Université de Bordeaux

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ONDES GRAVITATIONNELLES
&
TESTS DE LA RELATIVITÉ GÉNÉRALE

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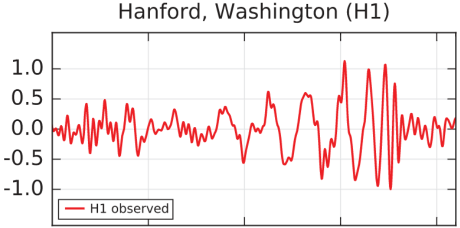
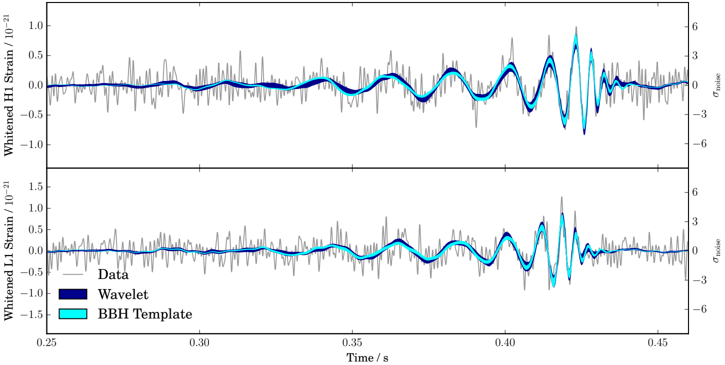
Binary black-hole event GW150914 [LIGO/Virgo collaboration 2016]





Quadrupole moment formalism [Einstein 1918; Landau & Lifchitz 1947]

$$4\pi R^2 \bar{\mathfrak{J}} = \frac{\varkappa}{40\pi}\left[\sum_{\mu\nu} \dddot{J}_{\mu\nu}^{\,2} - \frac{1}{3}\left(\sum_{\mu} \dddot{J}_{\mu\mu}\right)^2\right].$$

1. Einstein quadrupole formula

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)^{\text{GW}} = \frac{G}{5c^5}\left\{\frac{\mathrm{d}^3 Q_{ij}}{\mathrm{d}t^3}\frac{\mathrm{d}^3 Q_{ij}}{\mathrm{d}t^3} + \mathcal{O}\left(\frac{v}{c}\right)^2\right\}$$

2. Amplitude quadrupole formula

$$h_{ij}^{\text{TT}} = \frac{2G}{c^4 D}\left\{\frac{\mathrm{d}^2 Q_{ij}}{\mathrm{d}t^2}\left(t - \frac{D}{c}\right) + \mathcal{O}\left(\frac{v}{c}\right)\right\}^{\text{TT}} + \mathcal{O}\left(\frac{1}{D^2}\right)$$

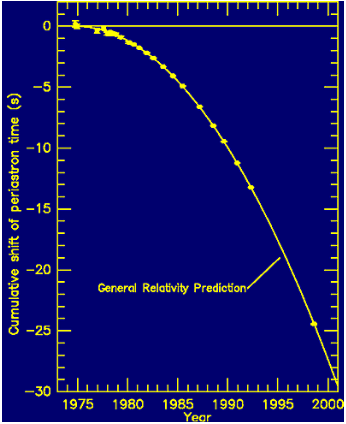
3. Radiation reaction formula [Chandrasekhar & Esposito 1970; Burke & Thorne 1970]

$$F_i^{\text{reac}} = -\frac{2G}{5c^5}\rho\, x^j \frac{\mathrm{d}^5 Q_{ij}}{\mathrm{d}t^5} + \mathcal{O}\left(\frac{v}{c}\right)^7$$

which is a 2.5PN $\sim (v/c)^5$ effect in the source's equations of motion

The quadrupole formula works for the binary pulsar

[Taylor & Weisberg 1982]



$$\dot{P} = -\frac{192\pi}{5c^5}\nu\left(\frac{2\pi G\,M}{P}\right)^{5/3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}} \approx -2.4\times 10^{-12}$$

[Peters & Mathews 1963; Esposito & Harrison 1975; Wagoner 1975; Damour & Deruelle 1983]

The quadrupole formula works for GW150914 !

