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WAVE CHAOS IN RAPIDLY ROTATING STARS

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Abstract. Acoustic waves inside centrifugally distorted stars are studied in the high frequency limit. In this regime, their propagation is governed by Hamiltonian equations. While the dynamical system is integrable for non-rotating spherically symmetric stars, the centrifugal distortion of rotating stars destroys integrability. We show the appearance of a KAM type chaos in polytropic rotating stars by computing Poincaré sections for increased values of the rotation rate. We also show how the structure of the phase space can serve as a guide for the study of p-modes in rapidly rotating stars.

1 Introduction

New methods to compute acoustic modes in centrifugally distorted stars are expected to allow progress in the seismology of rapidly rotating stars (Lignières et al. 2006, Reese et al. 2006). To complement the eigenmode approach, we consider here the small wavelength limit (as in geometric optics) where the acoustic wave is described locally by a plane wave, the propagation of which is governed by Hamiltonian equations. As in the field of quantum chaos or more generally wave chaos, the phase space structure of the Hamiltonian system can then be used to characterize the eigenmodes and the organization of the eigenfrequency spectrum. First results on the acoustic ray dynamics are presented here.

2 Ray dynamics

The WKB approximation consists in looking for wave-like solutions of the form $\Psi = A\exp(i\Lambda\Phi)$ to the linear equations governing the evolution of small amplitude perturbations. The functions A and Φ are supposed to vary on a length scale comparable with the scale of variation of the background state while Λ^{-1} is a small parameter characterizing the ratio between the wave length scale and that of the background. At the lowest order in Λ^{-1}, the evolution of the wave number $\vec{k} = \Lambda\vec{\nabla}\Phi$ at a point $\vec{x}$ moving with the group velocity is governed by the following Hamiltonian equations (Gough 1987) :

$$\frac{dk_i}{dt} = -\frac{\partial\omega}{\partial x_i}, \qquad \frac{dx_i}{dt} = +\frac{\partial\omega}{\partial k_i}$$

where the Hamiltonian $\omega = \sqrt{c_s^2 k^2 + \omega_c^2}$ is the wave frequency, c_s the sound speed and ω_c the cut-off frequency. Trajectories have been computed for uniformly rotating polytropic stellar models of indices $N = 3$ and $N = 1.5$. We restrict ourselves to trajectories contained in meridional planes which correspond to axisymmetric modes. We also neglected the cut-off frequency and determined the rebound conditions by a local analytical treatment at the $c_s = 0$ surface.

The four dimensional phase space has been studied using a Poincaré section defined as the surface located at a small constant distance from the stellar surface. Trajectories cut the Poincaré section at a colatitude θ and with a certain wave vector entirely determined by ψ the angle between the wave vector and the normal to the surface. The following Poincaré section displays the value of $\cos\theta$ and $\sin\psi$ for 150 different trajectories computed over a large number of time steps for the $N = 3$ case.

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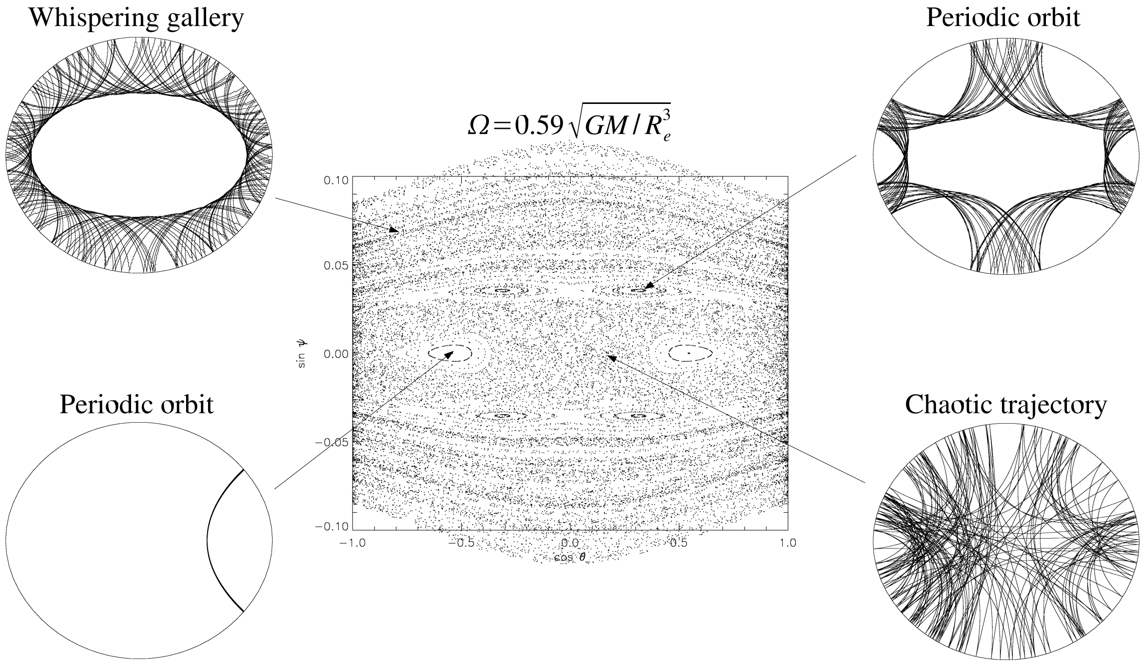

3 Transition to chaos

The Poincaré sections show how the initially integrable Hamiltonian system ($\Omega = 0$) becomes chaotic as rotation breaks spherical symmetry. The transition to chaos is of the KAM (Kolmogorov - Arnold - Moser) type as most of the tori of the integrable system survive for small rotation rates. These tori correspond to roughly horizontal lines (straight lines for $\Omega = 0$) running from $\cos\theta = 0$ to $\cos\theta = 1$ on the Poincaré section. They are associated with whispering gallery trajectories, an example being shown above for $\Omega = 0.59\sqrt{GM/R_e^3}$. In between these lines, island chains develop around stable periodic orbits while chaotic regions develop around unstable periodic orbits. Two examples of trajectories inside the islands are shown above as well as an example of a chaotic trajectory. As rotation increases, KAM tori are progressively destroyed and the chaotic regions become larger. Nonetheless, the two islands associated with trajectories concentrated at middle latitudes survive the deformation of the star while approaching the equatorial region.

4 Conclusion

The centrifugal distortion of rapidly rotating stars destroys the integrability of acoustic ray dynamics. In the domain corresponding to low degree number modes, the phase space structure is characterized by a large chaotic region and two stable islands associated with trajectories concentrated in mid-latitudes. New high frequency p-modes clearly associated with such trajectories have been found. We now plan to use quantum chaos techniques to systematically associate modes to region in the phase space. This would allow us to classify the modes and better understand their properties.

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References

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