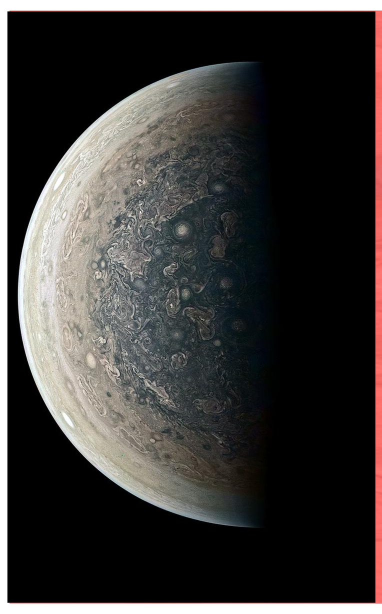

Layered Semi-Convection and Tides in Giant Planet Envelopes

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1 MOTIVATIONS

Tidal interactions drive the thermal, rotational and orbital evolution of planetary systems over astronomical time scales. However, how they operate depends on the details of the **internal structure** of the celestial bodies involved.

For giant planets, **layered semi-convection** (and associated **density staircases**) in their gaseous envelope is invoked to explain Saturn's luminosity excess [LC13] and the abnormally large radius of some hot Jupiters [CB07].

We study the **propagation of internal waves** in a region of layered semi-convection. The goal is then to understand the resulting **tidal dissipation** when these waves are excited by other bodies, such as moons.

MODEL

System: a small patch of the envelope of a giant planet containing a density staircase sustained by layered semi-convection. The planet rotates with an **angular velocity Ω**, and is submitted to a periodical tidal forcing of frequency ω.

Local Cartesian model: local box centered on a point M at the **colatitude** θ, with local radial (z), latitudinal (y), and azimuthal (x) directions.

Physical ingredients

- **Density staircase:** m convective steps of size d separated by stably-stratified interfaces of size l, total size D, mean buoyancy frequency N
- Restoring forces: **Coriolis** & buoyancy
- Dissipative mechanisms: viscosity and thermal diffusion