1. The GW frequency is given in terms of the chirp mass $\mathcal{M} = \mu^{3/5} M^{2/5}$ by

$$f = \frac{1}{\pi} \left[\frac{256}{5} \frac{G\mathcal{M}^{5/3}}{c^5} (t_\mathsf{f} - t) \right]^{-3/8}$$

2. Therefore the chirp mass is directly measured as

$$\boxed{\mathcal{M} = \left[\frac{5}{96} \frac{c^5}{G\pi^{8/3}} f^{-11/3} \dot{f} \right]^{3/5}}$$

which gives $\mathcal{M} = 30 M_\odot$ thus $M \geqslant 70 M_\odot$

3. The GW amplitude is predicted to be

$$h_\mathsf{eff} \sim 4.1 \times 10^{-22} \left(\frac{\mathcal{M}}{M_\odot} \right)^{5/6} \left(\frac{100\,\mathsf{Mpc}}{D} \right) \left(\frac{100\,\mathsf{Hz}}{f_\mathsf{merger}} \right)^{-1/6} \sim 1.6 \times 10^{-21}$$

4. The distance $D = 400\,\mathsf{Mpc}$ is measured from the signal itself

Total energy radiated away by GW150914

1. The ADM energy of space-time is constant and reads (at any t)

$$E_{\mathsf{ADM}} = (m_1 + m_2)c^2 - \frac{G m_1 m_2}{2r} + \frac{G}{5c^5}\int_{-\infty}^{t} \mathrm{d}t' \left(Q_{ij}^{(3)}\right)^2(t')$$

2. Initially $E_{\mathsf{ADM}} = (m_1 + m_2)c^2$ while finally (at time t_f)

$$E_{\mathsf{ADM}} = M_\mathsf{f} c^2 + \frac{G}{5c^5}\int_{-\infty}^{t_\mathsf{f}} \mathrm{d}t' \left(Q_{ij}^{(3)}\right)^2(t')$$

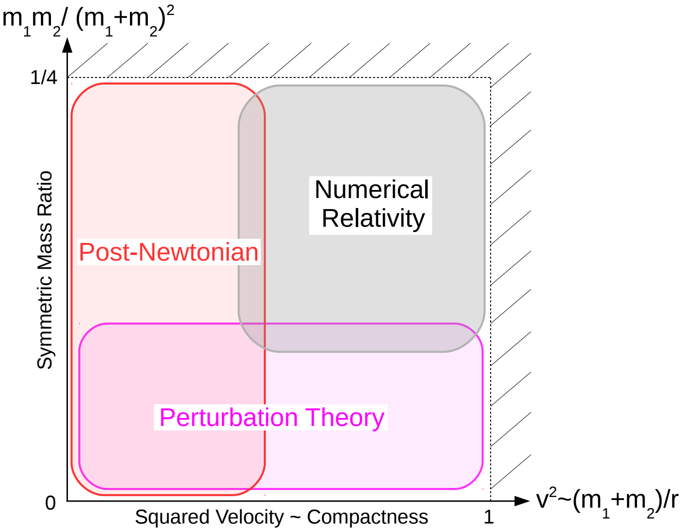
3. The total energy radiated in GW is

$$\boxed{\Delta E^{\mathsf{GW}} = (m_1 + m_2 - M_\mathsf{f})c^2 = \frac{G}{5c^5}\int_{-\infty}^{t_\mathsf{f}} \mathrm{d}t' \left(Q_{ij}^{(3)}\right)^2(t') = \frac{G m_1 m_2}{2 r_\mathsf{f}}}$$

