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TITAN'S FORCED ROTATION: A 3-DEGREE OF FREEDOM THEORY

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Abstract.
We present a 3-degree of freedom theory of Titan's forced rotation, as a rigid body. Such a study is possible thanks to the Cassini data of the gravitational potential of Titan. We use a semi-analytical model based on the recent analytical works of Henrard & Schwanen (2004), on a numerical integration and on an identification of the arguments as an integer combination of the fundamental ones. Titan's orbital motion is modelized with TASS theory (Vienne and Duriez, 1995). We find that the equilibrium obliquity is nearly zero, and that the fundamental periods of the free librations around the equilibrium are respectively 2.1, 167 and 306 years. Moreover, we enlight the influence of a 703-years period present in Titan's inclination, on its obliquity.

1 Introduction

As most of the major natural satellites, Titan is locked in a 1 : 1 spin-orbit resonance. More precisely, its least axis of inertia is always pointed to Saturn. Moreover, the node of Titan on its orbital plane precesses at the same rate as its orbital ascending node. These two commensurabilities constitute an equilibrium state known as Cassini state.

Thanks to Cassini-Huyghens mission, and especially to Cassini fly-bys, we know some coefficients of its gravity field, more particularly J_2 and C_{22} (Tortora et al. 2006), what allows a reliable study of its rotation.

Recently, analytical theories of the rotation of celestial bodies have been developed, by D'Hoedt & Lemaître (2004) for Mercury, locked in a 3 : 2 spin-orbit resonance, and by Henrard & Schwanen (2004) for the general case of the synchronous bodies, that was later developed for the particular cases of Io (2005a) and Europa (2005b, 2005c) by Henrard. In this study, we start from the work of Henrard & Schwanen (2004) and compare it to an original numerical study.

2 Analytical study

The analytical study starts from the following Hamiltonian:

$$\begin{aligned}\mathcal{H} = \frac{nP^2}{2} + \frac{nP^2}{8}\left[4-\xi_q^2-\eta_q^2\right]\left[\frac{\gamma_1+\gamma_2}{1-\gamma_1-\gamma_2}\xi_q^2+\frac{\gamma_1-\gamma_2}{1-\gamma_1+\gamma_2}\eta_q^2\right] \\ +n\left(\frac{d_0}{d}\right)^3\left(1+\delta_s\left(\frac{d_0}{d}\right)^2\right)\left[\delta_1(x^2+y^2)+\delta_2(x^2-y^2)\right]\end{aligned} \tag{2.1}$$

with the following action-angle canonical variables:

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$$p = l + g + h \qquad P = \frac{G}{nC}$$
$$r = -h \qquad R = \frac{G-H}{nC} = P(1-\cos K) = 2P\sin^2\frac{K}{2}$$
$$\xi_q = \sqrt{\frac{2Q}{nC}}\sin q \qquad \eta_q = \sqrt{\frac{2Q}{nC}}\cos q$$

where n is Titan's mean orbital motion, $q = -l$, and $Q = G - L = G(1-\cos J) = 2G\sin^2\frac{J}{2}$. The coefficients of the Hamiltonian are defined as follows:

$$\gamma_1 = J_2\frac{MR^2}{C} \qquad \delta_1 = -\frac{3}{2}\left(\frac{n^*}{n}\right)^2\gamma_1$$
$$\gamma_2 = 2C_{22}\frac{MR^2}{C} \qquad \delta_2 = -\frac{3}{2}\left(\frac{n^*}{n}\right)^2\gamma_2$$
$$\delta_s = \frac{5}{2}J_{2♄}\left(\frac{R_♄}{d_0}\right)^2$$

where $J_{2♄}$ is Saturn's J_2, d the distance Titan-Saturn, d_0 the mean value of d, n^* the mean motion associated to d_0, and the angles can be seen on the figure above, reproduced from (Henrard 2005a).

