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SPACE CLOCK MISSIONS AND ORBITOGRAPHY REQUIREMENTS

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Abstract. In this paper, we study in detail the requirements on orbitography compatible with operation of next generation space clocks at the required uncertainty, and based on a completely relativistic model. We show that the required accuracy goal can be reached with relatively modest constraints on the orbitography of the space clock, much less stringent than expected from "naive" estimates using the example of the ACES mission. Our results are generic to all space clocks and represent a significant step towards the generalised use of next generation space clocks in fundamental physics, geodesy, and time/frequency metrology.

1 Introduction

Over the last decade of the 20^{th} century and the first few years of the 21^{st}, the uncertainty of atomic clocks has decreased by over two orders of magnitude, passing from the low 10^{-14} to the below 10^{-16}, in relative frequency (Bize et al. 2005; Heavner et al. 2005; Oskay et al. 2006). Space applications in fundamental physics, geodesy, time/frequency metrology, navigation, etc... are among the most promising for this new generation of clocks. However this requires a precise knowledge of the clock precision. As an example, simple order of magnitude estimates of the relativistic gravitational frequency shift show that an 1m error on the position of the clocks leads to an error of $\sim 10^{-16}$ in the determination of their frequency difference.

In this paper, we study in more detail the requirements on orbitography compatible with operation of next generation space clocks at the required uncertainty, and based on a completely relativistic model. We use the example of the ACES (Atomic Clock Ensemble in Space) mission, an ESA-CNES project to be installed onboard the ISS (International Space Station) in 2014. For such a space station, one meter precision in position is difficult to obtain. We briefly describe the ACES mission and the relativistic model used for the clocks and the time transfer, followed by a description of a model for the orbitography error to be expected onboard the ISS, based on measurements using an in situ GPS receiver. Our main results are the calculation of the effect of that error on the determination of the relativistic frequency shift of the clocks and on the time transfer (MWL) for the ACES mission, where we show that the mission objectives can be achieved with relatively modest orbitography and, more generally, calculate the overall requirements on orbitography for the ACES mission. Our results are generic to all space clocks (not limited to the ACES mission) and represent a significant step towards the generalised use of next generation space clocks in fundamental physics, geodesy, and time/frequency metrology.

2 The ACES mission

The ACES project led by the CNES and the ESA aims at setting up on the ISS several highly stable clocks a cold atom clock PHARAO developed by CNES and a hydrogen maser (SHM developed by Neuchtel observatory) together with a microwave communication link. The objectives of the mission are to reach a time stability for ground to space comparisons of 0.3 ps at one ISS pass and 7 ps at one day. For our purposes we express the above requirements for the MWL in a simplified form by the temporal Allan deviation ($\sigma_x(\tau)$) :

$$\sigma_x(\tau) = 5.2 \cdot 10^{-12} \cdot \tau^{-\frac{1}{2}} \tag{2.1}$$

for one single passage (for integration times τ lower than 300 s) and by

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$$\sigma_x(\tau) = 1,7 \cdot 10^{-14} \cdot \tau^{\frac{1}{2}} \tag{2.2}$$

for longer integration times (for integration times τ greater than 300 s).

The time transfer is performed using a micro-wave two-way system, called Micro-Wave Link (MWL). Each reception or emission event is tagged by its coordinate time t_i (see Fig. 1). For instance, the f_1 frequency signal is emitted by the ground station at the coordinate time t_1 and received by the space station at t_2. The third frequency is added to measure the TEC in the ionosphere which allows the correction of the ionospheric delay. The combination of the observables coming from both signals f_1 and f_2 allows to evaluate the desynchronisation. By derivation, we obtain the frequency difference between the ground and the space clocks. We define $\overrightarrow{x_g}$, $\overrightarrow{v_g}$ and $\overrightarrow{a_g}$ respectively as the position, the velocity and the acceleration of the ground station, and $\overrightarrow{x_s}$, $\overrightarrow{v_s}$ and $\overrightarrow{a_s}$ respectively as the position, the velocity and the acceleration of the space station, in a non-rotating geocentric coordinate system (eg. GCRS as defined by the IAU).

