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C. Charbonnel, F. Combes and R. Samadi (eds)

RELATIVISTIC ASPECTS OF THE GAIA MISSION

Le Poncin-Lafitte, C.[1]

Abstract. Given the extreme accuracy reached in future global space astrometry, a mission such that GAIA will need a global relativistic modeling of observations. Outlining the importance of having a consistent relativistic approach all the way through the data analysis, we present also why GAIA observations will lead, in return, to an improvement of some General Relativity tests much beyond the current level.

1 Introduction

In 2000 the International Astronomical Union has adopted a general relativistic framework for modeling high-accuracy astronomical observations. Two fundamentals systems have been fixed: the Barycentric Celestial Reference System (BCRS) and the Geocentric Celestial Reference System (GCRS). The BCRS has been defined in such a way that it is covering the Solar System and observed sources. It is mainly very useful for the modeling of the dynamics of the solar system bodies and/or probe as well as the description of light propagation between light sources and an observer. The coordinate time of the BCRS is called the Barycentric Coordinate Time (TCB). It is however possible to use a scaled TCB for practical purpose, the so-called Dynamical Barycentric Time (TDB). The GCRS has been constructed in order that all gravitational fields generated by other bodies are seen as tidal potentials. The coordinate time of the GCRS is called Geocentric Coordinate Time (TCG). A scaled version of TCG exists and is called Terrestrial Time (TT) which is directly related to the International Atomic Time (TAI). The CGRS is suitable for the modeling of all physical processes in the immediate vicinity of the Earth. In addition, a local reference system, GCRS-like, can be constructed for any massless bodies, *i. e.* an observing satellite, and it is convenient to model any physical processes in the local vicinity of the satellite (Klioner 2004). By using this set of three relativistic reference systems, it is possible to draw a consistent relativistic scheme for the modeling of all kind of GAIA observations, as illustrated in Fig. 1.

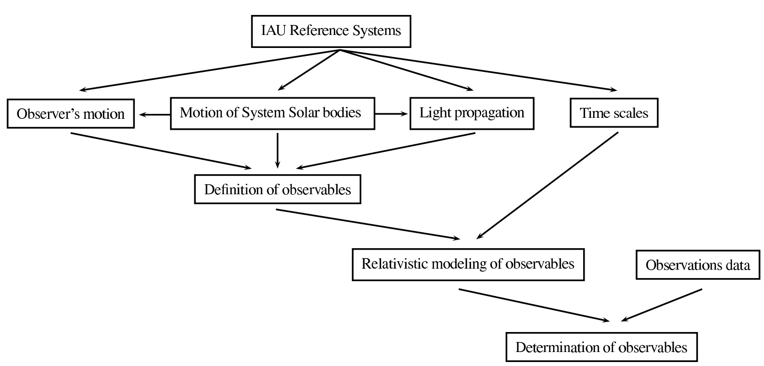

Fig. 1. Principles of relativistic modeling of astronomical observations.

[1] Observatoire de Paris, SYRTE CNRS/UMR 8630, 61 Avenue de l'Observatoire, F75014 Paris, France

2 modeling of the motion of the satellite/ Solar System bodies

It is well known that the equation of motion are ordinary differential equations of second order which can be solved numerically. The principal relativistic effects in the mass-monopole approximation for the gravitating bodies are the so-called Einstein-Infeld-Hoffmann equations of motion which can be written as follow

$$
\begin{aligned}
a^i_E \;=\; & -\sum_{B\neq E} G\,M_B\,\frac{r^i_{EB}}{r^3_{EB}} + \frac{G}{c^2}\sum_{B\neq E} M_B\,\frac{r^i_{EB}}{r^3_{EB}}\left\{\sum_{C\neq B}\frac{G\,M_C}{r_{BC}} + 4\sum_{C\neq E}\frac{G\,M_C}{r_{EC}}\right. \\
& \left. +\frac{3}{2}\frac{\left(r^j_{EB}\dot{x}^j_B\right)^2}{r^2_{EB}} - \frac{1}{2}\sum_{C\neq E,B} G\,M_C\,\frac{r^j_{EB}\,r^j_{BC}}{r^3_{BC}} - 2\,\dot{x}^j_B\,\dot{x}^j_B - \dot{x}^j_E\,\dot{x}^j_E + 4\,\dot{x}^j_E\,\dot{x}^j_B\right\} \\
& +\frac{1}{c^2}\sum_{B\neq E} G\,M_B\,\frac{r^j_{EB}}{r^3_{EB}}\left\{4\,\dot{x}^j_E - 3\,\dot{x}^j_B\right\}(\dot{x}^i_E - \dot{x}^i_B) - \frac{1}{c^2}\frac{7}{2}\sum_{B\neq E}\frac{G\,M_B}{r_{EB}}\sum_{C\neq E,B} G\,M_C\,\frac{r^i_{BC}}{r^3_{BC}} + \mathcal{O}(c^{-4}),
\end{aligned}
\tag{2.1}
$$

where capital latin subscripts B, C and E enumerate massive bodies, M_B is the mass of body B, $\mathbf{r}_{EB} = \mathbf{x}_E - \mathbf{x}_B$, a dot signifying time derivative with respect to TCB.

