The key role of critical layers for tidal dissipation in stars

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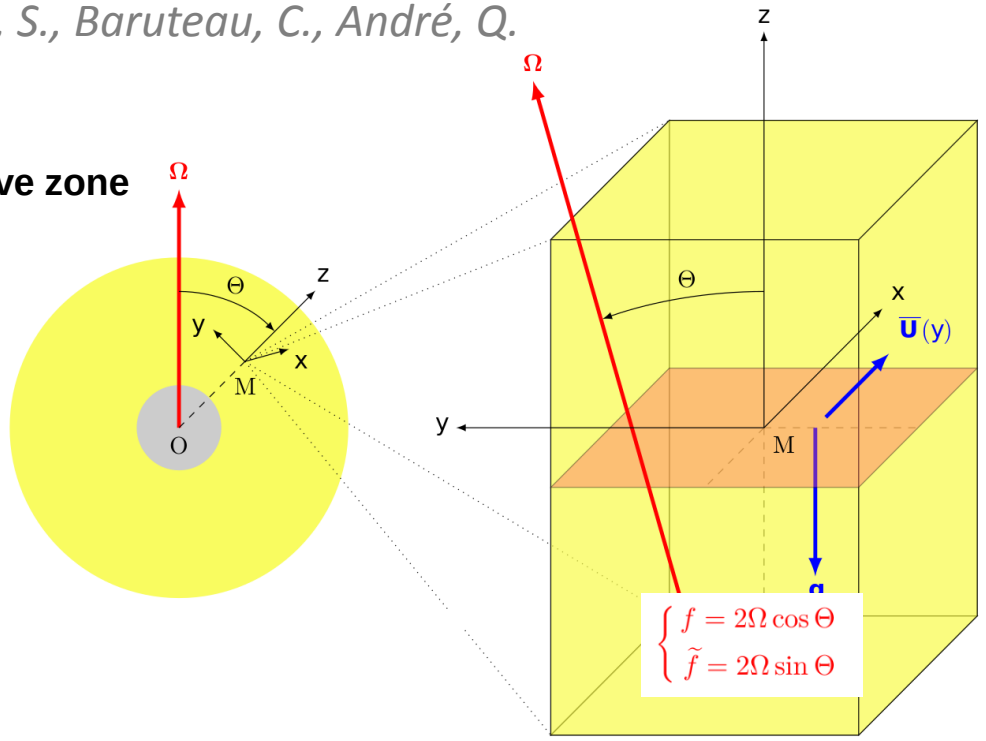
- **A prototype of differentially rotating convective zone**
  - Local Cartesian box
  - Convective medium
  - Rayleigh friction
  - Tidal forcing
  - **Latitudinal mean flow :** $\overline{U}(y)$
  - Boussinesq approximation
  - Coriolis but no centrifugal acceleration



Navier-Stokes + continuity equations : ⟶ **1-D Schrödinger-like equation on the latitudinal velocity :**

$$\frac{\mathrm{d}^2\hat{V}}{\mathrm{d}y^2} + \underbrace{\frac{k_{/\!/}}{4\mathcal{A}^2}\left[2i\widetilde{f}\frac{\mathrm{d}^2\overline{U}}{\mathrm{d}y^2}\mathcal{A} - 4\Sigma k_x\frac{\mathrm{d}\overline{U}}{\mathrm{d}y}\mathcal{B} + k_{/\!/}(\mathcal{B}^2 - 4\mathcal{A}\mathcal{C})\right]}_{\kappa(y)^2}\hat{V} = \frac{S(x,y,z)}{\mathcal{A}}$$

$\mathcal{A} = 0$ => critical layers (corotation)