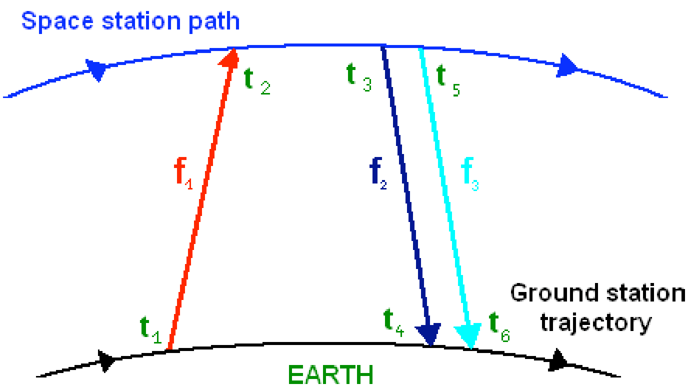

Fig. 1. MWL principle

In a general relativistic framework each clock produces its own local proper time, in our case τ_g and τ_s for the ground and space clocks respectively. The relation between the proper time and the coordinate time t is given to sufficient accuracy by Blanchet et al. (2001):

$$\frac{d\tau}{dt} = 1 - \left(\frac{U(t, \overrightarrow{x})}{c^2} + \frac{v^2(t)}{2c^2}\right) + O(c^{-4}) \tag{2.3}$$

where U is the Newtonian potential at the coordinate time t and the position $\overrightarrow{x}$ and v the velocity of the studied clock.

It is the derivative with respect to the coordinate time t of the relation which has to be studied for applications such as test of the gravitational redshift or geodesy:

$$\frac{d\tau^g}{dt} - \frac{d\tau^s}{dt} = \frac{1}{c^2} \cdot \left(U(t, \overrightarrow{x_s}) - U(t, \overrightarrow{x_g}) + \frac{v_s^2(t)}{2} - \frac{v_g^2(t)}{2}\right) + O(c^{-4}). \tag{2.4}$$

The ACES mission aims at obtaining the variation of the desynchronisation between ground and space clocks with time, that is to say, the function $\tau_g(t) - \tau_s(t)$. It is evaluated by combining the measurements performed on the ground and onboard the space station and a precise calculation of the signal propagation times. Then the expression of desynchronisation reads:

$$\begin{aligned} \tau^g(t_a) - \tau^s(t_a) = {} & \frac{1}{2}(\Delta\tau^s\left(\tau^s(t_2)\right) - \Delta\tau^g\left(\tau^g(t_4)\right) + T_{12} - T_{34} \\ & - \int_{t_1}^{t_2} (\frac{U(t, \overrightarrow{x_g})}{c^2} + \frac{v_g^2(t)}{2c^2})dt + \int_{t_3}^{t_4} (\frac{U(t, \overrightarrow{x_s})}{c^2} + \frac{v_s^2(t)}{2c^2})dt) \end{aligned} \tag{2.5}$$

where $t_a = (t_2 + t_4)/2$, and where $\tau^s(\tau^s(t_2))$ and $\tau^g(\tau^g(t_4))$ are the observables respectively from the ground and onboard the satellite at the coordinate times t_2 and t_4 and T_{ij} is defined as $T_{ij} = t_j - t_i$.

The integral terms result from proper time to coordinate time transformations. They are small corrections of order 10^{-12} s to the desynchronisation $\tau^g(t_a) - \tau^s(t_a)$. In (2.5) the difference $T_{12} - T_{34}$ needs to be calculated from the knowledge of satellite and ground positions and velocities (orbitography). The difference $T_{12} - T_{34}$ of upward and downward signals at f_1 and f_2 allows to eliminate to first order delaying and restraining factors such as range, troposphere or Shapiro effects (Shapiro 1964). Due to the asymmetry of the paths, that cancellation is not perfect, and there are some terms left which depend on orbitography as well as on the coordinate time interval T_{23} elapsed between reception and emission at the phase centre of the MWL antenna onboard the ISS. The aim of this work is to estimate with a simple orbital model, which levels of accuracy on orbitography and calibration of internal delays (knowledge of T_{23}) are required to reach the expected performances.