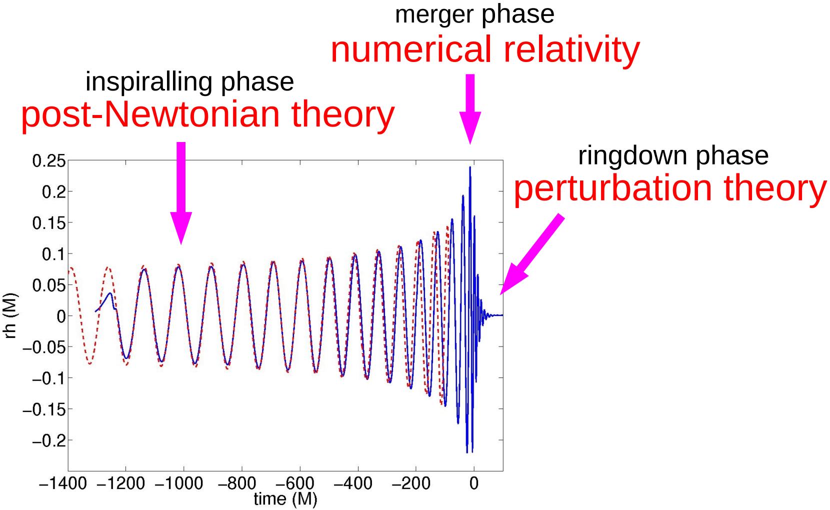
4. The total power released is

$$\boxed{P^{\mathsf{GW}} \sim \frac{3 M_\odot c^2}{0.2\,\mathsf{s}} \sim 10^{49}\,\mathsf{W} \sim 10^{-3}\,\frac{c^5}{G}}$$

Methods to compute GW templates



The gravitational chirp of compact binaries



The GW templates of compact binaries

1. In principle, the templates are obtained by matching together:
   - A high-order 3.5PN waveform for the inspiral [Blanchet *et al.* 1998, 2002, 2004]
   - A highly accurate numerical waveform for the merger and ringdown [Pretorius 2005; Baker *et al.* 2006; Campanelli *et al.* 2006; Hannam, Husa, Sperhake *et al.* 2008]
2. In practice, for black hole binaries (such as GW150914), effective methods that interpolate between the PN and NR play a key role in the data analysis
   - Hybrid inspiral-merger-ringdown (IMR) waveforms [Ajith, Hannam, Husa *et al.* 2011] are constructed by matching the PN and NR waveforms in a time interval through an intermediate phenomenological phase
   - Effective-one-body (EOB) waveforms [Buonanno & Damour 1999] are based on resummation techniques extending the domain of validity of the PN approximation beyond the inspiral phase
3. In the case of neutron star binaries (such as GW170817), the templates are entirely based on the 3.5PN waveform

Black holes have no hair

1. Exterior geometry of the rotating BH solution [Kerr 1963]

$$\begin{aligned} g_{00} &= -1 + \frac{M}{r} + \frac{Q_2 P_2(\cos\theta)}{r^3} + \frac{Q_4 P_4(\cos\theta)}{r^5} + \cdots \\ g_{0\varphi} &= \frac{J}{r^2} + \frac{J_3 \tilde{P}_3(\cos\theta)}{r^4} + \frac{J_5 \tilde{P}_5(\cos\theta)}{r^6} + \cdots \end{aligned}$$

2. The no hair theorem states (with $Q_0 = M$, $J_1 = J$ and $a = J/M$) [Hansen 1974]

$$\boxed{Q_\ell + \mathrm{i} J_\ell = M\,(\mathrm{i}a)^\ell}$$

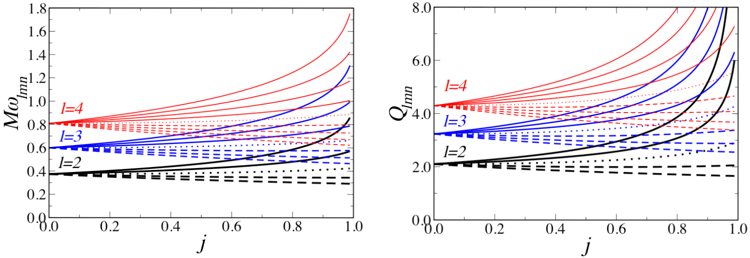
3. The quadrupole moment is determined from the mass and spin

$$\boxed{Q_2 = -M\,a^2 = -\frac{J^2}{M}}$$

Counting BH hairs with the ringdown radiation

- The merger of two black holes produces a distorted BH who emits "ringdown" radiation to shed hair
- The frequency modes of ringdown radiation [e.g. Berti, Cardoso & Will 2006]

$$\omega = \omega_{\ell mn} \left[1 + \frac{\mathrm{i}\pi}{2Q_{\ell mn}} \right]$$



