Numerical simulations of zero-Prandtl-number thermohaline convection

Journées de la Société Française d'Astronomie & d'Astrophysique

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Outline

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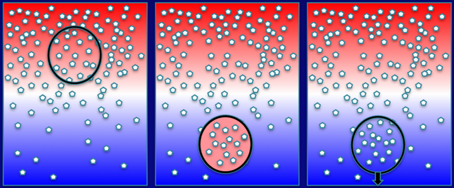
1. Theory of thermohaline convection
2. Numerical simulations

Description

- thermally stable *vs.* chemically unstable (${N_\mathrm{T}}^2 > 0$ and ${N_\mu}^2 < 0$)
- instability when $1 \leqslant R_0 \leqslant 1/\tau$, where $R_0 = -{N_\mathrm{T}}^2/{N_\mu}^2$ and $\tau = \kappa_\mu/\kappa_\mathrm{T}$



Garaud (2013)

- other relevant parameter: $Pr = \nu/\kappa_\mathrm{T}$

In stars

- occurs in late stages of evolution (e.g. $2\,{}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\mathrm{p}$)
- radiative levitation (Vauclair & Théado, 2012; Zemskova et al., 2014)
- infall material from planetary system (Vauclair, 2004; Deal et al., 2013)

Prescriptions

$$R_0 = -{N_\mathrm{T}}^2/{N_\mu}^2 \text{ and } \tau = \kappa_\mu/\kappa_\mathrm{T}$$

- Ulrich (1972):

$$D_\mu = C\kappa_\mathrm{T}/R_0$$

- Denissenkov (2010):

$$D_\mu = C\kappa_\mathrm{T}\frac{1-R_0\tau}{R_0-1}$$

- Brown et al. (2013):

$$D_\mu = C\kappa_\mathrm{T}\frac{\lambda^2}{k^2(\lambda+\tau k^2)}$$

with
$\lambda^3+k^2(1+Pr+\tau)\lambda^2+[k^4(\tau Pr+\tau+Pr)+Pr(1-\frac{1}{R_0})]\lambda+k^2Pr(k^4\tau+\tau-\frac{1}{R_0})=0$
and $(1+Pr+\tau)\lambda^2+2k^2(\tau Pr+\tau+Pr)\lambda+3k^4\tau Pr+Pr(\tau-\frac{1}{R_0})=0$

Problem

Insufficient to explain observed chemical abundances in red giants (Charbonnel & Zahn, 2007; Wachlin et al., 2014)

Very high thermal diffusivity

Radiative zones of RGB stars: typical parameter range

- $Pr \sim 10^{-8} - 10^{-6}$
- prohibitive computational cost for $Pr \ll 10^{-2}$
  $\rightarrow$ minimum value in Traxler et al. (2011) and Brown et al. (2013)

SPNA (Small-Péclet-number approximation, Lignières 1999)

- assumption: thermal background not modified by advection
- Taylor expansion in the Péclet number $Pe = u\ell/\kappa_\mathrm{T}$
- $Pr \ll 1$ regime can now be investigated

Other parameters

- $\phi = \frac{\kappa_\mu}{\nu} \left(= \frac{\tau}{Pr}\right) \sim 10^{-4} - 10^{-1}$
- $r = \frac{R_0 - 1}{1/\tau - 1} \sim R_0 \tau \left(= -\frac{{N_\mathrm{T}}^2 \kappa_\mu}{{N_\mu}^2 \kappa_\mathrm{T}}\right) \sim 10^{-6} - 1$
- still large computational cost for $\phi \ll 1$ and $r \ll 1$

Outline

Configuration

Numerical code: Snoopy (Lesur & Longaretti, 2011)

- spectral code
- periodic boundary conditions in all directions
- direct numerical simulations
- typical resolution: 256^3 distributed over 128 cores

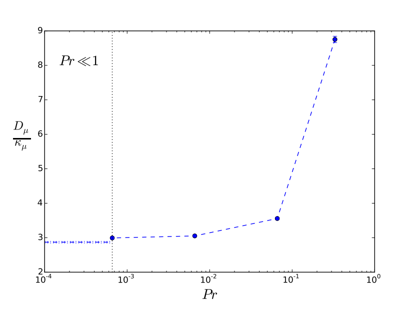
Physics

- uniform thermal and chemical background profiles
- equations solved for fluctuations
- full-Boussinesq or SPNA

Test of the SPNA for thermohaline convection

- full-Boussinesq simulations with $Pr \searrow$ (at constant $R_0\tau$ and ϕ)
- SPNA simulation (with same $R_0\tau$ and ϕ)

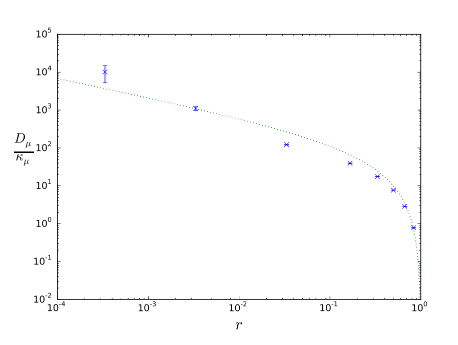
Results

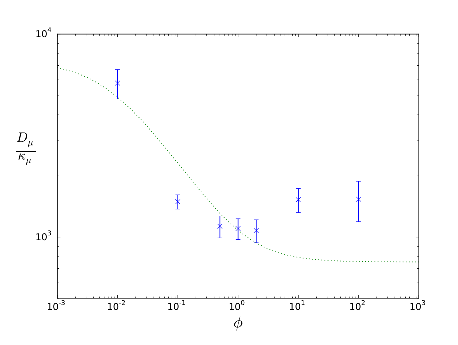


- SPNA works well
- asymptotic regime almost already reached at $Pr \sim 10^{-2}$

Zero-Prandtl-number simulations

Simulations (left $\phi = 1$, right $r = 3.33 \cdot 10^{-3}$)





Remarks

- follow trends given by Brown et al. (2013)
- sometimes significant discrepancies
- due to saturation mechanism?

Conclusion

Results

- validation of the SPNA for thermohaline convection
- simulations with realistic stellar paramaters

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Limitations

- small r and ϕ still expensive to simulate
- correspond to the boundary with a convective region $\rightarrow$ overshooting?

New features

- interaction with shear
- horizontal turbulence (Medrano et al., 2014)

Thank you.