For that purpose only the leading terms are required ie.

$$T_{12} - T_{34} = 2\frac{\overrightarrow{D}(t_4).\overrightarrow{v_g}(t_4)}{c^2} + \frac{\overrightarrow{D}(t_4).\overrightarrow{\Delta v}(t_4)}{c \cdot D(t_4)} T_{23} + O(\frac{1}{c^3}). \tag{2.6}$$

where $\overrightarrow{D}(t) = \overrightarrow{x_s}(t) - \overrightarrow{x_g}(t)$, $D(t) = ||\overrightarrow{D}(t)||$ and $\Delta\overrightarrow{v}(t) = \overrightarrow{v_g}(t) - \overrightarrow{v_s}(t)$. In summary, a reliable orbitography is required for two main reasons. On one hand to calculate precisely the corrections in equation (2.6). On the other hand, to evaluate correctly the terms on the right hand side of equation (2.4) ie. the second order Doppler and gravitational redshifts. In addition, we also need a precise knowledge of the time interval T_{23}, (ie. of the onboard internal delays) in order to be able to calculate the corresponding terms in (2.6) with sufficient accuracy. Equation (2.6) together with equation (2.4) for the gravitational redshift, is sufficient to derive the maximum allowed uncertainties on orbitography and internal delays in order to stay below the limits given by (2.1) and (2.2).

3 Orbit Determination Error Model

Now we investigate the effects of trajectory knowledge on the accuracy and the stability of the time transfer (see equation (2.6)) and on the estimation of the relativistic correction of the clock (see equation (2.4)).

For the time transfer (2.6) we have to consider the position of the antenna phase center, but it is the clock reference point trajectory which is important for equation (2.4). The trajectories of the antenna phase center and of the clock reference point are obtained the trajectory of ISS center of mass (orbit determination error), and a geometrical offset (vector ISS center of mass - reference point) which depends on the attitude and on the geometry of the ISS (on position errors in the ISS frame).

The differences between true and computed (using orbit determination and attitude) trajectories of the ISS center of mass have very specific structures. For example an eccentricity error gives no long term effects, but periodic errors can be important and the radial, along track and velocity errors are correlated. This means that position and velocities are not independent, and if possible, this has to be taken into account for a performance evaluation.

For weak eccentricity orbits, errors of the ISS center of mass position are given by the Hill model (or the Clohessy-Wiltshire model) which is an expansion of uncertainties with respect to a reference circular orbit. This error model takes into account the correlation between all orbitographic parameters. It gives their expressions in the local orbital frame ($\overrightarrow{R}$, $\overrightarrow{T}$ et $\overrightarrow{N}$) defined with $\overrightarrow{R}$ the unity vector between the Earth's center and the space station, $\overrightarrow{N}$ orthogonal to $\overrightarrow{R}$ and the inertial velocity and where $\overrightarrow{T}$ is orthogonal to $\overrightarrow{R}$ and $\overrightarrow{N}$. Then the position uncertainties along radial, tangential and normal axis are given as follows :

$$\begin{aligned} \text{radial axis} : \delta R(t) &= \frac{1}{2} X \cdot \cos(\omega t + \varphi_R) + c_R \\ \text{tangential axis} : \delta T(t) &= -X \cdot \sin(\omega t + \varphi_R) - \frac{3}{2}\omega \cdot c_R \cdot t + d_R \\ \text{normal axis} : \delta N(t) &= Y \cdot \cos(\omega t + \varphi_N) \end{aligned} \tag{3.1}$$

where X, Y, c_R and d_R are amplitude coefficients, and where ω is the orbital pulsation. Surface accelerations errors are not taken into account, because this model corresponds to a local error of the adjusted trajectory and adapted for short arc length.

An ISS orbitography restitution using an onboard GPS receiver gives the orders of magnitude of these coefficients. It typically leads to X and Y lower than ten meters, that is to say, a ten meters bound on the normal axis and a five meters bound on the which gives ten meters oscillations on the tangential axis. Moreover bias and linear terms (c_R and d_R) depend on the arc length. Basically the longer the observation duration is, the smaller these coefficient become. The major feature of the error model is to take into account the error correlations in the orbital plane. For instance, a positive radial bias leads to a negative error on the tangential velocity: the satellite is delayed with respect to the reference orbit.

We note $(\overrightarrow{X}(t),\ \overrightarrow{V}(t))$ the true trajectory of the reference point and $(\overrightarrow{X}'(t),\ \overrightarrow{V}'(t))$ the computed trajectory of the reference point. According to the studied case, the reference point is either the antenna phase center or the clock reference point. We also define $(\overrightarrow{X}_o(t),\ \overrightarrow{V}_o(t))$ the true trajectory and the true velocity of the center of mass of the station. These five trajectories are expressed in non-rotating geocentrical frame (GCRS, Geocentric Celestial Reference System) (Soffel et al. 2003).

Now we have to express the effects of station trajectory and time calibration uncertainties on time transfer and on gravitational redshift.