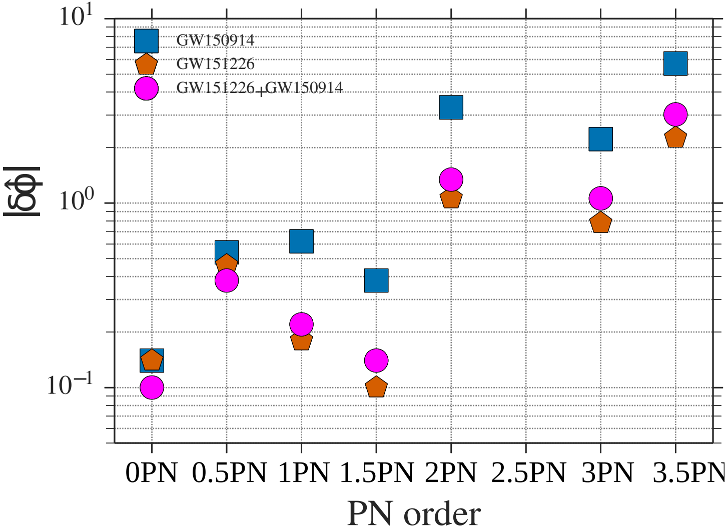
$$j = \frac{J}{M^2} = \frac{a}{M}$$

3.5PN energy flux of compact binaries

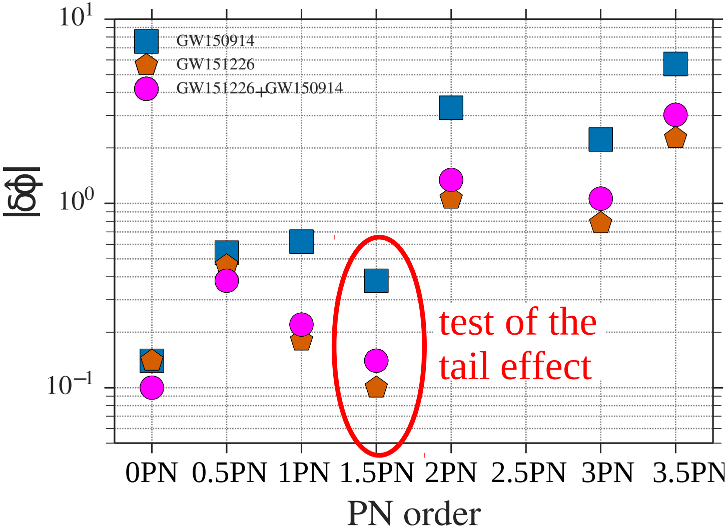
[Blanchet *et al.* 1998, 2002, 2004]

$$
\begin{aligned}
\mathcal{F}^{\mathrm{GW}} = \frac{32c^5}{5G}\nu^2 x^5 \Bigg\{ 1 &+ \overbrace{\left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x}^{\text{1PN}} + \overbrace{4\pi x^{3/2}}^{\text{1.5PN tail}} \\
&+ \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2 + \overbrace{\left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2}}^{\text{2.5PN tail}} \\
&+ \Bigg[\frac{6643739519}{69854400} + \overbrace{\frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_{\mathrm{E}} - \frac{856}{105}\ln(16\,x)}^{\text{3PN tail-of-tail}} \\
&\qquad + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3 \Bigg] x^3 \\
&+ \underbrace{\left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2}}_{\text{3.5PN tail}} + \mathcal{O}\left(\frac{1}{c^8}\right) \Bigg\}
\end{aligned}
$$

Measurement of PN parameters [LIGO/VIRGO 2016]



Measuring GW tails [Blanchet & Sathyaprakash 1994, 1995]



GW solutions in metric theories of gravity

1. Small perturbation of the metric around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \text{with} \quad |h_{\mu\nu}| \ll 1$$

2. Restrict attention to theories admitting GW solutions propagating at the speed of light: $c_g = 1$. Far from the sources the waves are planar, hence

$$\Box h_{\mu\nu} = 0 \quad \Longleftrightarrow \quad h_{\mu\nu} = h_{\mu\nu}(t-z)$$