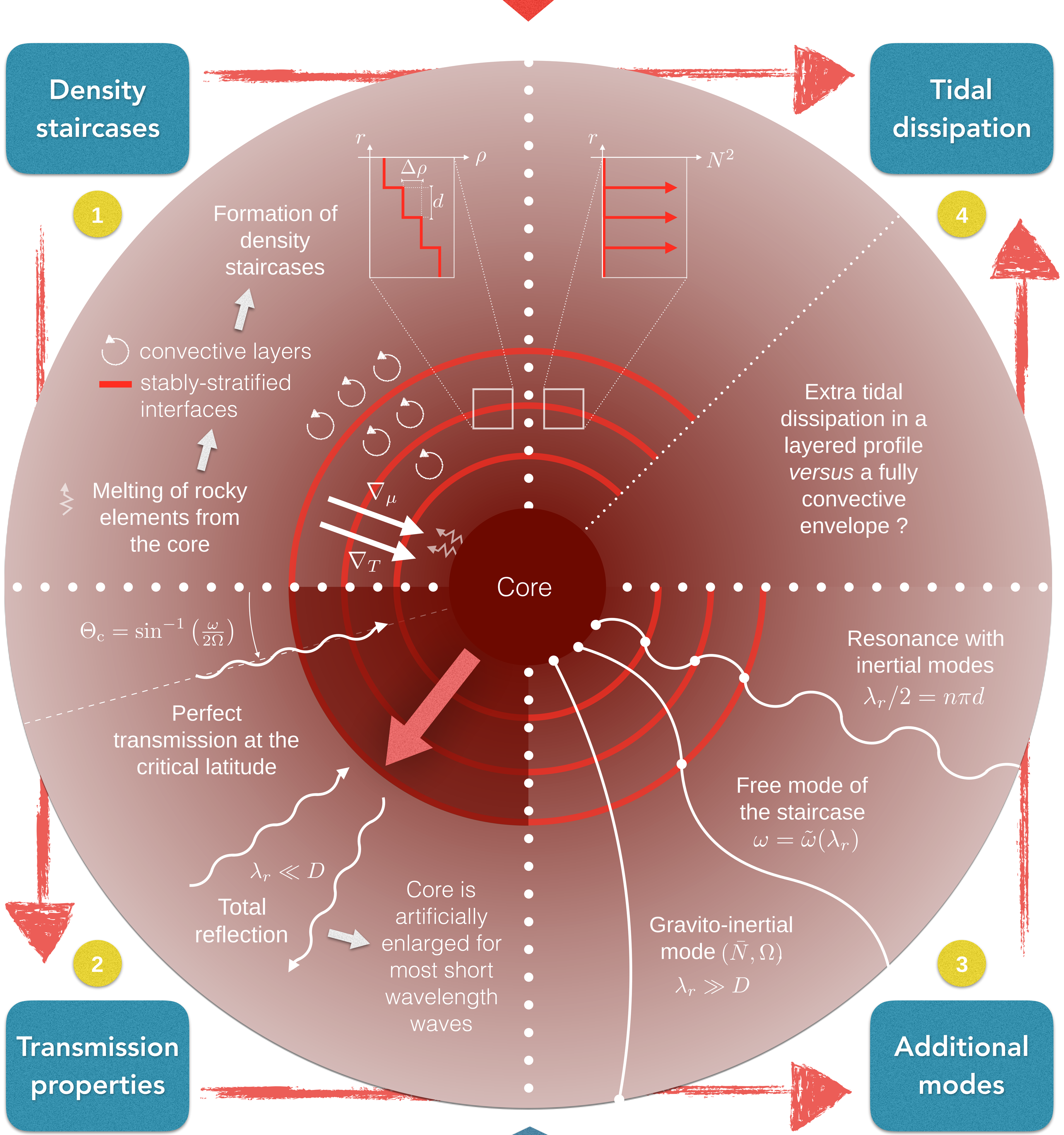
Propagation of internal waves

- Transmission coefficient T

Tidal response:

- Variations of velocity field, pressure, density, buoyancy
- rates of tidal dissipation: viscous dissipation D_{visc} and thermal dissipation D_{ther}, and frequency-integrated dissipation denoted by $\langle . \rangle$

? How density staircases in giant planet interiors affect the propagation of internal waves and modify the rates of tidal dissipation?

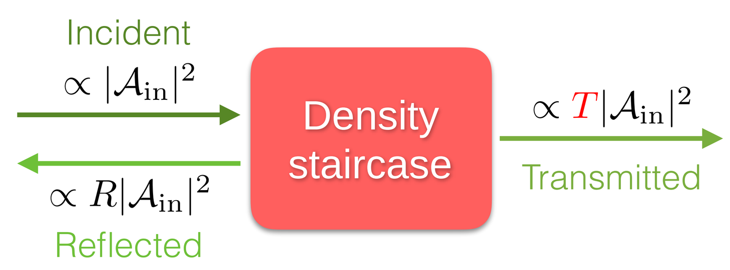


1 Melting of rocky elements from the core creates a stabilizing compositional gradient [M+15]. Its competition with the entropy gradient can **trigger layered semi-convection: density staircases are created. 2** In turn, **the propagation of internal waves is strongly affected**, e.g. almost all short wavelength waves are reflected at the top of the staircase, thus seeing an artificially enlarged core. **3** Resonances with free modes lead to the possibility that **new modes can be observed by seismology. 4** The resulting **tidal dissipation is modified versus a fully convective envelope**: dissipation spectra show particular resonances, and the rates of tidal dissipation change accordingly.

2 INTERNAL WAVES

Analytical model

- dispersion relation of the staircase generalizing [BQF15]: $\omega = \tilde{\omega}(\lambda)$
- transmission coefficient T generalizing [S16]:



Criteria for transmission

Perfect transmission

- large wave-length: $\lambda \gg D$
- resonance with inertial modes
- at the critical latitude
- free mode of the staircase (root of the dispersion relation)

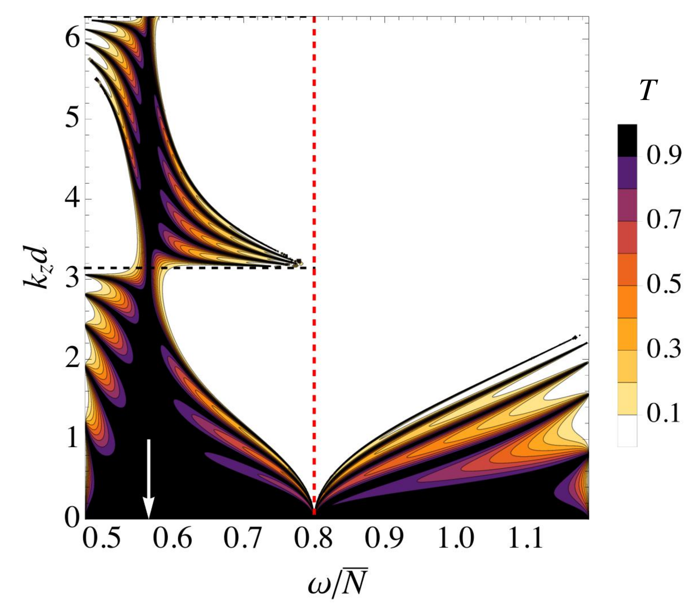


Fig. 1: Transmission coefficient as a function of wave's frequency and vertical wave number. White arrow: $\omega = 2\Omega\cos\theta$, dashed red line: $\omega = 2\Omega$. Each band of perfect transmission corresponds to a free mode of the staircase.