Of course, on the first hand, modern planetary ephemerides use Eq. (2.1) as a basic one and they are usually distributed in TDB time scale. On the other hand, one of the most controversial question is relativistic time scales and their relations. Indeed, one consequence of relativity is that the relation between one moment of time from one reference system to another can be constructed if and only if the spatial positions of the event are specified. A particular case is the transformation from TCG to TCB at the geocenter, which reads

$$
\frac{dTCG}{dTCB} = 1 + \frac{1}{c^2}\alpha(TCB) + \frac{1}{c^4}\beta(TCB) + \mathcal{O}\left(\frac{1}{c^5}\right), \tag{2.2}
$$

with

$$
\alpha(TCB) = -\frac{1}{2}v^2_E - \sum_A \frac{GM_A}{r_{GA}}, \tag{2.3}
$$

$$
\begin{aligned}
\beta(TCB) = & -\frac{1}{8}v^4_E + \frac{1}{2}\left(\sum_A \frac{GM_A}{r_{EA}}\right)^2 + \sum_A\left(\frac{GM_A}{r_{EA}}\sum_{B\neq A}\frac{GM_B}{r_{AB}}\right) \\
& + \sum_A \frac{GM_A}{r_{EA}}\left[4\mathbf{v}_A.\mathbf{v}_E - \frac{3}{2}v^2_E - 2v^2_A + \frac{1}{2}\mathbf{a}_A.\mathbf{r}_{EA} + \frac{1}{2}\left(\frac{\mathbf{v}_A.\mathbf{r}_{EA}}{r_{EA}}\right)^2\right],
\end{aligned}
\tag{2.4}
$$

where capital latin subscripts A, B and C enumerate massive bodies, E corresponds to the Earth, $\mathbf{v}_E = \dot{\mathbf{x}}_E$ and $\mathbf{a}_E = \ddot{\mathbf{x}}_E$.

Let us also stress that TDB has been redefined by the last IAU general assembly as a fixed linear transformation of TCB. It means that each planetary ephemeride, usually distributed in TDB, "realizes" the transformation (2.2). Since the official time scale chosen to process the GAIA data is TCB, It will be then natural to have an access to the transformations $TT \rightarrow TDB$ and back from the ephemeride used in the processing itself. Taking into account that it is not very complicated to do a Chebyshev polynomials representation of the numerical integration of Eq. (2.2), one can expect that in the future, GAIA mission is a good opportunity to lead the ephemeride providers to construct consistent relativistic four-dimensional planetary ephemerides.

3 modeling of positional observations

First of all, the equation of light propagation relative to the BCRS should be derived and solved. It can be seen that five vectors are needed (see Fig. 2): $\mathbf{s}$ is the unit observed direction, $\mathbf{n}$ is the unit vector tangential to the light ray at the moment of observation, σ is the unit vector tangential to the light ray at $t = -\infty$, $\mathbf{k}$ is the unit coordinate vector from the source to the observer and $\mathbf{l}$ is the unit vector from the Solar System barycenter to the light source. The modeling (GREM, Klioner 2003) consists then in a sequence of transformations between these vectors:

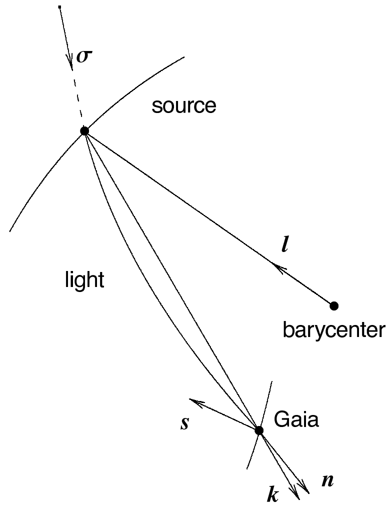

Fig. 2. Vectors used in the modeling of light propagation.

a) aberration: this step converts the observed direction to the source $\mathbf{s}$ into the coordinate velocity of the light ray $\mathbf{n}$ at the event of observation,

b) light deflection for source at past infinity: the vector $\mathbf{n}$ is converted into σ,

c) light deflection for finite sources: this step converts σ into the coordinate direction $\mathbf{k}$ going from the source to the observer,

d) parallax: $\mathbf{k}$ is converted into $\mathbf{l}$ going from the Solar System barycenter to the source,

e) proper motion: this step gives a description of the time dependence of $\mathbf{l}$ caused by the motion of the source with respect to the Solar System barycenter.