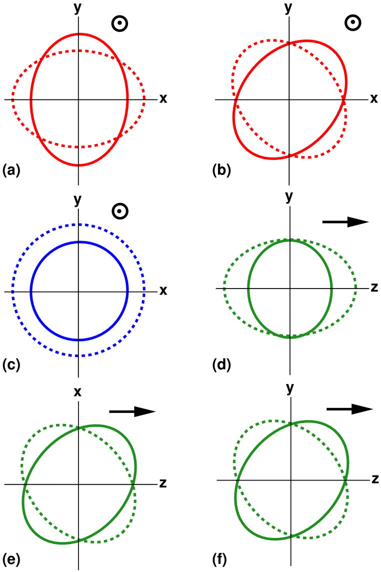
3. From the linearized Bianchi's identity obtain

$$\boxed{\Box R_{\mu\nu\rho\sigma} = 0 \quad \Longleftrightarrow \quad R_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma}(t-z)}$$

   showing that GWs have an invariant, coordinate-independent meaning

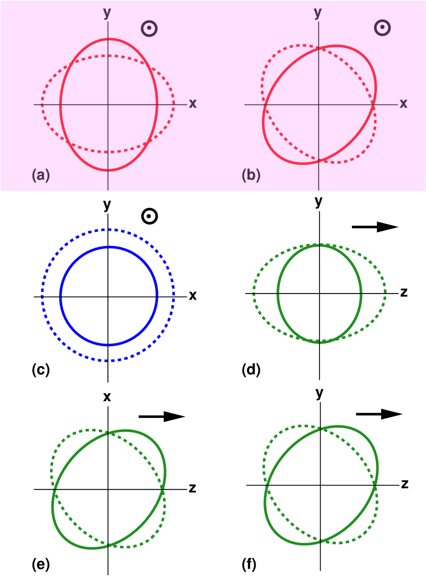
4. The six components R_{0i0j} (where $i,j = x,y,z$) represent six independent components (polarization modes)
5. In GR $R_{\mu\nu} = 0$ hence there are only two independent polarization modes

GW polarization modes in metric theories of gravity



- General Relativity
- Scalar-Tensor theory
  [*e.g.* Will 1993]
- Massive Gravity theory
  [*e.g.* de Rham 2014]
- Scalar-Vector-Tensor
  [Sagi 2010]

GW polarization modes in metric theories of gravity



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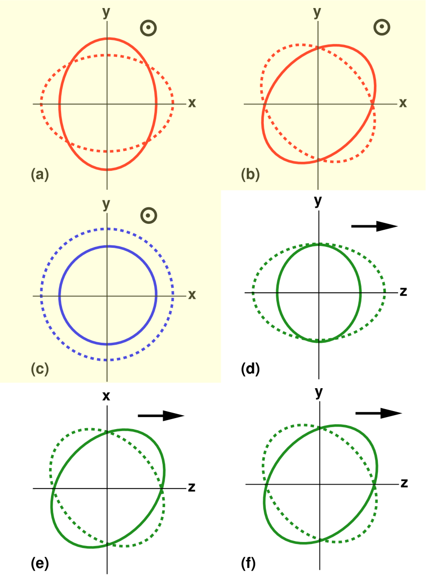

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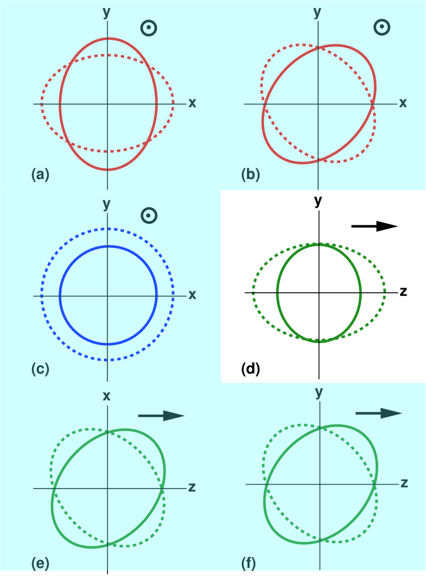

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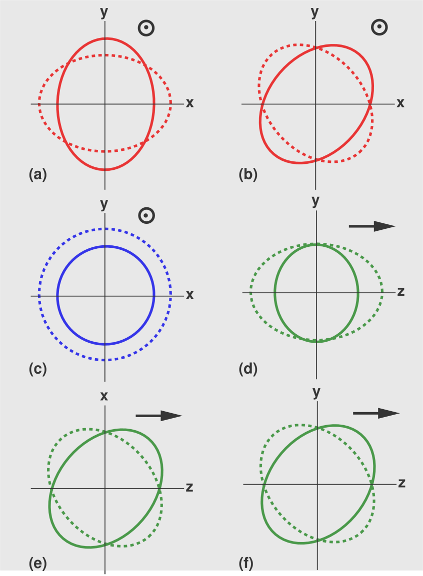