Total reflection

- short wave-length: $\lambda \ll D$

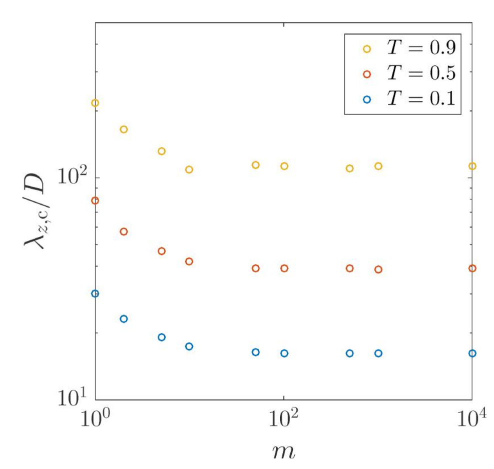


Fig. 2: Vertical cut-off wavelength as a function of number of steps, measured for 3 criteria on T.

4 TIDAL DISSIPATION

Numerical model

- **Spectral method** to solve the linearized Navier-Stokes equations for tidal gravito-inertial waves, with **periodic boundary conditions** in every direction
- **Buoyancy frequency profile** using smooth bumps of amplitude calculated by prescribing a mean value N

Dissipation spectra

- **Extra dissipation peaks** (excitation of free modes of the staircase)

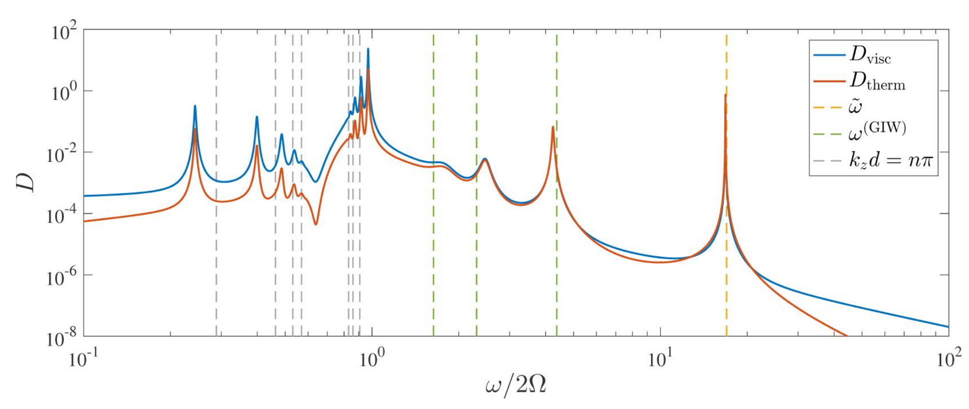


Fig. 3: Viscous (blue), and thermal (red) dissipation rates as a function of forcing frequency $\omega/2\Omega$. Each resonant peak corresponds to the excitation of a free mode of the staircase (see 2).

Frequency-integrated rates

- Exploration of parameter space: aspect ratio $\varepsilon = l/d$
- **High dissipation rates** in the relevant limit of small aspect ratios [LC12]

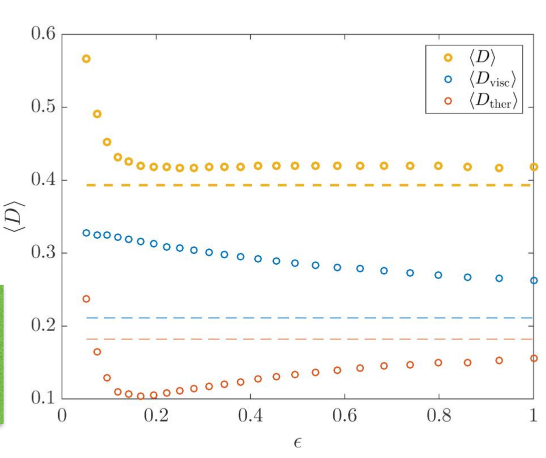


Fig. 4: Viscous (blue), thermal (red) and total (yellow) frequency-integrated dissipation rates as a function of aspect ratio.

3 CONCLUSIONS & PERSPECTIVES

Take away message

- **Internal waves are strongly affected** by the presence of a density staircase in a **frequency-, latitude- and wavelength-dependent manner.**
- **Layered semi-convection is a possible candidate that could explain high tidal dissipation rates** observed by e.g. [L+17].

Consequences for the seismology of planets

- New proposed criteria to **probe the core of giant planets** by seismology
- **New modes** can potentially be observed

To be done

- Calculation in a global spherical model
- Include non-linear effects, differential rotation and magnetic field

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