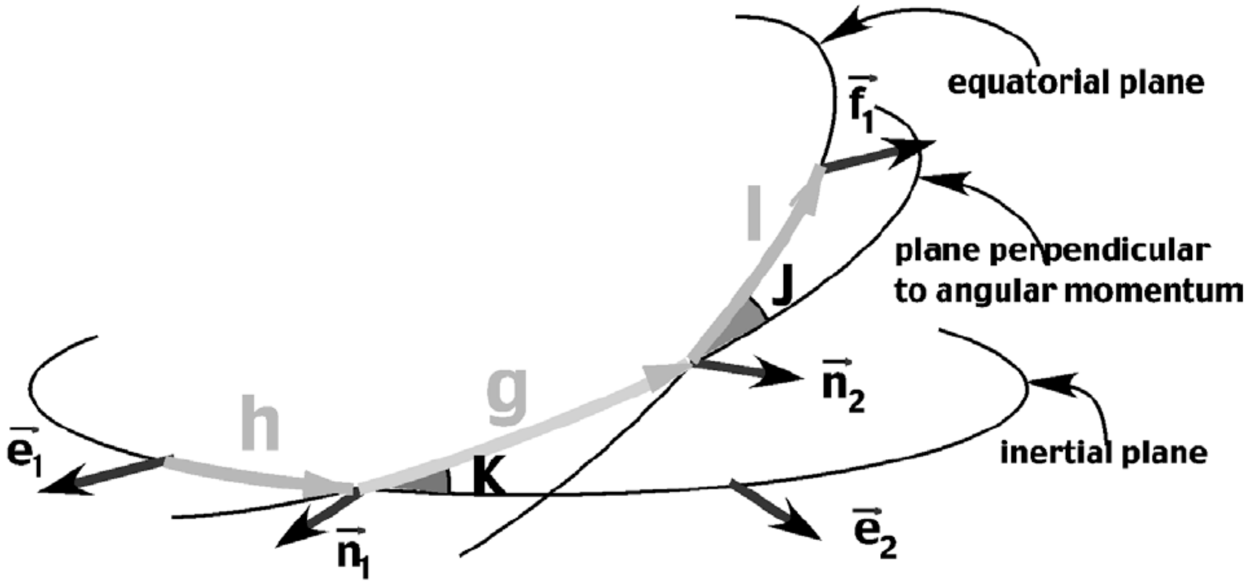

Fig. 1. The angles (reproduced from Henrard 2005a).

x and y are the first two coordinates of the center of Saturn in the frame $(\vec{f_1}, \vec{f_2}, \vec{f_3})$ bound to Titan. Then we use the model of Henrard & Schwanen (2004) to obtain the equilibrium (K^*, P^*) and the fundamental periods of the 3 proper librations around it: T_u, T_v and T_w. At the strict Cassini state, $\sigma = p - \lambda + \pi = 0$, $\rho = r + \Omega = 0$, $\xi_q = 0$ and $\eta_q = 0$ (ie the wobble angle J is null), where λ and Ω are respectively Titan's mean longitude and ascending node in an inertial frame (bound to Saturn). Moreover, Titan's orbit around Saturn is considered as circular with a constant inclination and a constant precession of the nodes.

Unfortunately, the quantity $\frac{C}{MR^2}$ remains unknown. $C = 0.4MR^2$ corresponds to a spherical undifferentiated body, its real value should in fact be lower. We use both $\frac{C}{MR^2} = 0.31$ and $\frac{C}{MR^2} = 0.35$. For J_2 and C_{22}, we use the values given by (Tortora et al. 2006), ie $J_2 = 3.15 \times 10^{-5}$ and $C_{22} = 1.1235 \times 10^{-5}$.

Tab.1 gives the values we obtained using the analytical model.

3 Numerical study

We perform numerical computations starting from the Hamiltonian (2.1), but with complete ephemeris for Titan, instead of assuming its orbit as circular. We use TASS1.6 theory. Our initial conditions are taken very

Table 1. Equilibrium and fundamental periods of libration analytically obtained.