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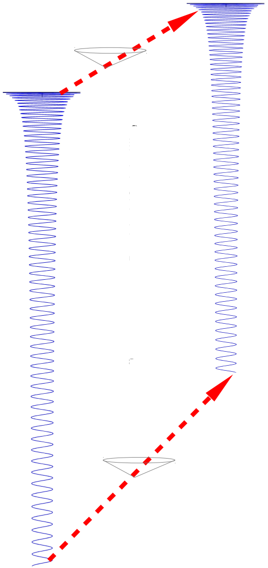
Bounding the mass of the graviton [Will 1998]



- Dispersion relation for a massive graviton

$$\boxed{\frac{v_g^2}{c^2} = 1 - \frac{m_g^2 c^4}{E_g^2}} \quad \text{with} \quad E_g = \hbar\omega_g$$

- The frequency of GW sweeps from low to high frequency during the inspiral and the speed of GW varies from lower to higher (close to c) speed at the end
- The constraint is [LIGO/Virgo 2016]

$$\boxed{m_g \lesssim 10^{-22}\,\text{eV}} \quad \Leftrightarrow \quad \lambda_g \gtrsim 0.02\ \text{ly}$$

Nonlinear ghost-free massive (bi-)gravity

[de Rham, Gabadadze & Tolley 2011; Hassan & Rosen 2012]

- The theory (called dRGT) is defined non-perturbatively as

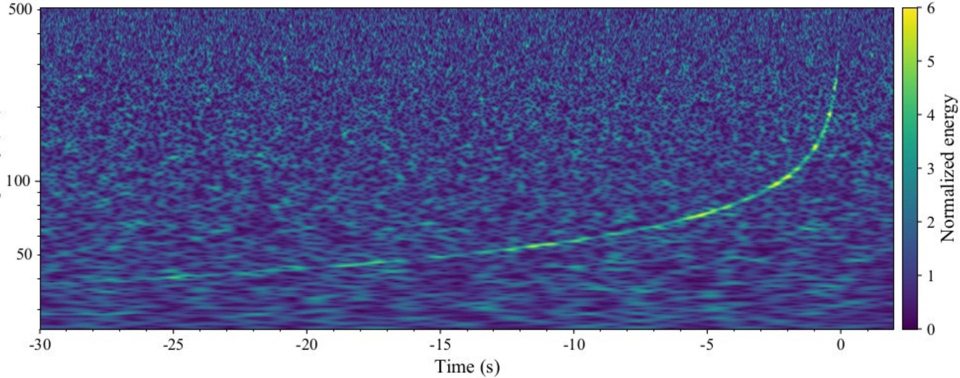
$$S = \int \mathrm{d}^4 x \left[\frac{M_g^2}{2} \sqrt{-g} R_g + \overbrace{m^2 \sqrt{-g} \sum_{n=0}^{4} \alpha_n e_n(X)}^{\text{ghost-free interaction mass term}} + \frac{M_f^2}{2} \sqrt{-f} R_f \right]$$

where the interaction between the two metrics is defined from the elementary symmetric polynomials $e_n(X)$ of the square root matrix $X = \sqrt{g^{-1} f}$

- Such massive gravity theories modify GR at large cosmological distances and are motivated by the problem of the cosmological constant

$$\boxed{\Lambda \sim \frac{m_\mathrm{g}^2 c^2}{\hbar^2}} \quad \Rightarrow \quad m_\mathrm{g} \sim 3.7\,10^{-33}\,\mathrm{eV}$$