Tidal forcing

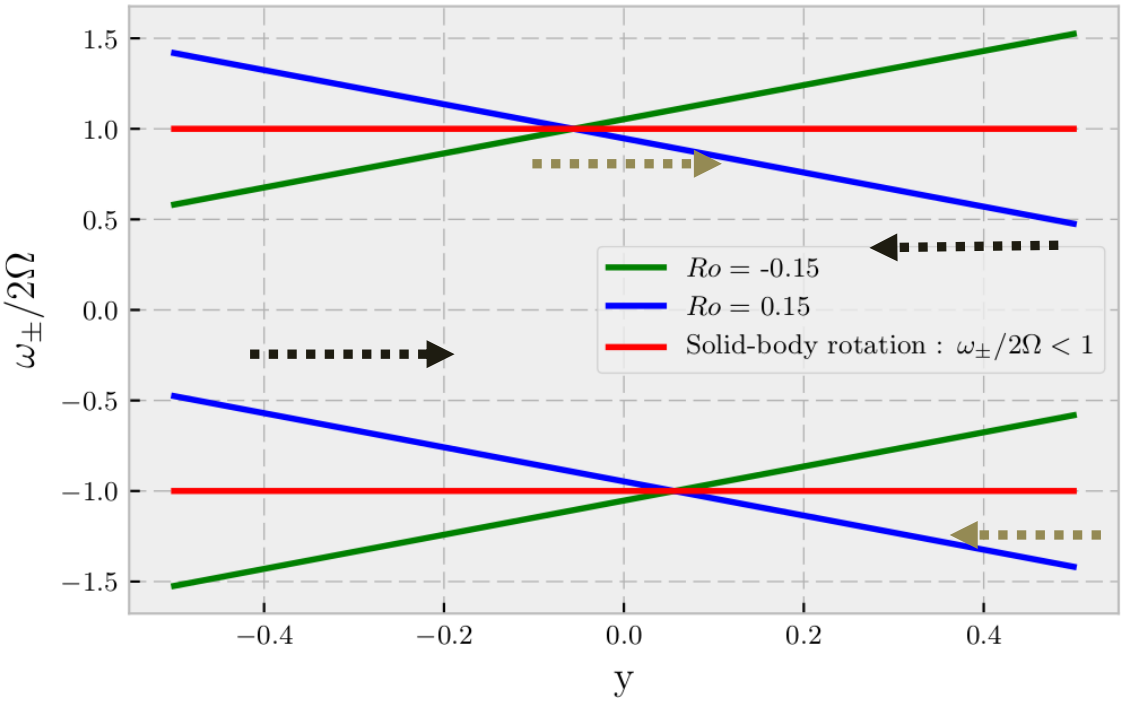
Waves propagation domain

- **The 2-D Poincaré equation in V(y,z) have to be of hyperbolic type to have wave propagation.**

$$\Rightarrow \quad \frac{\omega_\pm(y)}{2\Omega} = -yk_x Ro \pm \sqrt{\frac{1}{2}\left[1 - Ro\cos\theta + \sqrt{(Ro-\cos\theta)^2 + \sin^2\theta}\right]}$$

for a linear shear : $\overline{U}(y) = \Lambda y$

and $$Ro = \frac{1}{2\Omega}\frac{\mathrm{d}\overline{U}}{\mathrm{d}y} = \frac{\Lambda}{2\Omega}$$



- <u>Waves propagation</u> :
  - D modes → propagation in the full domain
  - DT* modes → propagation in a narrow domain

* DT : Differential rotation with a turning surface *[Baruteau and Rieutord 2013]*

Waves behaviour at critical layers

- **Development of the Schrödinger-like equation at critical layers :**

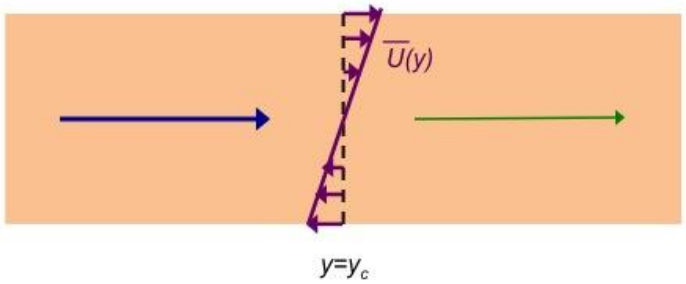
Complex number depending on the Rossby number Ro

$$\hat{V}''(y) + \frac{\chi}{(y - y_c)^2}\hat{V}(y) = 0$$

♠ if $|\mathrm{Im}(\chi)| \ll |\mathrm{Re}(\chi)|$, 2 regimes :

- $\mathrm{Re}(\chi) \gg 1/4$, stable
- $\mathrm{Re}(\chi) \ll 1/4$, unstable

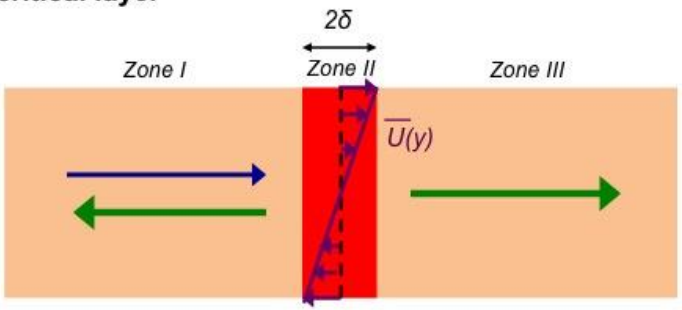
*** Stable critical layer**



Damping

Tidal inertial waves transfer their momentum to the mean flow

*** Shear unstable critical layer**



Damping or **possible over-reflection/transmission**

In the case of over-reflection/ transmission, the shear provides energy to tidal waves

Incident tidal inertial waves

Reflected/transmitted waves