On one hand, according to the equation (2.5), the error in the time transfer is related with the uncertainties of $T_{12} - T_{34}$ entering in the desynchronisation . It can be obtained from the simplified equation (2.6) and is then dependant of the ground and space stations trajectories knowledge, of the value of T_{23} and of the uncertainty on this parameter. As said before, a precise knowledge of the time interval T_{23} is related to the internal delays calibration. If we suppose the uncertainty on ground station position is negligible with respect to the ISS position errors, the knowledge of the vector $\overrightarrow{D}$ is related to the uncertainty on the position on the space station reference point which is the antenna phase center. Then we have $\delta\overrightarrow{D} = \overrightarrow{X_aX_a'}$. The error on $T_{12} - T_{34}$ can be written as:

$$\delta\left(T_{12} - T_{34}\right) = 2\frac{\overrightarrow{X_aX_a'}.\overrightarrow{v_g}}{c^2} + \frac{\overrightarrow{D}.\overrightarrow{\Delta v}}{c \cdot D}\delta T_{23} + \left(\frac{\overrightarrow{X_aX_a'}.\overrightarrow{\Delta v}}{c \cdot D} - \frac{\overrightarrow{D}}{c \cdot D}.\frac{d\overrightarrow{X_aX_a'}}{dt} - \frac{\overrightarrow{D}.\overrightarrow{\Delta v}}{c \cdot D}\frac{\|\overrightarrow{X_aX_a'}\|}{D}\right)T_{23} \tag{3.2}$$

On the other hand, the computation of the clock relativistic correction along a trajectory is defined by equation (3.1). It depends on the position and the velocity of the reference point, in this case the clock. We need to express the error on the reference point frequency - that is to say the frequency difference between the true clock position and the computed clock position - in order to compare its Modified Allan stability with the specifications. The gravitational potential can be evaluated on a given trajectory with a sufficient precision (Wolf & Petit 1995) using gravity models (eg. GRIM5 or EGM96).The error on the frequency shift at the clock position is given by:

$$\delta(\frac{d\tau}{dt})_{\overrightarrow{X_c}} = (\frac{d\tau}{dt})_{\overrightarrow{X_c}} - (\frac{d\tau}{dt})_{\overrightarrow{X_c'}} = -\frac{1}{c^2}\left(U(t,\overrightarrow{X_c}) - U(t,\overrightarrow{X_c'}) + \frac{V_c^2 - V_c'^2}{2}\right) \tag{3.3}$$

Using the fact that $\overrightarrow{X}_o$ is the solution of the differential equation

$$\frac{d^2\overrightarrow{X}_o}{dt^2} = \overrightarrow{\Gamma}_P + \overrightarrow{\Gamma}_S \tag{3.4}$$

where $\overrightarrow{\Gamma}_P$ is the acceleration due to gravitational potential and $\overrightarrow{\Gamma}_S$ is the acceleration due to other effects (e.g. surface forces like air drag and radiation pressure), in (3.3), we obtain:

$$(\frac{d\tau}{dt})_{\overrightarrow{X}} - (\frac{d\tau}{dt})_{\overrightarrow{X}_o} = -\frac{1}{c^2}\left[\frac{d\overrightarrow{V}_o}{dt}.\overrightarrow{X_oX} + \overrightarrow{V_o}.\frac{d\overrightarrow{X_oX}}{dt} + \frac{1}{2}(\frac{d\overrightarrow{X_oX}}{dt})^2 - \overrightarrow{\Gamma}_S.\overrightarrow{X_oX}\right] \tag{3.5}$$

In order to simplify the equation (3.5), we evaluate the order of magnitude of the different contributors appearing in this equation. To investigate the importance of the term $\frac{\overrightarrow{\Gamma}_S.\overrightarrow{X_cX_c'}}{c^2}$, the drag has been modelled along a reference orbit of the ISS, for various altitudes. A period with important solar activity has been chosen in order to evaluate the worst case. To estimate its effect on formula (3.5), the acceleration has been multiplied by a 10 meter bias or a 10 meter sinusoidal function at orbital period, corresponding to possible attitude and orbit errors effects of the ISS. The corresponding Allan variance of fractional frequency stays below 10^{-21}, which is totally negligible here. This term has also no effect on the frequency accuracy. The effect of other surface accelerations like solar radiation pressure is also negligible.