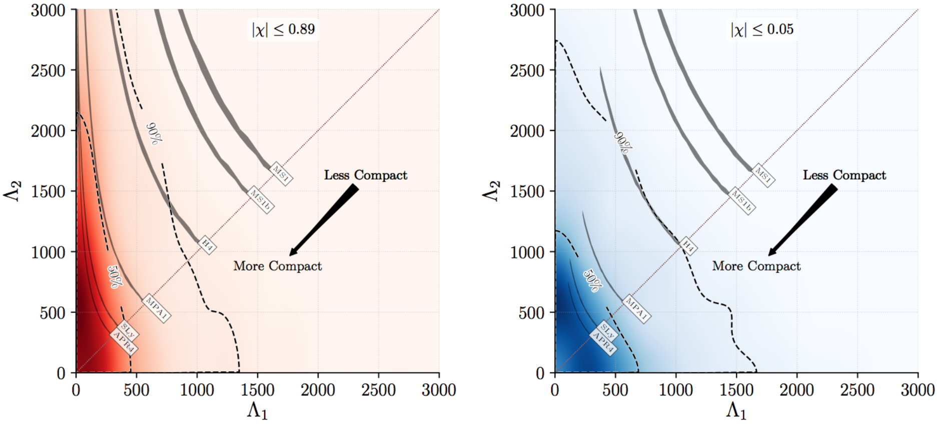
Binary neutron star event GW170817 [LIGO/Virgo 2017]



- The signal is observed during $\sim 100\,\mathrm{s}$ and ~ 3000 cycles and is the loudest gravitational-wave signal yet observed with a combined SNR of 32.4
- The chirp mass is accurately measured to $\mathcal{M} = \mu^{3/5} M^{2/5} = 1.98\,M_\odot$
- The distance is measured from the gravitational signal as $D = 40$ Mpc

Constraining the neutron star equation of state

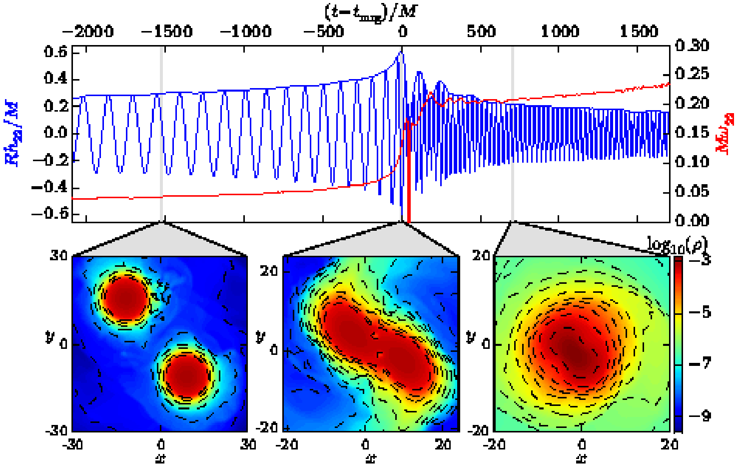
[LIGO/Virgo 2017]



$$\Lambda = \frac{2}{3} k_2 \left(\frac{c^2 a}{G m} \right)^5$$

Post-merger waveform of neutron star binaries

[Shibata *et al.*; Rezzolla *et al.* 1990-2010s]



Gravitational echoes [Cardoso *et al.* 2016]

- Suppose that the object formed by the merger of two BHs contains a material surface between the horizon at $2M$ and the photon sphere at $3M$,

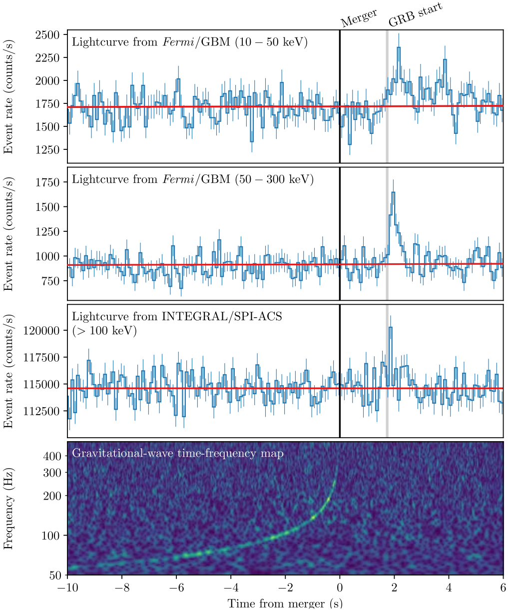
$$R = 2M + \varepsilon \quad \text{with} \quad \varepsilon \ll M$$