The most complicated part of the modeling lies on the description of light deflection. To reach the microarcsecond accuracy, it is needed to take into account the effects of monopole field of major celestial bodies as well as the quadrupole field of giant planets and gravitomagnetic fields due to the translational motion of deflecting bodies.

4 Synchronization of the onboard clock

Another important aspect is the conversion of BCRS time intervals $dTCB$ into the corresponding GAIA proper time intervals dTG. The general form of this transformation is similar to Eq. (2.2), but has to be calculated along the worldline of GAIA. It reads

$$\frac{dTG}{dTCB} = 1 + \frac{1}{c^2}\alpha'(TCB) + \frac{1}{c^4}\beta'(TCB) + \mathcal{O}\left(\frac{1}{c^5}\right) , \tag{4.1}$$

with

$$\alpha'(TCB) = -\frac{1}{2}v_G^2 - \sum_A \frac{GM_A}{r_{GA}} , \tag{4.2}$$

$$\begin{aligned}\beta'(TCB) = & -\frac{1}{8}v_G^4 + \frac{1}{2}\left(\sum_A \frac{GM_A}{r_{GA}}\right)^2 + \sum_A \left(\frac{GM_A}{r_{GA}} \sum_{B\neq A} \frac{GM_B}{r_{AB}}\right) \\ & + \sum_A \frac{GM_A}{r_{GA}} \left[4\mathbf{v}_A.\mathbf{v}_G - \frac{3}{2}v_G^2 - 2v_A^2 + \frac{1}{2}\mathbf{a}_A.\mathbf{r}_{GA} + \frac{1}{2}\left(\frac{\mathbf{v}_A.\mathbf{r}_{GA}}{r_{GA}}\right)^2\right] ,\end{aligned} \tag{4.3}$$

where all quantities, indexed with capital latin subscript G, refer to the satellite. GAIA will be observable from Earth ground stations several hours peer day. During all visibility periods, the clock of GAIA will be synchronize with the ground. Essentially, GAIA onboard clock will generate some time packet OBT_k, which differs from the "ideal" proper time TG because technical errors of the clock, and this information will be send to the ground station which will produce a time tag in Universal Coordinate Time UTC_k. The whole story is to be able to give a relation between all pairs $(OBT_K\,; UTC_K)$ and the corresponding relativistic pairs (TG_K, TCB_k) One possible approach is illustrated in Fig. 3 where numerous time scales and relativistic transformations are involved.

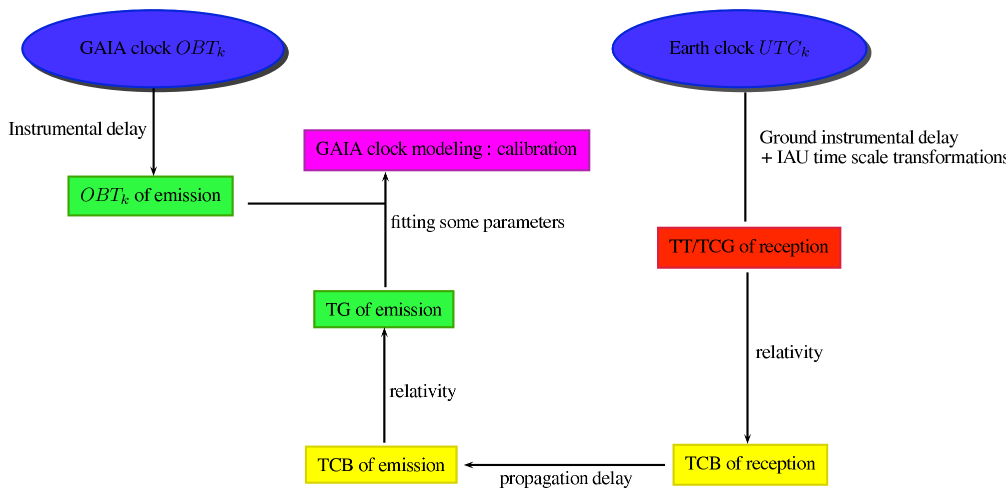

Fig. 3. Relativitistic scheme of onboard clock synchronization

5 Testing General Relativity?

Because the standard reduction modeling of GAIA is deeply based on General Relativity basic principles, GAIA data can be used to test many aspects of relativity itself. It is difficult to describe here all possible tests, so let us only give the main contributions of GAIA to fundamental physics:

- the PPN parameter γ will be measured with an accuracy between 10^{-6} and 5×10^{-7} in a wide range of angular distances from the Sun which constitutes a complete test of light deflection in Solar System,
- the PPN parameter β will be measured with an accuracy close to 10^{-4} from the observations of asteroïds (Hestroffer et al. 2007),
- local gravitational light deflection due to giant planet will be measured: the monopole deflection, the deflection due to translational motion of the planets and the deflection due to the quadrupole field of Jupiter (GAREX experiment of Crosta & Mignard 2006, Le Poncin-Lafitte & Teyssandier 2008).

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