The residual term of the second order Doppler shift $\frac{1}{2c^2}\left[(\frac{d\overrightarrow{X_oX_c}}{dt})^2 - (\frac{d\overrightarrow{X_oX_c'}}{dt})^2\right]$ must be computed with the GCRS trajectories for attitude or orbital errors. The corresponding Allan variance is bounded by $10^{-16} \cdot \tau^{-1}$ for a 10 meter sinusoidal function at orbital period. This effect is also negligible. However because of the power of two, this term does not have a zero mean. The magnitude of this frequency bias can be evaluated as equal to $1.7 \cdot 10^{-21}$. As far as the ACES mission is concerned, this effect can also be neglected. So only the component of the clock error parallel to the velocity of the ISS plays a role. This can be understood considering for example a purely positive radial component. In this case we underestimate the gravitational potential but overestimate the velocity (at constant ω), so the two cancel.

4 Numerical Results On Clock Comparison

In this section we use the previously described error model to calculate realistic requirements for time transfer and gravitational frequency shift. For this purpose, we consider an ephemeris of ISS. First we study the time transfer between the International Space Station and a ground station based in Toulouse, France. For this purpose, we first consider the error equation (3.2) on the time transfer. We choose the signs of the independent parameters (X, T_{23} and δT_{23}) so as to maximize the resulting temporal Allan deviation. The calculated deviations have to be compared with the MWL's specifications (2.1). Assuming we have no error on T_{23} (ie. δT_{23} =0 s), for all values of factor X (or Y) of equation (3.1), it is possible to determine the maximum value of the time interval T_{23} for which the temporal Allan deviation remains under the specifications. With numerous values of X, we calculate a bound which marks out two different areas: the allowed uncertainties area in which each couple (X, T_{23}) gives a deviation staying under the specifications, and the prohibited area.

Figure (2) shows that, the smaller the time interval is, the less precise the space station position knowledge is required. This result provides a way to combine upwards and downwards signals in order to allow the maximum uncertainty on space station position to comply with the specifications. The most favourable situation to combine upwards and downwards signals is when the reception at the antenna phase center of the space station corresponds to the emission at the same place ie. $t_2 = t_3$. This way of combining signals is named the "Λ configuration". To work with parameter X in the asymptotic area requires T_{23} to be under 10^{-6} s.

Then if we plot the maximum value of δT_{23} for all values of the factor X, there will appear two asymptotic values we cannot cross if we want to stay under the specifications. Basically a compromise between the knowledge of the space station trajectory and the precision of the internal delays calibration must be achieved owing to the maximization of the Allan deviation. We will evaluate the maximum allowed errors on these two parameters if no other errors are present.

First we search for the asymptotic value of factor X which complies with the specifications for all phases values (φ_R, φ_N) when we have no error on T_{23}. The asymptotic value for orbitography is obtained for X = 2150 which corresponds to a 2.1 km error on the normal and tangential axis, and to an 1 km error on the radial axis (see Fig. (2)).

The asymptotic value of the time calibration does not depend on orbitographic uncertainties. So it is independent of the phases (φ_R, φ_N). We find that δT_{23} must stay under $5.2 \cdot 10^{-8}$ s (see Fig. (3)). Moreover the accuracy on the time transfer is not a problem because all the terms of the equation (3.2) have zero mean for one passage.

The requirements for several passes have also been investigated. In this case, the calculated deviations have to be compared to the specifications given by (2.2). The results of this work showed that the requirements on orbitography and time calibration are less stringent for several passes than for a single pass. Therefore if specifications are respected for a single pass, specifications for longer integration times are most likely also respected as the requirements on the uncertainty on T_{23} are less stringent in that case.

Now we evaluate requirements on orbitography considering the gravitational frequency shift. We search for the maximum value of X to comply with the long term specifications (2.2). First term of (3.5) is evaluated

