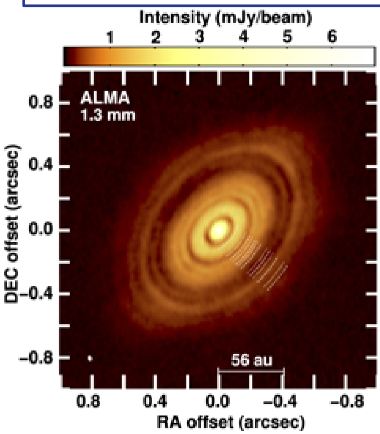
Multi-body figures of equilibrium in axial symmetry

Baptiste BOUTIN-BASILLAIS[1,2]
Jean-Marc HURÉ[1,2]

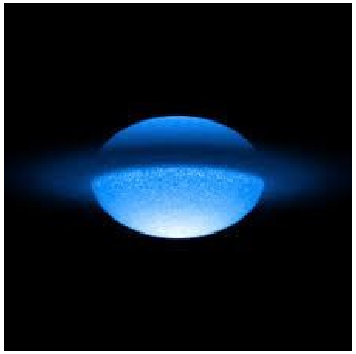
1 : Université de Bordeaux, LAB, UMR 5804,F-33615, Pessac, France
2 : CNRS, LAB, UMR 5804, F-33615, Pessac, France

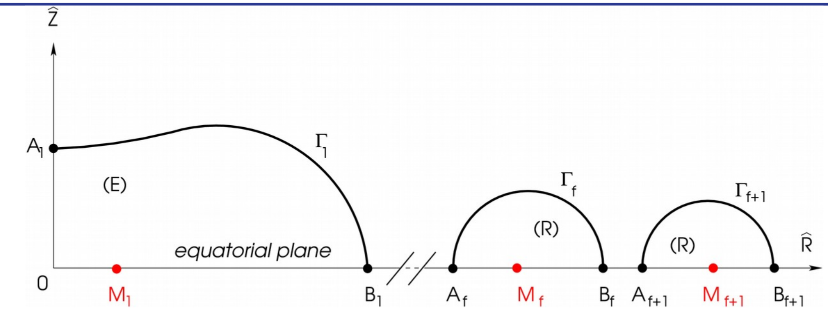
Abstract We present an efficient **multi-body code** based on the Self-Consistent Field (SCF) method which aims at studying self-gravitating **polytropic stars and rings** in mutual **gravitational** interaction. It works for any positive polytropic index, **rotation law** and configuration (axis ratios and relative separation). We have investigated a wide range of equilibria involving 2 up to 8 bodies. We have further attempted to reproduce two observed systems : the very young protoplanetary disk around **HL Tau** and the **Be-star Achernar**.



Achernar ▶

◀ HL Tau





Typical configuration for a multi-body system made of an **ellipsoid** (E) and concentric **rings** (R). The SCF-method relies on two reference point A and B, where the fluid enthalpy vanishes, and a third point M where the enthalpy is maximum.

Overview The theory of **figures of equilibrium figures** is studied since Newton and Cassini to explain the shape of the Earth. It has been widely applied to understand stellar formation, **stellar structure**, galaxy dynamic and the properties of massive disks and tori. Articles by Hachisu in the 80's are cornerstones about numerical solutions.

Most works are devoted to **single body systems**. Hachisu et al. (1986) studied two-body equilibria and Ansorg et al. (2003) computed single body solutions on the verge of splitting into several bodies.

Here, we upgrade the **DROP** code (Huré & Hersant 2017) to treat **multiple-body** configurations.

