this mode → www.tinyurl.com/GMMirouh

Example of axisymmetric modes

We follow a gravito-inertial DT mode while decreasing E at constant Pr (i.e. $\nu \searrow$ & $\kappa \searrow$).



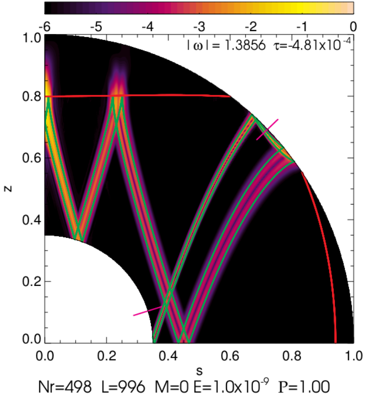


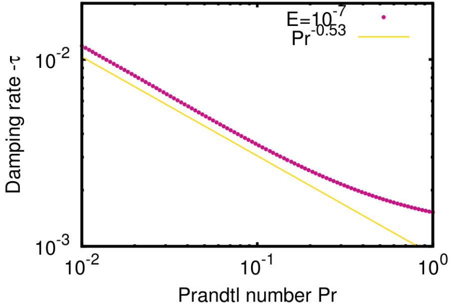
Asymptote: $-\tau \propto \mathrm{E}^{0.45}$

Video of the evolution of this mode → www.tinyurl.com/GMMirouh

Example of axisymmetric modes

We follow a gravito-inertial DT mode while increasing Pr at constant E (i.e. $\kappa \searrow$ only).

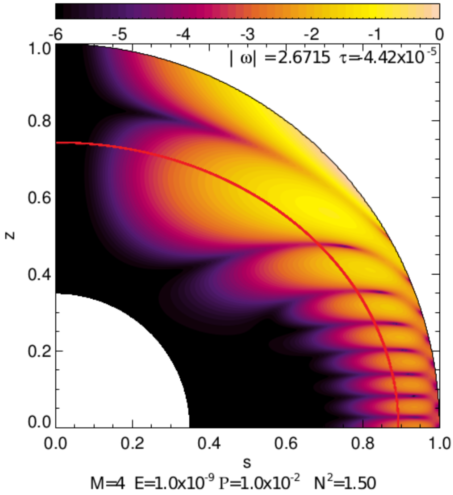




Asymptote: $-\tau \propto \mathrm{Pr}^{-0.53}$

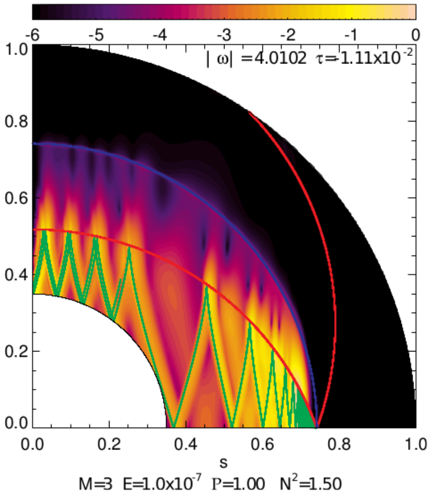
Video of the evolution of this mode → www.tinyurl.com/GMMirouh

Example of non-axisymmetric modes



$\rightarrow$ **regular modes**

structure independent from E



$\rightarrow$ **corotation resonances**

$\exists r \mid \omega + m\Omega(r) = 0 \rightarrow \boldsymbol{v}_\phi = 0$

Conclusions

So far,

- we computed the modes of oscillations on a differentially-rotating flow, and their characteristics in the non-dissipative limit;
- we predict accurately the extent of the propagation domain;
- we can predict the presence and the strength of characteristics, depending on the mode frequency.

We need to

- use quadruple precision in our code to study corotation resonances
- find a general characterization of the modes' focusing on singular structures
- finish a 30-page paper for *Journal of Fluid Mechanics* (coming soon!)

Thanks for listening !