	$\frac{C}{MR^2} = 0.31$	$\frac{C}{MR^2} = 0.35$
K^*	1.1205×10^{-2} rad	1.1205×10^{-2} rad
R^*	6.277×10^{-5}	6.277×10^{-5}
P^*	1	1
T_u	2.0945 y	2.2258 y
T_v	167.366 y	188.9876 y
T_w	306.624 y	346.2365 y

near the equilibrium estimated in the analytical study, but not at the exact equilibrium. The reason is that we want to identify the free solution so as to numerically determine the fundamental periods associated. Then, a frequency analysis is being performed to identify the solutions.

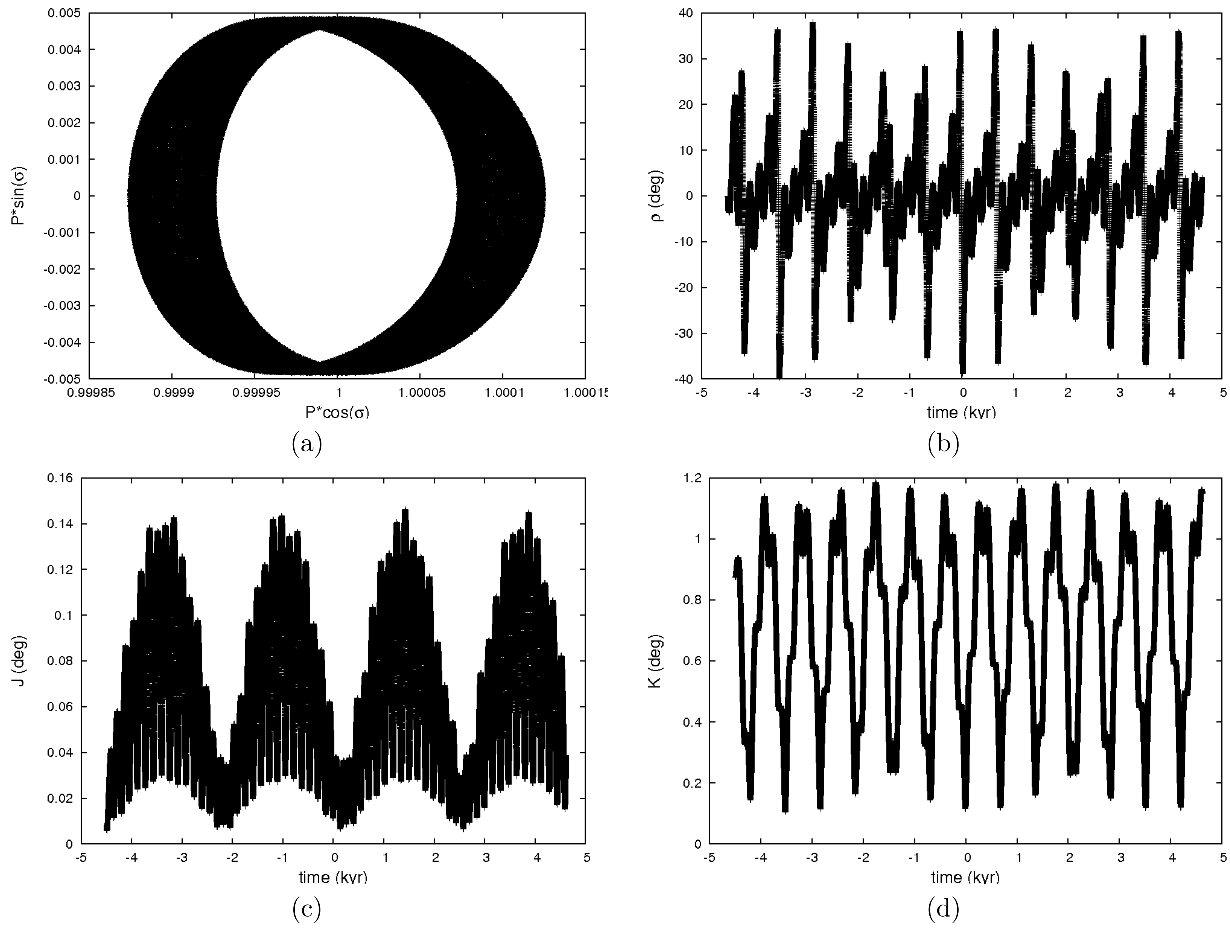

Fig. 2. Numerical simulation of Titan's obliquity over 9000 years, with $\frac{C}{MR^2} = 0.31$.