- For instance ε could be the Planck length ℓ_P and be related to quantum effects occuring at the horizon scale
- In that case the ringdown radiation at frequency $f_\text{ringdown} \sim \pi/M$ should be followed by an echo at high frequency $f_\text{echo} \sim \pi/\tau_\text{echo}$ with

$$\tau_\text{echo} \sim M \left| \ln\left(\frac{\varepsilon}{M}\right) \right|$$

- Echos may also reveal the existence of ultra compact exotic objects with radius between the Buchdahl limit $R_\text{B} = \frac{9}{4}M$ and the photon sphere $3M$

Multi-messenger astronomy with GWs [LIGO/Virgo 2017]



Test of the strong equivalence principle

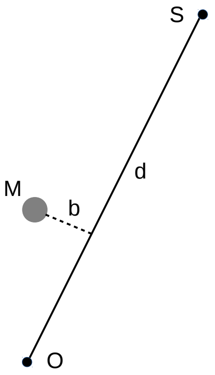
[see e.g. Desai & Kahya 2016]

1. The test involves the cumulative Shapiro time delay due to the gravitational potential of the dark matter distribution
2. The violation of the equivalence principle is quantified by a PPN like parameter γ_a depending on the type of radiation $a = \text{GW}, \text{EM}$. For a spherical mass distribution

$$\Delta t^a_{\text{Shapiro}} = (1+\gamma_a)\frac{GM}{c^3}\ln\left(\frac{d}{b}\right)$$

3. The main contributions come from the galaxy NGC4993 and our own Galaxy with mass $M_{\text{MW}} = 5.6\,10^{11}\,M_\odot$. Assuming an isothermal density profile for dark matter this yields about 400 days delay in GR
4. The observed difference in arrival time $\Delta t = 1.7\,\text{s}$ yields

$$\boxed{|\gamma_{\text{GW}} - \gamma_{\text{EM}}| \lesssim 10^{-7}}$$



Dark energy after GW170817 [Bettoni *et al.* 2017; Creminelli & Vernizzi 2017]

1. The observed time delay between GW170817 and the GRB constrains

$$|c_{\mathrm{g}} - c_{\mathrm{em}}| \lesssim 10^{-15} c$$

2. Consider models of dark energy and modified gravity characterized by a single scalar degree of freedom (Horndeski theory)

$$\begin{aligned} L &= G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R \\ &\quad - 2G_{4,X}(\phi, X)\left(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right) + G_5(\phi, X)E^{\mu\nu}\phi_{\mu\nu} \\ &\quad + \frac{1}{3}G_{5,X}(\phi, X)\left(\Box\phi^3 - 3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi^{\mu\nu}\phi_{\mu\rho}\phi^{\rho}_{\nu}\right) \end{aligned}$$

3. Imposing the speed of GWs to be one ($i.e.$ $c_{\mathrm{g}} \equiv c_{\mathrm{T}} = 1$) drastically reduces the space of allowed theories

$$\boxed{L_{c_T=1} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + B_4(\phi)R}$$

4. In beyond-Horndeski theory [Gleyzes, Langlois, Piazza & Vernizzi 2015] another type of term is also allowed

Dark matter emulators after GW170817

[Boran, Desai, Kahya & Woodard 2018; Wang *et al.* 2018]

1. Gravitational waves couple to the Einstein-frame metric $g_{\mu\nu}$ produced by GR without dark matter
2. Ordinary matter couples to the Jordan-frame metric $\tilde{g}_{\mu\nu}$ which is a disformally transformed metric that would be produced by GR with dark matter

$$\boxed{\tilde{g}_{\mu\nu} = e^{2\phi}(g_{\mu\nu} + U_\mu U_\nu) - e^{-2\phi} U_\mu U_\nu}$$

3. GW170817 excludes dark matter emulators such as
   - TeVeS [Bekenstein 2004]
   - MOG [Moffat 2006]
4. However other dark-matter motivated theories are still viable
   - Bi-MOND [Milgrom 2009]
   - Nonlocal MOND [Deffayet, Esposito-Farèse & Woodard 2011]
   - Khronon theories [Blanchet & Marsat 2011; Sanders 2011]
   - Dipolar dark matter [Blanchet & Le Tiec 2008; Blanchet & Heisenberg 2017]