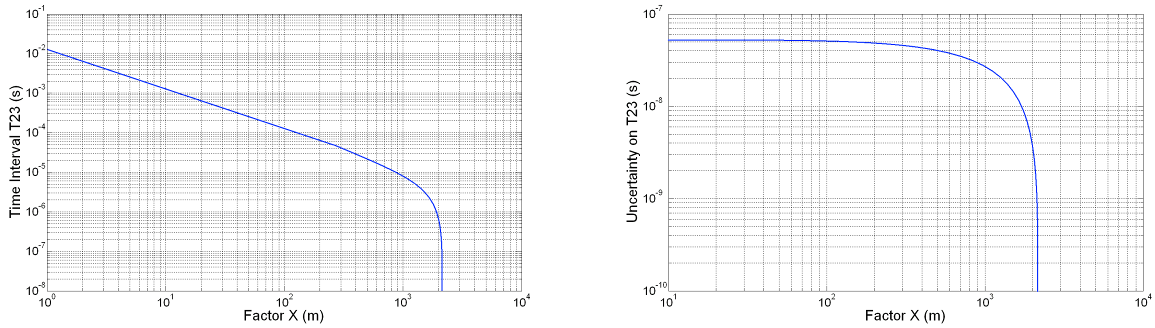

Fig. 2. left: maximum allowed value of T_{23} as function of the scale factor X to comply with the specifications, assuming $\delta T_{23} = 0$ / right: maximum allowed value of δT_{23} as function of the scale factor X to comply with the specifications, assuming $T_{23} = 0$

with the error model (3.1), and its Allan deviation is calculated for different values of X. For integration time greater than one thousand seconds, these Allan deviations are independent of the phases φ_R and φ_N.

Figure (3) shows that, if the factor X is equal to 16 m ie. if we have an eight meter error on the radial axis and sixteen meter error on the tangential axis, then we comply with the specifications. The requirement on the factor Y is two orders of magnitude less stringent than on the factor X. This is mainly due to the projection of the position error along the ISS center of mass velocity (see equation (3.5)).

The accuracy requirement of ACES is 10^{-16} in relative frequency over ten days. From the integral of equation (3.5) this implies that the position error ($\overrightarrow{X_c X_c'}$ in (3.5)) cumulated over ten days needs to remain below one kilometer including for example the linear term along the tangential axis in (3.1)). This has to be compared with the typical ten meter error on the tangential axis. Therefore, the accuracy requirement should not raise any difficulty.

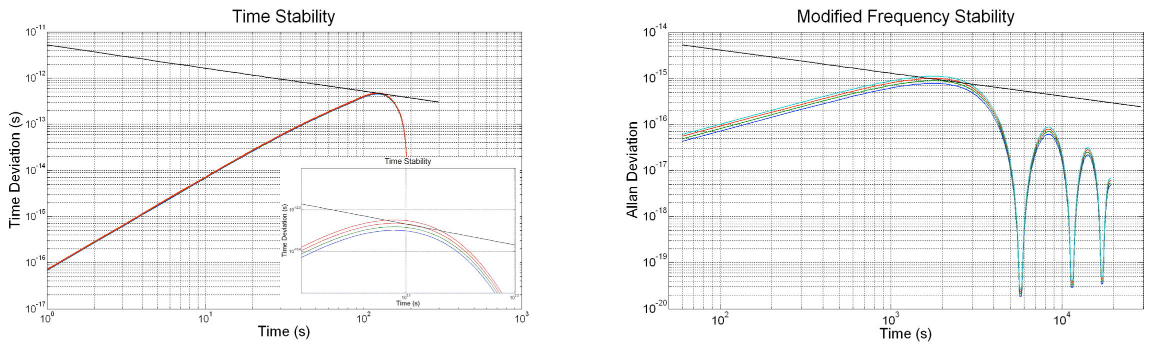

Fig. 3. left: temporal Allan deviations for X = 0, $T_{23} = 0$ s and $\delta T_{23} = [51, 52, 53, 54]$ ns /right: modified Allan deviations of the redshift error for X=14,16,18,20 m

5 Conclusions

The formulations of time transfer and clock relativistic effects errors were described and applied on standard errors corresponding to orbit determination and geometry. We also evaluated the order of magnitude of the main effects. Investigating the requirements for the ACES mission provides a way to combine upwards and downwards signals (the 'Λ configuration'). Thus the requirements on orbitography and time calibration have been identified to reach stability specifications. They are summarized in table (1).

The periodic error on the radial axis and on the tangential axis must stay respectively below eight and sixteen meters. The uncertainty on the normal axis is two orders of magnitude less stringent. Moreover the error on internal delays (δT_{23}) must not exceed fifty two nanoseconds. At last the requirement on the tangential drift (below one kilometer in ten days) is easily reached in order to comply with the accuracy specification (10^{-16} in relative frequency at ten days).

In conclusion the requirements on orbitography are significantly less stringent than the initial 'naive' estimate (one meter error for 10^{-16} in relative frequency) which is mainly due to partial cancellation between the gravitational redshift and the second order Doppler effect.

Table 1. Requirements on parameters

X (m)	Y (m)	δT_{23} (ns)
16	2150	52

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