Fig. 2 and 3 give some solutions of the numerical integration with the two values of $\frac{C}{MR^2}$, while Tab.2 and 3 give examples of solutions for $\frac{C}{MR^2} = 0.31$. The importance of the period of 703 years in the solution of ρ is striking. It is due to a forcing effect by the proper mode of TASS1.6 Φ_6, present in Titan's inclination. Moreover, we can see the high values taken by the wobble J for $\frac{C}{MR^2} = 0.35$ (Fig.3c). We can infer that we are in a quasi-resonant state that could force the free libration.

Tab. 4 gives a comparison between the numerical and the analytical results for $\frac{C}{MR^2} = 0.31$. We see a very good agreement for the fundamental periods of libration. On the contrary, a significant difference exists on the mean obliquity K^*. This might be due to a too simple analytical model.

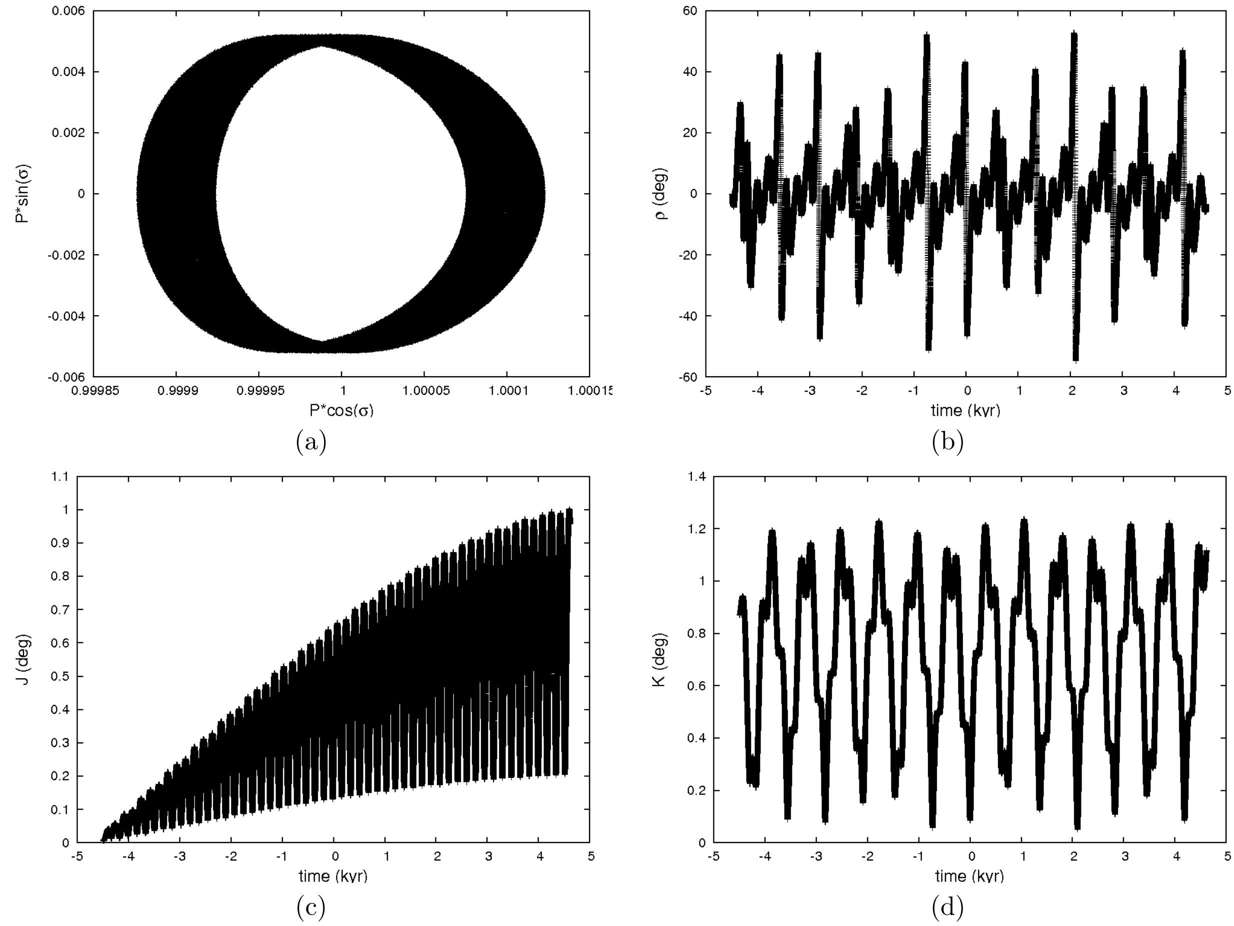

Fig. 3. Numerical simulation of Titan's obliquity over 9000 years, with $\frac{C}{MR^2} = 0.35$.

Table 2. Quasiperiodic decomposition of ρ, for $\frac{C}{MR^2} = 0.31$. The series are in sine.

	amplitude	phase (°)	period (y)	identification
1	0.18089837	175.64	167.49723	ϕ_v (free)
2	0.15667339	-170.90	703.52446	$-\Phi_6$ (forced)
3	0.11829380	-175.18	135.28724	$\phi_v - \Phi_6$ (free)
4	0.09023900	-161.92	351.75789	$2\Phi_6$ (forced)
5	0.07735641	-166.02	113.46712	$\phi_v - 2\Phi_6$ (free)
6	0.05226443	-152.60	234.50407	$-3\Phi_6$ (forced)
7	0.05058400	-156.89	97.70793	$\phi_v - 3\Phi_6$ (free)
8	0.03311443	-147.66	85.79329	$\phi_v - 4\Phi_6$ (free)
9	0.03060799	-143.30	175.88361	$-4\Phi_6$ (forced)

4 Conclusion