Numerical treatment

Set of equations

We solve the **Bernoulli equation**

$$H+\Psi+\Phi=cte,$$

together with the **Poisson equation**

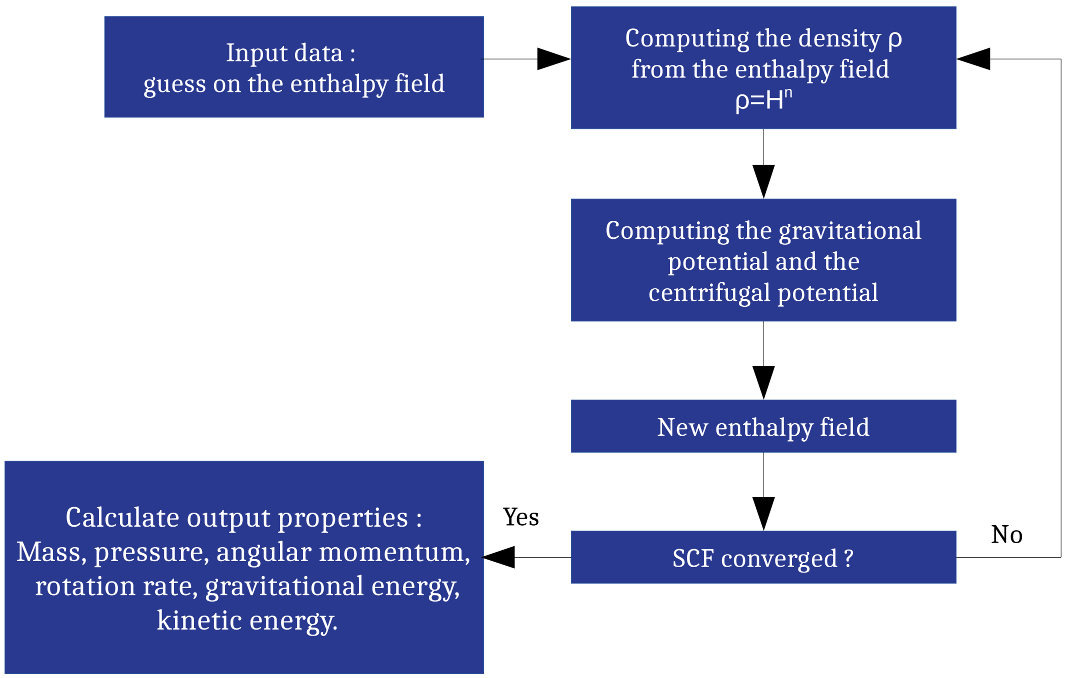
$$\Delta\Psi=4\pi\rho$$

in a scale free manner, Hachisu (1986).

The equation-of-state is **polytropic**. The enthalpy field H is then linked to the density ρ through : $\rho=H^n$

We can compute various configurations. The code outputs all physical properties such as the fluid **mass** M, the pressure field P or even the gravitational energy. The **Virial test** is fulfilled ~ $\mathbf{10^{-4}}$ typically

SCF cycle

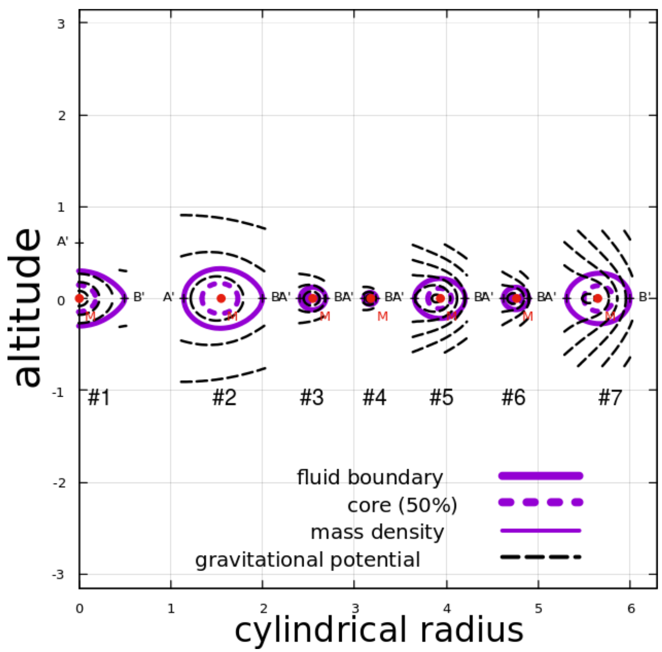


Numerical method We use one computational box per body with a linear **radial stretching**, appropriate for oblate structures. Individual Poisson equations are solved with **multigrid** at second order with **Dirichlet and Neumann boundary conditions**. Fluid boundaries are detected with a **Freeman chain code** and accounted for in the determination of all global quantities.

Application to two observed systems

HL Tauri and its protoplanetary disk

The disk around the star HL Tauri is very young with several **overdensities** organized themself in **tori** of matter. We were able to reproduce this system with a central **ellipsoid** surrounded by 6 concentric **tori** in **solid rotation**. The **mass** of the disk is believed to be 0.1 to 0.5 mass of the central star and we can't neglect the effect of **self-gravitating** interactions over the gravitational pull of the star. The relative mass of each body follow the measurements made in Carrasco-González et al. (2016).



The Be star Achernar

Achernar is a **rapid rotator** with axis ratio of **0.66**. Due to its high rotation speed, Achernar displays a **pulsating circumstellar disk** in the equatorial plane (Vinicius et al., 2016). This periodic property is caused by the transition between the B phase (no disk) and the Be phase (presence of a disk). We obtain a similar configuration using an ellipsoid with an axis ratio of 0.66 surrounded by an almost-in-contact small torus. We have modelled the transition between the B phase and the Be phase with a sequence of equilibria where the torus is getting smaller and smaller and vanishes at some point.