This work gives a first study of Titan's rotation, where Titan is seen as a rigid body. We obtain a quasiperiodic decomposition of the forced solution, that can be splitted from the free solution, in which Titan's obliquity plays an overwhelming role. Moreover, we find a good agreement between the free librations around the equilibrium, analytically and numerically evaluated. However, we have a slight difference in the equilibrium obliquity. Finally, we cannot exclude a resonance between the proper mode Φ_6 and Titan's wobble.

The next fly-bys of Cassini spacecraft should give us more information on Titan's gravitational field, and so we should be able to perform a more accurate study on its rotation, that could include direct perturbations on

Table 3. Quasiperiodic decomposition of K, for $\frac{C}{MR^2} = 0.31$. The series are in cosine.

	amplitude $\times 10^2$	phase ($^\circ$)	period (y)	identification
1	1.25481164	8.68×10^{-10}	-2.65×10^{13}	constant
2	0.68465799	-170.92	703.51272	$-\Phi_6$ (forced)
3	0.17842225	175.02	167.49146	ϕ_v (free)
4	0.10246867	-161.88	351.76856	$-2\Phi_6$ (forced)
5	0.07264971	-15.67	219.80041	$\phi_v + \Phi_6$ (free)

Table 4. Comparison between our analytical and numerical results, for $\frac{C}{MR^2} = 0.31$.

	analytical	numerical	difference
K^* (rad)	1.1204859×10^{-2}	$1.25481164 \times 10^{-2}$	12%
T_u (y)	2.094508	2.09773	0.15%
T_v (y)	167.36642	167.49723	0.08%
T_w (y)	306.62399	306.33602	0.09%

the other Saturnian satellites. These perturbations are known to be small (see for instance Henrard 2005c) and should be negligible compared to the uncertainties we have on Titan's gravitational parameters. After that, the next step is to consider Titan as a multilayer non-rigid body and to study the consequences of its internal dissipation on the rotation.

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